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Research Article

Improving Bus Operations through Integrated Dynamic Holding Control and Schedule Optimization

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Bus bunching can lead to unreliable bus services if not controlled properly. Passengers will suffer from the uncertainty of travel time and the excessive waiting time. Existing dynamic holding strategies to address bus bunching have two major limitations. First, existing models often rely on large slack time to ensure the validity of the underlying model. Such large slack time can significantly reduce the bus operation efficiency by increasing the overall route travel times. Second, the existing holding strategies rarely consider the impact on the schedule planning. Undesirable results such as bus overloading issues arise when the bus fleet size is limited. This paper explores analytically the relationship between the slack time and the effect of holding control. The optimal slack time determined based on the derived relationship is found to be ten times smaller than in previous models based on numerical simulation results. An optimization model is developed with passenger-orient objective function in terms of travel cost and constraints such as fleet size limit, layover time at terminals, and other schedule planning factors. The optimal choice of control stops, control parameters, and slack time can be achieved by solving the optimization. The proposed model is validated with a case study established based on field data collected from Chengdu, China. The numerical simulation uses the field passenger demand, bus average travel time, travel time variance of road segments, and signal timings. Results show that the proposed model significantly reduce passengers average travel time compared with existing methods.

1. Introduction

Maintaining the reliability and efficiency of bus services is critical to ensure their competitiveness and popularity over the use of private cars. The ideal situation is that all buses can keep up to the schedule and travel evenly along the bus route at a predetermined headway (headway is defined as the time gap between two consecutive buses). In reality, however, due to the existence of various perturbations, it is difficult for buses to keep their headways to the designated value. Eventually, some buses bunch up together and start to travel in pairs. This phenomenon is referred to as bus bunching. Fundamentally, the bus bunching phenomenon is triggered by external randomness and amplified by bus system's volatile nature. In a stochastic traffic environment, buses travel in different speeds and cater to different number of waiting passengers because of external randomness (e.g., changing traffic conditions, different driving behaviors, and fluctuating passenger demands), which causes buses' headways to deviate from the target value. Subsequently, due to the volatility of bus system, any headway deviation tends to increase over time and result in bunching phenomenon at last. This volatile nature is firstly explained by Newell [1]: due to the fact the number of waiting passengers at a stop is proportional to bus headway, a delayed bus with larger headway has to dwell at the stop for a longer time to collect passengers, resulting in a further vehicle delay. Meanwhile, the following bus has fewer passenger to serve and travels relatively faster. Consequently, the process of headway deviation will continuously accelerate under positive feedback loop and lead to bunching phenomenon at last.

Bus bunching is unexpected because it increases passengers' waiting time, leads to crowding conditions in slow buses, and makes passengers' travel time unpredictable. There are mainly two reasons for the increment of waiting time. First, as illustrated by Welding [2], the average waiting time of all passengers increases with the increment of headway deviation, which is named as ordinary waiting time (OWT) in this paper. Second, due to bus capacity limit, some slow buses may become overloaded at some stops, and parts of the waiting passengers will suffer a much longer waiting time to wait for the next bus. We call it as extra waiting time (EWT). Various bus control strategies are proposed to mitigate bus bunching, such as transit signal priority, bus speed regulation, stop-skipping strategy, and etc., of which, holding control strategy is supposed to be one of the most basic and effective one.

Traditionally, transit agencies insert slack times into bus schedules, and require all buses to depart on time at control stops (Barnett [3], Rossetti and Turitto [4]). However, the slack times required are usually very huge and will significantly reduce the efficiency of bus operation. Nowadays, with the rapid development of automated vehicle location (AVL), automated passenger counter (APC) and smartcard payment in transit system, agencies are able to monitor passenger demands, traffic conditions, and passenger loads more efficiently, and then control bus holdings adaptively in real time (Hanaoka [5]). Basically, adaptive holding control strategies can be categorized as optimization-based holding control and dynamic holding control.

Optimization-based holding control use control laws to minimize certain cost functions. Existing methods differ in different components of the optimization model.

(i) Scenarios: Sánchez-Martínez et al. [6] consider dynamic running times and passenger demands. Wu et al. [7] propose optimal holding control considering vehicle overtaking and distributed passenger boarding behavior. Sánchez-Martínez [8] et al. formulate event-driven holding control to adapt buses to the expected changes in running times and demand during events.

(ii) Objective functions: Barnett [3] and Zhao et al. [9] consider the passenger waiting time. Hall et al. [10], Delgado et al. [11] take transfer time into account when measuring the waiting time. Asgharzadeh & Shafahi [12] minimize passengers' total travel time which consist of passengers' ordinary waiting time, extra waiting time when the bus is at full capacity, and the waiting time of onboard passengers.

(iii) Constraints: Delgado et al. [13] consider the vehicle capacity constraint; while Eberlein et al. [14] consider the safe headway constraint.

(iv) Solution algorithms: Cortés et al. [15] propose generic algorithms to expedite model solving process. Koffman [16] developed a simulation model to evaluate the control effect of various control methods and solved the optimal control scheme accordingly. Chen et al. [17] present a multiagent reinforcement-learning framework to optimization bus operations in real time. Yu et al. [18] develop a SVM model to predict bus travel time and dwell time and minimize passengers' waiting time with the improved holding strategy.

Optimization-based holding control may face computational issues when solving their complicated formulations for the optimal solutions. Furthermore, the agencies can only observed the performance (e.g., accuracy and reliability) after the implementation of the control strategies and may lead to unforeseeable control effect.

Dynamic holding control strategies apply negative feedback control to make bus system self-regulating to reduce headway deviation or schedule deviation. By communicating with control center, buses can continuously collect real-time data about the deviation of headway and schedule of its own and surrounding vehicles and dynamically determine their holding time to create a negative force to counteract those deviations. The advantages of dynamic holding control include the following: (1) all control parameters can be calibrated offline with archived bus AVL data and smartcard data and do not need to be calibrated in real time. (2) Each bus can dynamically generate its holding plan locally and efficiently without complex optimizations. (3) Performance measures like the schedule deviation, headway deviation, average waiting time and average in-vehicle travel time can be predicted beforehand.

Dynamic holding strategy is usually headway-based or schedule-based. Fu & Yang [19] investigate two different holding control models. The first model generates holding time based on the headway to the proceeding bus. The second model uses both preceding and following headway to determine the holding time. Daganzo [20] developed a forward-looking method to keep bus headways adhering to a predefined target headway. Holding time for buses is dynamically determined based on the deviation of their forward headways from targeted headway, where buses with smaller headways will be assigned with longer holding times, and vice versa. The method of convolution is introduced to simplify the modeling process, which is then widely used for the modeling of dynamic holding by other researchers. Daganzo & Pilachowski [21] further improved the control performance by proposing a two-way-looking control model. Bus cruising speed is continuously adjusted according to both forward headway and backward headway. Results showed that this two-way-looking method managed to maintain headway stability with faster bus travel speed than in forward-looking method. Bartholdi & Eisenstein [22] proposed a backward-looking control model that did not require predefined target headway and the information of passenger demands. This method has been proved to be effective in low demand by a field test. As pointed out by Xuan et al. [23], the above three headway-based methods can only maintain buses with stable headways, but do not ensure the adhesion to bus schedules. Xuan et al. [23] proposed a general holding control model and proved that all previous dynamic control methods are special forms of this one. In order to simplify the formulation and calibration, a one-parameter version called "simple control strategy" was developed. The "simple control strategy" can maintain both headway adherence and schedule adherence with relative smaller slack times than the previous methods. Liang et al. [24] developed a zero-slack version of two-way-looking control model. Simulation results shown that this control model further reduced passengers average travel time compared with Daganzo & Pilachowski [21]'s method. Zhang & Lo [25] propose a two-way-looking self-equalizing control method for both deterministic and stochastic running times. It is proved that the proposed control method keeps bus headway self-equalized under deterministic travel time and reduces the variance of headway to a certain value when travel time is stochastic.

