## Machine Learning Assisted Mesh Adaptation for Geophysical Fluid Dynamics

## Siyi Li\*, Eleda Johnson\*, Joseph G. Wallwork <sup>§</sup>, Stephan C. Kramer<sup>†</sup> and Matthew D. Piggott<sup>†</sup>

\*,<sup>†</sup> Department of Earth Science and Engineering, Imperial College London, SW7 2AZ, UK e-mail: siyi.li20@imperial.ac.uk, web page: http://amcg.ese.ic.ac.uk/

<sup>§</sup> Met Office, FitzRoy Road, Exeter EX1 3PB, UK

## ABSTRACT

Numerical simulations play a central role in understanding the impact and risks of pressing global engineering problems, such as the scale-up challenges of energy generation from complex, non-linear renewable sources including wind and tidal. Effectively discretizing over multiple spatial scales, as inherent in such geophysical fluid dynamics problems, can come at a high computational cost when targeting a reasonable level of accuracy for meaningful results. Mesh adaptation can improve the accuracy of numerical simulations by modifying the discretized structure. Guiding the mesh adaptation process with a goal-based approach can focus the discrete resolution distribution where it most directly contributes to improving the accuracy of the renewable energy problem being addressed. In addition to mesh adaptation, identifying opportunities to augment the numerical methods with machine learning workflows has potential to further reduce computational overhead by automating the process and incorporating prior knowledge.

We review work extending Wallwork et al 2022<sup>1</sup> by substituting simple surrogate CNN and GNN machine learning methods for the costly dual-weighted residual error estimation step in a goal-based mesh adaptation workflow applied to numerical simulations motivated by tidal energy applications. The steady-state tidal turbine array test case and promising results as outlined in Wallwork et al 2022<sup>1</sup> serve as a foundation for investigating faster data-driven methods to replace the highly accurate dual-weighted error estimation step. We directly use the renewable energy scale-up goal of maximizing tidal turbine array power generation as the error estimation functional driving the mesh adaptation process. We explore surrogate architectures which incorporate additional patch-based or nearest neighbour information and have a reasonable chance of generalization. The discussion is focused on trade-offs between accuracy preservation and efficiency gain for the machine learning based surrogate methods.

## REFERENCES

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