Other researchers further investigate the above-mentioned dynamic holding methods to make them suitable for specific scenarios. Argote-Cabanero et al. [26] generalized Xuan et al.'s [23] "simple control strategy" into multiline systems by using both dynamic holding control and en-route driver guidance. The proposed control strategy applies to bus systems that mix with headway-based and schedule-based bus lines. Control effects are tested by both simulation and field study. Nesheli & Ceder [27] use real-time operational tactics to increase the actual occurrence of synchronized transfers and therefore reduce the magnitude and uncertainty of passengers' travel time. Estrada et al. [28] present dynamic cruising speed control methods and signal priority timing strategies to regulate bus operations. The proposed methods are based on the basic control logics of Daganzo [20]'s holding control strategy, but further consider the vehicle capacity constraints.

Studies are also focused on solving some key problems in implementing holding controls, such as the method of choosing control stops, trip time and optimal slack. Eberlein et al. [14] develop a deterministic quadratic program in rolling horizon scheme to formulate dynamic holding, and then test the holding model by simulations. Results suggest that holding buses at the first stop is of the highest efficiency, since even dispatching headways help to slow down the growth of headway variation along the route. Oort et al. [29] study how the choice of trip time, location and amount of control stops affect the reliability and efficiency of long-headway bus services. Simulation shows that a good combination of optimal trip time value and well selected control points can significantly reduce the additional travel time, where the optimal trip time usually ranges from 30-60 percentile value. In addition, bus route with two control stops achieves better control effects than that of one, but further increasing the number of control stops do not significantly improve the effect. Fu & Yang [19] develop both forward-looking and two-way-looking holding strategies. With simulation analysis, they find that usually two control stops are need to ensure better control effect, one at the terminal and the other at a high-demand stop near the middle of the route. By using $D/G/c$ queue model, Zhao et al. [30] generate a theoretical model that address the optimization problems of slack time for a schedule-based bus route with 1 bus and 1 control stop. They also present approximation algorithm for more general situations. Simulation shows that the proposed model can well describe how different value of slack time affects passengers' waiting time and delay.

Table 1 concludes recent works on the topic of holding control strategies. Recent research mainly carries out studies about holding control from three aspects: First, further improving control effect with new control logic, like forward-looking control, backward-looking control etc.; second, using holing control to tackle specific problems, like holding in multilines, holding to synchronize transfers; third, optimizing some key parameters about bus holding, like the optimization of slack time, trip time, number, and location of control stops etc. It is well known that there are tradeoffs between the reliability and efficiency of bus operation when implementing dynamic holding control. Although large slack time, multicontrol stops and strong control coefficient helps to maintain bus operation of high reliability, they may seriously reduce the system's efficiency. Recent research mainly studies the optimization of slack time, number and location control stops and other parameters by simulations or empirical analyses. In addition, rare studies consider about the planning issue (e.g., determining bus schedule and dispatching headway when the fleet size and bus capacity are limited) in their holding strategies. This paper aims to integrate holding control and schedule planning together to achieve better control effects. The contributions of this paper are as follows: First, planning factors like fleet size, bus capacity, dispatching frequency and layover time are considered, which is critical for field operations. Second, mathematical formulas are generated to describe how control strength, slack time and the choice of control stop affect the reliability and efficiency of dynamic holdings. Third, formulas are proposed to quantify passengers' ordinary waiting time, extra waiting time and in-vehicle travel time. Fourth, an optimization model is presented to solve optimal control coefficients, slack time and the number and location of control stops.

2. Notation

The frequently used parameters in this work are listed in Table 2.

3. Modeling of Dynamic Holding Control

The general control model proposed by Xuan et al. [23] is widely studied to solve bus bunching problems. This paper adopts the basic control logic of Xuan et al. [23]'s work to formulate dynamic holding control method, and further improves the control efficiency by reducing the slack time needed. First, for the convenience of discussion, Xuan et al. [23]'s method is briefly introduced as background. Then mathematical analysis is conducted to discuss the reliability of dynamic holding in heterogeneous situations. Last, the concept of Equivalent Control Parameter (ECP) is proposed to quantitatively measure the impact of small slack time on the reliability and efficiency of dynamic holding.

3.1. Background. Equations (1) ~ (3) are often used to formulate bus motions (e.g., Daganzo [20], Xuan et al. [23], etc.).

$$t_{n,s+1} = t_{n,s} + \beta_s H + d_s + c_s \tag{1}$$

$$t_{n,s} = t_{n-1,s} + H \tag{2}$$

$$a_{n,s+1} = a_{n,s} + \beta_s h_{n,s} + D_{n,s} + c_s + \gamma_{n,s+1} \tag{3}$$

Equations (1) and (2) represent the scheduled bus motion, and (3) describes real bus motion. Combining (1), (2), and (3),

Table 1: Summary of Recent Dynamic Holding Strategies.

Authors	Approach	Highlight
Fu & Yang [19]	FWL+TWL	The optimal number and location of control points, and the optimal control strength are studied with simulations.
Daganzo [20]	FWL	The method of convolution is introduced to simplify the modeling process.
Daganzo & Pilachowski [21]	TWL	A cruising speed control method is proposed with the two-way-looking control logic.
Bartholdi & Eisenstein [22]	BWL	The proposed holding strategy does not require headway or schedule information.
Xuan et al. [23]	SB	A general holding control model is generated which represents a family of different control methods.
Liang et al. [24]	TWL	A zero-slack version of holding strategy is proposed.
Zhang & Lo [25]	TWL	A self-equalizing holding strategy with two-way-looking control logic is proposed.
Argote-Cabanero et al. [26]	SB	Holding control is generalized into multi bus lines.
Nesheli & Ceder [27]	FWL	Methods like holding control, boarding-limit control, and stop-skipping control are used to synchronize transfers for multi bus lines.
Estrada et al. [28]	TWL	Cruising speed control and signal priority control methods are proposed based on two-way-looking control logic.
Eberlein et al. [14]	—	The optimal location of control stop is analyzed by simulations.
Oort et al. [29]	—	Illustrating how the choice of trip time, location and amount of control stops affect the reliability and efficiency of long-headway bus services.
Zhao et al. [30]	SB	Mathematical analysis is carried out to address the optimization problem of slack time.

Notes: ‘FWL’ stands for ‘Forward-Looking’, ‘BWL’ stands for ‘Backward-looking’, ‘TWL’ stands for ‘Two-way-looking’, and ‘SB’ stands for ‘Schedule-based’.

Table 2: Summary of primary parameters.

Parameter	Definition
n	Bus number
s	Stop number
N	The amount of buses
S	The amount of stops
$t_{n,s}$	The scheduled arrival time of bus n at stop s
$a_{n,s}$	The actual arrival time of bus n at stop s
$\varepsilon_{n,s}$	The deviation from scheduled arrival time of bus n at stop s. $\varepsilon_{n,s} = a_{n,s} - t_{n,s}$
$h_{n,s}$	The headway between bus n and bus $(n-1)$ at stop s. $h_{n,s} = a_{n,s} - a_{n-1,s}$
c_s	The average cruising time between stop s and stop $(s+1)$.
$\gamma_{n,s+1}$	The random noise in the cruising time of bus n between stop s and stop $(s+1)$
$D_{n,s}$	The holding time applied to bus n at stop s
d_s	The amount of slack time inserted in the schedule at stop s
λ_s	The passenger arriving rate at stop s
T_b	The passenger boarding rate
β_s	A dimensionless measure for the demand rate at stop s. $\beta_s = T_b\lambda_s$
H	The target headway

the deviation from scheduled arrival time can be formulated as follows:

$$\varepsilon_{n,s+1} = \varepsilon_{n,s} + \beta_s\left(\varepsilon_{n,s} - \varepsilon_{n-1,s}\right) + \gamma_{n,s+1} + \left(D_{n,s} - d_s\right) \quad (4)$$

Xuan et al. [23] generate the holding time $D_{n,s}$ as a linear function of the schedule deviation of all buses at stop s:

$$D_{n,s} = \max\left(0, d_s - \left[\left(1+\beta_s\right)\varepsilon_{n,s} - \beta_s\varepsilon_{n-1,s}\right) + \sum_i f_i\varepsilon_{n-i,s}\right) \quad (5)$$

Assuming the slack time d_s is large enough, (4) can be simplified as (6):

$$\varepsilon_{n,s+1} = \sum_i f_i\varepsilon_{n-i,s} + \gamma_{n,s+1} \quad (6)$$

With numerical studies, Xuan et al. [23] notice that the control efficiency mainly depends on coefficient f_0. Therefore, a so called "simple control strategy" is generated as (7) and (8). The relationship of schedule variance between two consecutive stops can be expressed as (9). Note that parameter f_s represents the coefficient f_0 designated to bus stop s. For ease of expression, f_s rather than $f_{0,s}$ is used.

$$D_{n,s} = \max\left(0, d_s - \left[\left(1+\beta_s\right)\varepsilon_{n,s} - \beta_s\varepsilon_{n-1,s}\right] + f_s\varepsilon_{n,s}\right) \quad (7)$$

$$\varepsilon_{n,s+1} = f_s\varepsilon_{n,s} + \gamma_{n,s+1} \quad (8)$$

$$\sigma_{\varepsilon,s+1}{}^2 = f_s{}^2\sigma_{\varepsilon,s}{}^2 + \sigma_{s+1}{}^2 \quad (9)$$

So far, slack time d_s is assumed to be a large enough number. However, a large value of slack time leads to long average dwell time at stop. Xuan et al. [23] let $d_s = 3\sigma_{D,s}$ to guarantee a 99.87% confidence level of positive holding time, where $\sigma_{D,s}$represents the deviation of holding time.

$$\sigma_{D,s}^2 = \left(1+\beta_s - f_s\right)^2\sigma_{\varepsilon,s}{}^2 + \beta_s{}^2\sigma_{\varepsilon,s}{}^2 \quad (10)$$

3.2. Reliability Analysis. Xuan et al. [23] prove the "simple control strategy" to be a reliable control method under homogeneous circumstance, where inputs like c_s, $\gamma_{n,s+1}$, β_s are identical for all bus stops and road segments. We generalize their conclusions into heterogeneous situations.

Let us expand the right-hand side of (8) iteratively as follows:

$$\begin{aligned}\varepsilon_{n,s+1} &= f_s\varepsilon_{n,s} + \gamma_{n,s+1} = f_s\left(f_{s-1}\varepsilon_{n,s-1} + \gamma_{n,s}\right) + \gamma_{n,s+1} \\ &= \left(\prod_{k=1}^{s} f_k\right)\cdot\varepsilon_{n,1} + \sum_{i=2}^{s}\prod_{j=i}^{s}\left(f_j\cdot\gamma_{n,i}\right) + \gamma_{n,s+1}\end{aligned} \quad (11)$$

Based on (11), the expected value of schedule deviation $\varepsilon_{n,s+1}$ can be formulated as $E(\varepsilon_{n,s+1}) = (\prod_{k=1}^{s} f_k)\cdot\varepsilon_{n,1}$. Its variance is as follows:

$$\begin{aligned}\operatorname{var}\left(\varepsilon_{n,s+1}\right) &= \left(\prod_{k=1}^{s} f_k{}^2\right)\cdot\operatorname{var}\left(\varepsilon_{n,1}\right) \\ &\quad + \sum_{i=2}^{s}\left(\prod_{j=i}^{s} f_j{}^2\right)\cdot\operatorname{var}\left(\gamma_{n,i}\right) + \operatorname{var}\left(\gamma_{n,s+1}\right)\end{aligned} \quad (12)$$

Where $\varepsilon_{n,1}$ is the schedule deviation that can be measured as the difference between bus 's actual dispatching time and scheduled dispatching time at the first stop. Furthermore, $\operatorname{var}(\varepsilon_{n,1}) = 0$ since $\varepsilon_{n,1}$ is a deterministic value. Equation (12) can be rewritten as follows:

$$\operatorname{var}\left(\varepsilon_{n,s+1}\right) = \sum_{i=2}^{s}\left(\prod_{j=i}^{s} f_j{}^2\right)\cdot\operatorname{var}\left(\gamma_{n,i}\right) + \operatorname{var}\left(\gamma_{n,s+1}\right) \quad (13)$$

Given that $\gamma_{n,s+1}$ follows the same distribution, i.e., $\operatorname{var}(\gamma_{1,s+1}) = \operatorname{var}(\gamma_{2,s+1}) = \ldots = \operatorname{var}(\gamma_{n,s+1})$, we assign $\operatorname{var}(\varepsilon_{1,s+1}) = \operatorname{var}(\varepsilon_{2,s+1}) = \ldots = \operatorname{var}(\varepsilon_{n,s+1}) = \sigma_{\varepsilon,s+1}{}^2$.

Table 3: Control effects in different conditions.

No.	Condition	ECP	Schedule deviation after control
1	$d_s - [(1+\beta_s)\varepsilon_{n,s} - \beta_s\varepsilon_{n-1,s}] + f_s\varepsilon_{n,s} \geq 0$	f_s	$f_s\varepsilon_{n,s}$
2	$d_s - [(1+\beta_s)\varepsilon_{n,s} - \beta_s\varepsilon_{n-1,s}] + f_s\varepsilon_{n,s} < 0$	$f_s{}' = -\dfrac{d_s - [(1+\beta_s)\varepsilon_{n,s} - \beta_s\varepsilon_{n-1,s}]}{\varepsilon_{n,s}}$	$f_s{}'\varepsilon_{n,s}$

Furthermore, since $\varepsilon_{n,s+1}$ is the summation of many independent random variables, $\varepsilon_{n,s+1}$ can be assumed to be normally distributed according to the central limit theorem, i.e., $\varepsilon_{n,s+1} \sim N((\prod_{k=1}^{s} f_k) \cdot \varepsilon_{n,1}, \sigma_{\varepsilon,s+1}{}^2)\ \forall n$.

Let $F = \max(|f_1|, |f_2| \ldots |f_s|)$, and M be a large enough number to keep $M > \mathrm{var}(\gamma_{n,s+1})\ \forall s$. Then we have:

$$\begin{aligned} \sigma_{\varepsilon,s+1}{}^2 &= \sum_{i=2}^{s}\left(\prod_{j=i}^{s} f_j{}^2\right) \cdot \mathrm{var}(\gamma_{n,i}) + \mathrm{var}(\gamma_{n,s+1}) \\ &< \sum_{i=0}^{s} F^{2i} \cdot M = \frac{1-F^{2(s+1)}}{1-F^2} \cdot M \approx \frac{M}{1-F^2} \end{aligned} \tag{14}$$

Equation (14) shows that the schedule variance $\mathrm{var}(\varepsilon_{n,s+1})$ is bounded when $|F| < 1$. Therefore, as long as we ensure $|f_s| < 1$ for each control stop s, buses will adhere to their schedules. It should be noted that regardless of whether the control parameter f_s is positive or negative, the reliability of the system remains the same as long as their absolute values are the same. However, (9) shows that when f_s is negative, a larger slack time is needed to ensure the effectiveness of the control. Therefore, a positive f_s is always selected in practice.

3.3. Optimization of Slack Time. As aforementioned, Xuan et al. [23] let $d_s = 3\sigma_{D,s}$ to keep holding time positive. However, as to be shown in the case study, such magnitude of slack time is too large to keep bus operations of high efficiency.

To reduce slack time, the term $d_s - [(1+\beta_s)\varepsilon_{n,s} - \beta_s\varepsilon_{n-1,s}] + f_s\varepsilon_{n,s}$ is allowed to be negative in some cases. This leads to $D_{n,s} = 0$ in (7). By introducing the concept Equivalent Control Parameter (ECP) in this paper, we redefine (9) to correlate slack time d_s with schedule variance $\sigma_{\varepsilon,s+1}{}^2$. Then the best slack time can be obtained by solving an optimization problem.

Table 3 shows the choice of ECP in different conditions. In condition 1, the predefined control parameter f_s does not lead to negative holding time. Thus the bus holding time can be determined by f_s. In condition 2, negative holding time occurs if the value of control parameter still sticks to f_s. In reality, however, the actual holding time will be 0 according to (5). In this case, if we replace the original control parameter f_s witt $f_s{}'$, condition 2 can be converted to condition 1, satisfying $d_s - [(1+\beta_s)\varepsilon_{n,s} - \beta_s\varepsilon_{n-1,s}] + f_s\varepsilon_{n,s} = 0$.

Statistics tell us that we can calculate the variance of variable X as $\mathrm{var}(X) = E(X^2) - E(X)$. Therefore, the variance of the schedule deviation with holding control can be expressed as follows:

$$\begin{aligned} \sigma_{\varepsilon,n}^{AC2} &= \int_{-\infty}^{+\infty}\int_{-\infty}^{Bound}\left[(f_s\varepsilon_{n,s})^2 - f_s\varepsilon_{n,s}\right] \mathrm{f}(\varepsilon_{n,s}) \\ &\quad \cdot \mathrm{f}(\varepsilon_{n-1,s})\, \mathrm{d}(\varepsilon_{n,s})\, \mathrm{d}(\varepsilon_{n-1,s}) \\ &\quad + \int_{-\infty}^{+\infty}\int_{Bound}^{+\infty}\left[(f_s{}'\varepsilon_{n,s})^2 - f_s{}'\varepsilon_{n,s}\right] \mathrm{f}(\varepsilon_{n,s}) \\ &\quad \cdot \mathrm{f}(\varepsilon_{n-1,s})\, \mathrm{d}(\varepsilon_{n,s})\, \mathrm{d}(\varepsilon_{n-1,s}) \\ f_s{}' &= -\frac{\{d_s - [(1+\beta_s)\varepsilon_{n,s} - \beta_s\varepsilon_{n-1,s}]\}}{\varepsilon_{n,s}} \\ Bound &= \frac{[d_s - (1+\beta_s - f_s)\varepsilon_{n,s}]}{\beta_s} \\ \varepsilon_{n,s} &\sim N(0, \sigma_{\varepsilon,s}^2),\ \varepsilon_{n-1,s} \sim N(0, \sigma_{\varepsilon,s}^2) \\ &\text{where } \mathrm{f}(\varepsilon_{n,s}) \text{ and } \mathrm{f}(\varepsilon_{n-1,s}) \text{ are probability density functions} \end{aligned} \tag{15}$$

Then, (9) can be revised as follows:

$$\sigma_{\varepsilon,s+1}{}^2 = \sigma_{\varepsilon,n}^{AC2} + \sigma_{s+1}{}^2 \tag{16}$$

Equations (15) and (16) correlate the magnitude of slack time to its corresponding impacts on the schedule variations. The next section will show how to optimize control coefficients based on these two s.

So far we assumed that all bus stops are control stops. At control stops, buses will be held for some extra time after passengers finish boarding to help buses meet their schedules. Too many control stops can significantly reduce the average bus travel speeds. This assumption can be relaxed. Equation (4) shows that the schedule deviations of different buses are interdependent without holding control. Bus's schedule deviation at stop $s+1$ depends on the schedule deviations of both the bus itself and its preceding bus at stop s. If we implement control strategy at normal stops as the following, we can regain the independent feature.

$$D_{n,s} = \max(0, d_s + \beta_s\varepsilon_{n-1,s}) \tag{17}$$

The schedule variance at stop $s+1$ can then be expressed without terms related to the preceding bus (n-1):

$$\sigma_{\varepsilon,s+1}{}^2 = (1+\beta_s)^2\sigma_{\varepsilon,s}{}^2 + \sigma_{s+1}{}^2 \tag{18}$$

Based on trial simulations, the holding time shown in (17) is usually very small and can be ignored. Therefore, we can use s (16) and (18) to deduce the stop-by-stop schedule variances for every control stop and normal stop.

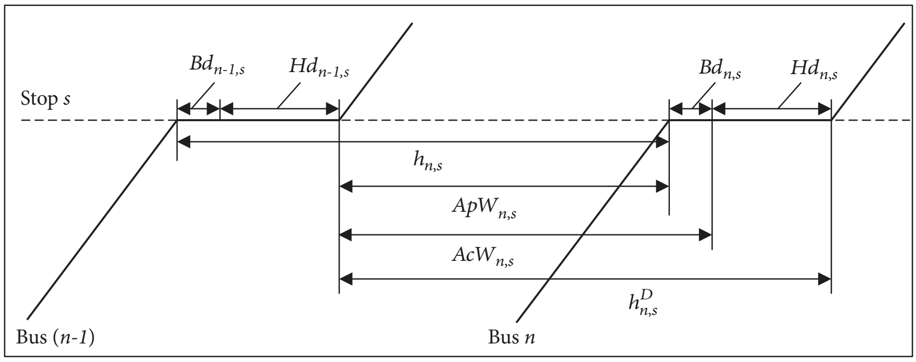


Figure 1: Illustration of passenger waiting time at stop.

4. Optimization of Bus Operation

Transit agencies need to provide reliable and efficient bus services to passengers with limited resources. In this section, we optimize the bus dispatch frequency and holding control methods to minimize passenger travel time when bus capacity and vehicle fleet size are limited.

4.1. Passenger Ordinary Waiting Time. When bus capacity is unlimited, all waiting passengers can hop on the first bus they meet, and we call their average waiting time as ordinary waiting time (OWT) in this circumstance.

Figure 1 depicts two consecutive buses dwellings at stop s. As shown, the bus dwelling time consists of the time for boarding ($Bd_{n,s}$) and holding ($Hd_{n,s}$) procedure. Existing researches assumed that passengers arrive within $h_{n,s}$ will board bus n. The OWT for all buses can be calculated as $\overline{T}_{Wait,s} = (H/2)(1 + \sigma_{h,s}^2/H^2)$ (Welding [2]), where $\sigma_{h,s}^2$ is the headway variance for all buses at stop s. Given $h_{n,s} = H + \varepsilon_{n,s} - \varepsilon_{n-1,s}$, $\sigma_{h,s}^2$ can be calculated as $\sigma_{h,s}^2 = 2\sigma_{\varepsilon,s}^2$.

In practice, only passengers who arrive during $AcW_{n,s}$ have to wait for boarding. $AcW_{n,s}$ starts from the departure time of bus $(n-1)$ and ends at the end of bus n 's boarding procedure. Passengers who arrive during bus 's holding procedure can directly get onboard without waiting. Furthermore, since the value of $Bd_{n,s}$ is much smaller than that of $AcW_{n,s}$, $AcW_{n,s}$ can be approximated as $ApW_{n,s}$. Then the OWT at stop s can be calculated as follows:

$$\overline{T}_{Wait,s} = \frac{\mathrm{E}\left(ApW_{n,s}\right)}{2}\left[1 + \frac{\mathrm{var}\left(ApW_{n,s}\right)}{\mathrm{E}\left(ApW_{n,s}\right)^2}\right]\cdot\frac{\mathrm{E}\left(ApW_{n,s}\right)}{\mathrm{E}\left(h_{n,s}^D\right)} \qquad (19)$$

Where $\mathrm{E}(ApW_{n,s})$ stands for the average value of $ApW_{n,s}$, $\mathrm{E}(ApW_{n,s}) = (H - d_s)$ if we ignore the existence of boarding procedure. The average value of departure headway $h_{n,s}^D$ is $\mathrm{E}(h_{n,s}^D) = H$. In (19), the weight of the OWT is assigned as $\mathrm{E}(ApW_{n,s})/\mathrm{E}(h_{n,s}^D)$ because among all passengers arriving during $h_{n,s}^D$, only passengers who arrive during $ApW_{n,s}$ have to wait to board. Since $ApW_{n,s}$ is decided by bus $(n-1)$'s leaving time and bus n 's arriving time, $\mathrm{var}(ApW_{n,s}) = \sigma_{\varepsilon,n}^{AC^2} + \sigma_{\varepsilon,s}^2$. Finally, (19) can be expressed as follows:

$$\overline{T}_{Wait,s} = \frac{H - d_s}{2}\left[1 + \frac{\sigma_{\varepsilon,n}^{AC^2} + \sigma_{\varepsilon,s}^2}{\left(H - d_s\right)^2}\right]\cdot\frac{H - d_s}{H} \qquad (20)$$

The OWT for passengers at all stops is just the weighted summation of OWT at each stop.

$$\overline{T}_{Wait} = \frac{\left(\sum_{i=1}^{S}\lambda_i\overline{T}_{Wait,i}\right)}{\left(\sum_{j=1}^{S}\lambda_j\right)} \qquad (21)$$

4.2. Passenger extra waiting time. Due to the limit on bus capacity, some passengers may not be able to board the first bus they meet when the bus is overloaded, and therefore will experience an extra waiting time (EWT) for waiting the next bus. Supposing that all buses run along with schedules, bus's loads after leaving stop s are as follows:

$$p_{n,s}^t = p_{n,s-1}^t + H\lambda_s - \sum_{i=1}^{s} H\lambda_i l_{i,s} \qquad (22)$$

Where λ_s is the average passenger arrival rate at stop s, and $l_{i,s}$ indicates the proportion of passengers who board at stop i and alight at stop s. However, when buses deviate from schedules, the actual passenger load becomes:

$$p_{n,s}^a = p_{n,s-1}^a + h_{n,s}\lambda_s - \sum_{i=1}^{s} h_{n,i}\lambda_i l_{i,s} \qquad (23)$$

By combining (22) and (23), the passenger load deviation can be expressed as follows:

$$\varepsilon_{n,s}^p = \varepsilon_{n,s-1}^p + \lambda_s\left(\varepsilon_{n,s} - \varepsilon_{n-1,s}\right) - \sum_{i=1}^{s}\lambda_i l_{i,s}\left(\varepsilon_{n,i} - \varepsilon_{n-1,i}\right) \qquad (24)$$

If we respectively replace $p_{n,s-1}^a$ and $\varepsilon_{n,s-1}^p$ in the right-hand side of (23) and (24) in an iterative way, we finally have the following:

$$p^a_{n,s} = \sum_{i=1}^{s} h_{n,i}\lambda_i \left(1 - \sum_{j=1}^{s} l_{ij}\right) \quad (25)$$

$$\varepsilon^p_{n,s} = \sum_{i=1}^{s} \left(\varepsilon_{n,i} - \varepsilon_{n-1,i}\right)\lambda_i \left(1 - \sum_{j=1}^{s} l_{ij}\right) \quad (26)$$

With (25), we have the average passenger load for all buses at stop s:

$$\overline{p_s} = H \cdot \sum_{i=1}^{s} \lambda_i \left(1 - \sum_{j=1}^{s} l_{ij}\right) \quad (27)$$

Passenger loads variance can be calculated by (26):

$$\begin{aligned} \sigma^2_{\varepsilon^p,s} &= \sum_{i=1}^{s} 2\sigma^2_{\varepsilon,i}\lambda_i^2 \left(1 - \sum_{j=1}^{s} l_{ij}\right)^2 \\ &= \sum_{i=1}^{s} \sigma^2_{h,i}\lambda_i^2 \left(1 - \sum_{j=1}^{s} l_{ij}\right)^2 \end{aligned} \quad (28)$$

However, as illustrated in Figure 1, the number of boarding passengers is actually decided by the departure headway $h^D_{n,s}$ rather than the arriving headway $h_{n,s}$. Therefore, we replace the variance of arriving headway $\sigma^2_{h,i}$ by the variance of departure headway $\sigma^2_{lh,s}$ in (28), where we have $\sigma^2_{lh,s} = 2\sigma^{AC^2}_{\varepsilon,s}$ when stop s is a control stop, and $\sigma^2_{lh,s} = 2(1+\beta_s)^2\sigma^2_{\varepsilon,s}$ if stop s is a normal stop. Then (28) can be revised as follows:

$$\begin{aligned} \sigma^2_{\varepsilon^p,s} &= \sum_{i=1}^{s} \sigma^2_{lh,i}\lambda_i^2 \left(1 - \sum_{j=1}^{s} l_{ij}\right)^2 \\ \sigma^2_{lh,s} &= 2\sigma^{AC^2}_{\varepsilon,s} \quad \text{if stop } s \text{ is a control point} \\ \sigma^2_{lh,s} &= 2\left(1+\beta_s\right)^2\sigma^2_{\varepsilon,s} \quad \text{if stop } s \text{ is a normal stop} \end{aligned} \quad (29)$$

Supposing that all buses are identical with capacity C_a, bus n 's residual capacity at stop s will follow $C^R_{n,s} \sim N(C_a - \overline{p_s}, \sigma^2_{\varepsilon^p,s})$ $\forall n$. $C^R_{n,s} > 0$ means that there are still $C^R_{n,s}$ residual capacity left in bus n after it leaves stop s; while $C^R_{n,s} < 0$ indicates that bus n is fully loaded and $-C^R_{n,s}$ passengers will experience EWT for the next bus.

We consider $C_a - \overline{p_s} < 0$ as an unacceptable condition, because the number of waiting passengers will keep growing over time. Therefore we let the average EWT to be M_{ExWait} in this situation, where M_{ExWait} is a large enough number. When $C_a - \overline{p_s} \geq 0$, for those who have to wait for the following buses, they may manage to board the next bus $n+1$, or some of them may have to wait even longer if bus $n+1$ is also fully loaded. Table 4 enumerates all possible cases.

Based on Table 4, we can calculate the average value of EWT.

In case 1, $-C^R_{n,s}$ passengers cannot board bus n. The average number of passengers who suffer the EWT in case 1 can be calculated as follows:

$$\begin{aligned} &\overline{P}^{case1}_s \\ &= \int_{-\infty}^{0} -C^R_{n,s} f\left(C^R_{n,s}\right) d\left(C^R_{n,s}\right) \int_{-C^R_{n,s}}^{\infty} f\left(C^R_{n,s}\right) d\left(C^R_{n+1,s}\right) \end{aligned} \quad (30)$$

Suppose that passengers will experience an extra H waiting time for every one extra bus, the total EWT in case 1 becomes the following:

$$\begin{aligned} T^{total,case1}_{s,ExWait} = &\int_{-\infty}^{0} -C^R_{n,s} H f\left(C^R_{n,s}\right) d\left(C^R_{n,s}\right) \\ &\cdot \int_{-C^R_{n,s}}^{\infty} f\left(C^R_{n+1,s}\right) d\left(C^R_{n+1,s}\right) \end{aligned} \quad (31)$$

Since the average number of passengers arriving between two consecutive buses is $\lambda_s H$, the average EWT in case 1 can be calculated as follows:

$$\begin{aligned} \overline{T}^{case1}_{s,ExWait} &= \frac{T^{total,case1}_{s,ExWait}}{(\lambda_s H)} \\ &= \frac{\int_{-\infty}^{0} -C^R_{n,s} H f\left(C^R_{n,s}\right) d\left(C^R_{n,s}\right) \int_{-C^R_{n,s}}^{\infty} f\left(C^R_{n+1,s}\right) d\left(C^R_{n+1,s}\right)}{(\lambda_s H)} \end{aligned} \quad (32)$$

Similarly, we have the following s for case 2.1 and case 2.2.

$$\overline{P}^{case2.1}_s = \frac{\int_{-\infty}^{0} -C^R_{n,s} f\left(C^R_{n,s}\right) d\left(C^R_{n,s}\right) \int_{-\infty}^{0} f\left(C^R_{n+1,s}\right) d\left(C^R_{n+1,s}\right) \int_{-C^R_{n,s}}^{\infty} f\left(C^R_{n+2,s}\right) d\left(C^R_{n+2,s}\right)}{(\lambda_s H)} \quad (33)$$

$$\overline{T}^{case2.1}_{s,ExWait} = \frac{\int_{-\infty}^{0} -C^R_{n,s} \cdot 2H f\left(C^R_{n,s}\right) d\left(C^R_{n,s}\right) \int_{-\infty}^{0} f\left(C^R_{n+1,s}\right) d\left(C^R_{n+1,s}\right) \int_{-C^R_{n,s}}^{\infty} f\left(C^R_{n+2,s}\right) d\left(C^R_{n+2,s}\right)}{(\lambda_s H)} \quad (34)$$

$$\overline{P}^{case2.2}_s = \frac{\int_{-\infty}^{0} -C^R_{n,s} f\left(C^R_{n,s}\right) d\left(C^R_{n,s}\right) \int_{0}^{C^R_{n,s}} f\left(C^R_{n+1,s}\right) d\left(C^R_{n+1,s}\right) \int_{-(C^R_{n,s}+C^R_{n+1,s})}^{\infty} f\left(C^R_{n+2,s}\right) d\left(C^R_{n+2,s}\right)}{(\lambda_s H)} \quad (35)$$

Table 4: All possible cases of EWT.

	Case 1	Case 2		Case 3
Case Depiction	Bus $n+1$ can take all the passengers who cannot board bus n.	Only part of the passengers who cannot board bus n are able to board bus $n+1$, and the remaining passengers have to wait bus $n+2$ for boarding. **Case 2.1** Bus $n+1$ is already full before collecting those passengers and therefore all passengers have to wait for bus $n+2$.	**Case 2.2** Bus $n+1$ can collect some of the passengers, and all the other passengers have to wait for bus $n+2$.	When some of the passengers who cannot board bus n have to wait for more than 2 buses for boarding.
Condition	(1) $C_{n,s}^R < 0$ (2) $-C_{n,s}^R \le C_{n+1,s}^R$	(1) $C_{n,s}^R < 0$ (2) $C_{n+1,s}^R \le 0$ (3) $-C_{n,s}^R \le C_{n+2,s}^R$	(1) $C_{n,s}^R < 0$ (2) $0 < C_{n+1,s}^R < -C_{n,s}^R$ (3) $-C_{n,s}^R \le C_{n+1,s}^R + C_{n+2,s}^R$	(1) $C_{n,s}^R < 0$ (2) $-C_{n,s}^R > C_{n+1,s}^R + C_{n+2,s}^R$
Average Number of Passengers	$\overline{P}_s^{case1}$ *Equation (30)*	$\overline{P}_s^{case2.1}$ *Equation (33)*	$\overline{P}_s^{case2.2}$ *Equation (35)*	$\overline{P}_s^{case3}$ *Equation (37)*
Average Extra Waiting Time	$\overline{T}_{s,ExWait}^{case1}$ *Equation (32)*	$\overline{T}_{s,ExWait}^{case2.1}$ *Equation (34)*	$\overline{T}_{s,ExWait}^{case2.2}$ *Equation (36)*	$\overline{T}_{s,ExWait}^{case3}$ *Equation (37)*

$$\overline{T}_{s,ExWait}^{case2.2} = \frac{\int_{-\infty}^{0}\left[C_{n+1,s}^R H - 2H\left(C_{n,s}^R + C_{n+1,s}^R\right)\right] \mathrm{f}\left(C_{n,s}^R\right) \mathrm{d}\left(C_{n,s}^R\right) \int_{0}^{C_{n,s}^R} \mathrm{f}\left(C_{n+1,s}^R\right) \mathrm{d}\left(C_{n+1,s}^R\right) \int_{-(C_{n,s}^R + C_{n+1,s}^R)}^{\infty} \mathrm{f}\left(C_{n+2,s}^R\right) \mathrm{d}\left(C_{n+2,s}^R\right)}{(\lambda_s H)} \tag{36}$$

Case 3 is the situation when some passengers have to wait for more than three buses for boarding, and we suppose that all passengers who suffer case 3 will experience a $3H$ EWT. Then the average EWT in this case can be calculated as follows:

$$\overline{T}_{s,ExWait}^{case3} = \frac{3H\overline{P}_s^{case3}}{(\lambda_s H)}$$

$$\overline{P}_s^{case3} = \int_{-\infty}^{0} -C_{n,s}^R \mathrm{f}\left(C_{n,s}^R\right) \mathrm{d}\left(C_{n,s}^R\right) - \overline{P}_s^{case1} - \overline{P}_s^{case2.1} - \overline{P}_s^{case2.2} \tag{37}$$

It should be noted that all cases above are mutually exclusive events. Therefore the average EWT for passengers at all stops can be formulated as follows:

$$\overline{T}_{ExWait} = \frac{\sum_{i=1}^{S-1} \lambda_i \overline{T}_{i,ExWait}}{\sum_{j=1}^{S-1} \lambda_j}$$
$$s.t. \begin{cases} \overline{T}_{s,ExWait} = \left(\overline{T}_{s,ExWait}^{case1} + \overline{T}_{s,ExWait}^{case2.1} + \overline{T}_{s,ExWait}^{case2.2} + \overline{T}_{s,ExWait}^{case3}\right) & \text{if } C_a - \overline{p_s} > 0 \\ \overline{T}_{s,ExWait} = M_{ExWait} & \text{if } C_a - \overline{p_s} < 0 \end{cases} \tag{38}$$

4.3. Passenger In-Vehicle Travel Time (IvTT). With (3), we can get the bus travel time from stop s to stop $s+1$:

$$T_{n,s} = a_{n,s+1} - a_{n,s} = \beta_s h_{n,s} + D_{n,s} + c_s + \gamma_{n,s+1} \tag{39}$$

The average value of $T_{n,s}$ is as follows:

$$\overline{T}_s = \beta_s H + d_s + c_s \tag{40}$$

Consider the scenario that passengers travel from stop s to stop $s+m$ as an example for illustration. If we assume all waiting passengers at stop s can get onboard immediately after the bus arrival. The average IvTT for those passengers can be calculated as $\sum_{k=s}^{s+m} \overline{T}_i = \sum_{k=s}^{s+m}(\beta_k H + d_k + c_k)$. The average IvTT for passengers at all stops is as follows:

$$\overline{T}_{inveh} == \frac{\sum_{i=1}^{S-1} \sum_{j=i+1}^{S} \sum_{k=i}^{j-1} (\beta_k H + d_k + c_k)}{\sum_{m=1}^{S} \lambda_m} \tag{41}$$

In reality, the waiting passengers cannot get onboard immediately, but keep on boarding during the boarding and holding procedure as shown in Figure 1. Therefore the average IvTT they spend at stop s is less than $\beta_s H + d_s$. We approximate the average value of IvTT at stop s as half of the average bus dwelling time, i.e., $(\beta_s H + d_s)/2$. Then (39) can be revised as follows:

$$\overline{T}_{inveh} == \frac{\sum_{i=1}^{S-1} \sum_{j=i+1}^{S} \sum_{k=i}^{j-1} \lambda_i l_{i,j} \left(\beta_k H + d_k + c_k\right) - \sum_{p=1}^{S-1} \lambda_p \left(\beta_p H + d_p\right)/2}{\sum_{m=1}^{S} \lambda_m} \quad (42)$$

4.4. Fleet Size Limit. When all buses run according to schedules, the total travel time cost for one bus run is as follows:

$$T_{total} = \sum_{i=1}^{S-1} \left(\beta_i H + d_i + c_i\right) + L \quad (43)$$

Where L is the layover time to recover buses from schedule deviations and to provide breaks for drivers. The mean value of T_{total} is $\overline{T}_{total} = \sum_{i=1}^{S-1}(\beta_i H + d_i + c_i) + L$. Supposing that the bus fleet size is N_{fleet}, we can calculate the target headway as follows:

$$H = \frac{\left[\sum_{i=1}^{S-1} \left(\beta_i H + d_i + c_i\right) + L\right]}{N_{fleet}} \quad (44)$$

4.5. Optimization Model. Tradeoff exists when implementing dynamic holding strategy. Although bus holding has the potential to reduce passenger waiting time by lessening bus headway deviation (Welding [2]), sometimes excessive holding increases passengers' total travel time because of the following reasons. First, the added slack time increases buses' average dwelling time, thereby leading to long in-vehicle travel time for onboard passengers. Second, as illustrated by (44), bus dispatching headway increases with the value of slack time when fleet size is limited. As explained by Welding [2], passenger ordinary waiting time increases with average headway (i.e., dispatching headway) and decreases with headway deviation. When the benefit (i.e., reduction of headway deviation) of dynamic holding cannot compensate for the loss (i.e., increment of dispatching headway) caused by it, passenger ordinary waiting time will increase. Moreover, large dispatching headway also increase the likelihood of extra waiting time suffered by waiting passengers, especially if the supply of bus service is less than passenger demand.

The proposed optimization fully considers the positive and negative effect of dynamic holding on bus operation, and minimize average passenger travel time by solving the optimal locations of control stop, and the corresponding slack time and control parameter designated for each control stop. Limited bus capacity and fleet size are considered in this paper. As shown in function (45), passenger travel time consists of ordinary waiting time, extra waiting time and in-vehicle travel time as discussed before. Studies show that the perceived waiting time by passengers are usually much longer than the actual waiting time due to factors like adverse waiting environment, waiting anxiety, etc. (Psarros et al. [31], Mishalani et al. [32]). Therefore, waiting penalty ζ_{wait} is added to the objective function to measure this perceiving difference. In this paper, we set the value of ζ_{wait} to be 2.1 as recommended in Transit Capacity and Quality of Service Manual [33].

$$\min \overline{T}_{total} = \zeta_{wait} \left(\overline{T}_{Wait} + \overline{T}_{ExWait}\right) + \overline{T}_{InVeh} \quad (45)$$

Some major constraints are summarized as follows:

$$H = \frac{\left[\sum_{i=1}^{S-1} \left(\beta_i H + d_i + c_i\right) + L_{\min} + 3\sigma_{\varepsilon,S}\right]}{N_{fleet}} \quad (46.a)$$

$$\sigma_{\varepsilon,s+1}{}^2 = \begin{cases} \sigma_{\varepsilon,n}^{AC^2} + \sigma_{s+1}{}^2 & \text{if } \eta_s = 1 \\ \left(1+\beta_s\right)^2 \sigma_{\varepsilon,s}{}^2 + \sigma_{s+1}{}^2 & \text{if } \eta_s = 0 \end{cases} \quad (46.b)$$

$$\sigma_{lh,s}^2 = \begin{cases} 2\sigma_{\varepsilon,s}^{AC^2} & \text{if } \eta_s = 1 \\ 2\left(1+\beta_s\right)^2 \sigma_{\varepsilon,s}^2 & \text{if } \eta_s = 0 \end{cases} \quad (46.c)$$

$$\eta_s = \begin{cases} 0 & \text{if stop } s \text{ is acontrol point} \\ 1 & \text{if stop } s \text{ is anormal stop} \end{cases} \quad (46.d)$$

Where (46.a) is the formula to calculate dispatching headway, (46.b) depicts the propagation of schedule variance along the bus route, (46.c) further illustrates how to calculation schedule various for both control stops and normal stops, the parameter η_s in (46.d) indicates whether stop s is a control stop or not. Note that η_s, d_s and f_s $(s = 1, 2, \ldots, S)$ are decision variables that need to be solved, and all other variables such as $\sigma_{\varepsilon,s}^2$, $\sigma_{lh,s}^2$ and H etc. are determined with certain value of η_s, d_s and f_s $(s = 1, 2, \ldots, S)$.

4.6. Parameter Calibration. We show how to calibrate the average cruising time c_s and the variance of cruising time $\sigma_{s+1}{}^2$ between stop s and stop $(s+1)$ with AVL data and signal timing data. Suppose that there are $N_{s,seg}$ road segments and $N_{s,int}$ intersections between stop s and stop $(s + 1)$. The average travel time $\overline{T}_{s,i}^{seg}$ and variance of travel time $\sigma_{s,i}^{seg^2}$ $(i = 1 \ldots N_{s,seg})$ for each segment can be calculated by using AVL data. In terms of signal delay, we assume that buses will not queue up at intersections in this experiment, then the average delay $\overline{DE}_{s,j}^{int}$ and variance of delay $\sigma_{s,j}^{int^2}$ at intersections can respectively be calculated with (47) and (48), where $RT_{s,j}^{int}$ is the red time and $CL_{s,j}^{int}$ is the cycle length.

$$\overline{DE}_{s,j}^{int} = \frac{\left(RT_{s,j}^{int}\right)^2}{2CL_{s,j}^{int}} \tag{47}$$

$$\sigma_{s,j}^{int\,2} = \frac{\left(RT_{s,j}^{int}\right)^3}{3CL_{s,j}^{int}} - \left(DE_{s,j}^{int}\right)^2 \tag{48}$$

The average value and variance of cruising time are the summation of the values of all segments and intersections between stop s and stop$(s+1)$, respectively.

$$c_s = \sum_{i=1}^{N_{s,seg}} \overline{T}_{s,i}^{seg} + \sum_{j=1}^{N_{s,int}} \overline{DE}_{s,j}^{int} \tag{49}$$

$$\sigma_{s+1}{}^2 = \sum_{i=1}^{N_{s,seg}} \sigma_{s,i}^{seg2} + \sum_{j=1}^{N_{s,int}} \sigma_{s,j}^{int\,2} \tag{50}$$

5. Case Study

5.1. Data Collection. The case study uses field data collected from the Bus Route 56 in Chengdu, China. This bus route is one of the busiest in Chengdu, which serves more than 50000 passengers per day. The route runs across the city in the north-south direction, and connects several universities, large-scale business areas, hospitals and residential districts. We select 14 major bus stops located in the downtown area of the city as shown in Figure 2. In the simulation, we assume that these 14 stops form a loop. AVL data and smartcard data are collected during morning rush hour (7:30-8:30) from 17th to 21st (Monday to Friday) October 2016. Signal timing information is collect on 18th October 2016. The appendix list all the data results.

1. **AVL Data:** Each piece of AVL data contains information including bus ID, route ID, GPS location, GPS speed and timestamp. Buses regularly send a piece of AVL data to the control center every ten seconds. With large amount of AVL data available, the mean value and variance of cruise time can be calibrated for each road segment by using (47) ~ (50).

5.2. Simulation Framework. A simulation platform is developed to test the proposed control method. As shown in Figure 3, this platform mainly consists of five components: bus, terminal, intersection, bus stop and road segment.

(i) **Bus:** Figure 3 illustrates how buses travel along the bus route. First, buses depart the terminal and travel in the outbound direction. After reaching the farthest bus stop, buses turn around and start to travel in the inbound direction. Finally, buses return to the terminal, waiting there for the next round of operation. As we can see, there are 4 buses waiting at terminal, 2 buses travelling on road segments, 2 buses dwelling at stops, 2 buses waiting at intersections. Therefore the fleet size is 10.

(ii) **Terminal:** Each waiting bus at the terminal will be assigned a bar to record its status of layover time. The whole length of the bar represents the required layover time as aforementioned in (43). The green part of the bar indicates the elapsed time, and the grey part is the remaining layover time. Agency can release a bus from the terminal only when the following two conditions are met: First, the time since the last departure exceeds the predetermined bus headway. Second, there is (are) bus(es) at the terminal whose status bar is all green.

(iii) **Intersection:** All intersections are signalized and operate in fixed-time. Dedicated bus lanes are employed to avoid bus queuing problem at intersections. The key signal control parameters include cycle length, phase plan, offset and splits (splits are the portion of time allocated to each phase at an intersection).

(iv) **Bus Stop:** Passengers arrive at the bus stop at a fixed arrival rate. After a bus arrive at the stop, the waiting passengers start to board the bus at a fixed boarding rate until there are no more waiting passengers or the bus is fully loaded. The bus will leave the bus stop when both the boarding procedure and holding procedure are over.

(v) **Road Segment:** A road segment is a portion of the bus route which is separated by intersections and bus stops. Each road segment is assigned with two parameters: average cruising time of the road segment, and standard deviation of the cruising time.

Figure 4(a) shows a simulation result of how the route 56 performs under an uncontrolled situation. As shown, a total of 13 buses are assigned to this bus route, i.e., the fleet size is 13. At first, buses depart the terminal station in even headways. However, as time goes by, buses fail to maintain their headway to the target value when they travel in different speed and delay at intersections for different period of time. What's more, buses with larger headways serve more passengers and will lag further behind their preceding vehicle, and vice versa. Finally, buses bunch up together and move in pairs. More interestingly, when we observe the two rounds of operation of bus 1, it is easy to see that after bus 1 finish one round of operation, it has to take a layover time before depart for the next round.

Figure 4(b) shows the detailed view of the zoomed-in area in Figure 4(a). The horizontal lines at bus stops indicate buses' dwelling procedure, where longer line means longer dwelling time. Signal timings at intersections are represented by green and red horizontal lines. The green line and red line stand for green time and red time respectively. Figure 4(b) is a good example to show how bus bunching phenomenon occurs. Due to the large time gap between bus 13 and bus 1, many passengers are waiting at stop 8 and stop 9 for boarding, and therefore lead to an increase of dwelling time for bus 1. In contrast, the dwelling time of bus 2 is much shorter than that

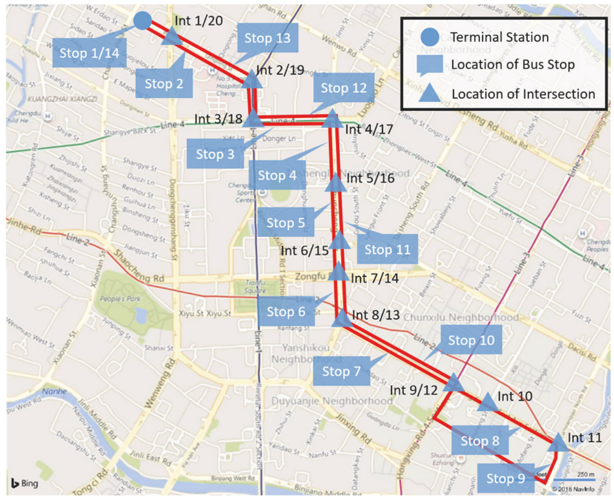


FIGURE 2: Map of bus route 56 in the Downton Area, Chengdu, China.

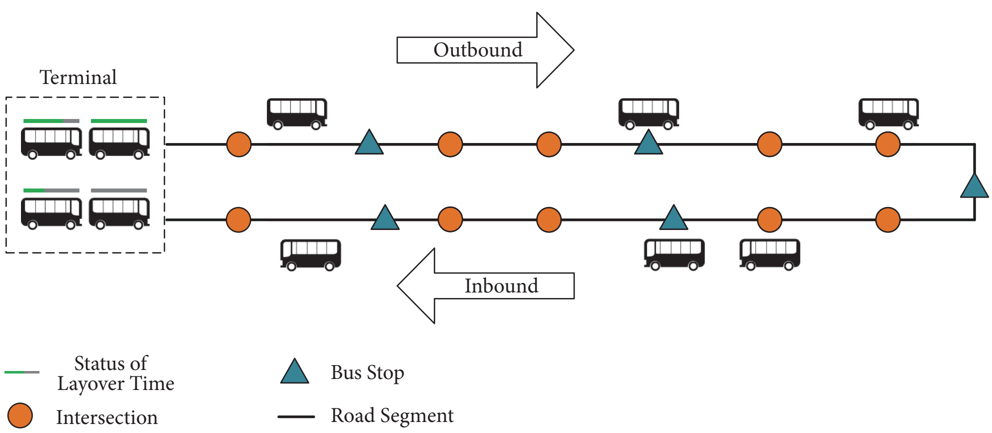
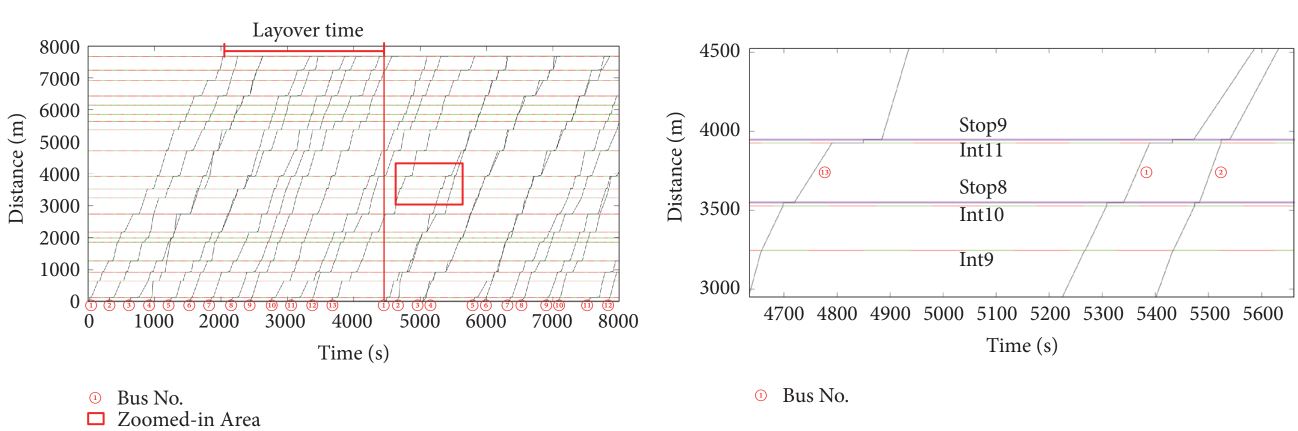


FIGURE 3: Illustration of the simulation platform.



(a) Bus trajectories

(b) Zoomed-in view of bus trajectories

FIGURE 4: Illustration of the simulation results.

of bus 1, which is because only a small number of passengers arrive at stop 8 and stop 9 after bus 2 left. As a result, the headway between bus 1 and bus 2 will shrink over time until they finally bunch.

5.3. Simulation Result Analysis. In this section, the proposed simulation platform is calibrated with collected data from route 56 to evaluate different control methods.

As shown by (45) and (46.a), (46.b), (46.c), and (46.d), the proposed optimization model solve the best control scheme by choosing the prime location of control stops (η_s), and solving the optimal value of control parameters f_s and holding slack time d_s for each control stop. It should be noted that the proposed model is of very high flexibility which allows control stops to be unevenly distributed, and allows different control parameter and holding slack time for different control stops. However, to make it easier for comparison between our control model and other control methods, we simplify the optimization problem as follows:

1. Control stops are designated every 3 stops. Therefore, we have $\eta_s = 1$ ($s = 3, 6, 9, 12$), and $\eta_s = 0$ for the other stops.
2. The control parameters f_s at all control stops are of the same value, and the value of f_s ranges from 0.1 to 0.9 with a precision of 0.1;
3. We let $d_s = \alpha_s \sigma_{D,s}$, where α_s ranges from 0.1 to 3.0 with precision of 0.1. All control stops share the same value of α_s.

Figure 5 illustrates the predicted and simulated passenger average travel time (ATT) with different control parameter settings. The control coefficient f_s ranges from 0.1 to 0.9, and slack time coefficient α_s ranges from 0.1 to 3.0. The green surface represents the predicted ATT when bus overloading is minor events, i.e., $C_a - \overline{p_s} > 0\ \forall s$. Both red and brown surface are ATT results generated by the simulations. It can be observed that the red surface matches well with the green surface with relative error less than 5%. We also use the simulations to generate the brown surface which represents significant overloading conditions when some bus stops satisfy $C_a - \overline{p_s} < 0$. The brown surface is outside of the green surface area. The value of ATT increases quickly when bus overloading become frequent events. Figure 5 illustrates that the proposed passenger-oriented performance measures can make good prediction on bus operational performance when buses are controlled under the proposed holding control strategy. Therefore, the optimization model shown in (43) and (44) can be a useful tool to optimize control parameters in order to minimize passenger average travel time.

Figure 5 also provides a comparison between the results from Xuan et al. [23]'s control method and the proposed control method in terms of passenger average travel time. Xuan et al. [23] set $\alpha_s = 3$ in their research to guarantee the predesigned holding control will always work in the valid range. However, their strategy requires very large slack time at control stops, which can lead to undesirable passenger travel costs. As shown in Figure 5, the minimum ATT that can be achieved by Xuan et al. [23] is 1256s when $\alpha_s = 3$ and $f_s = 0.9$.

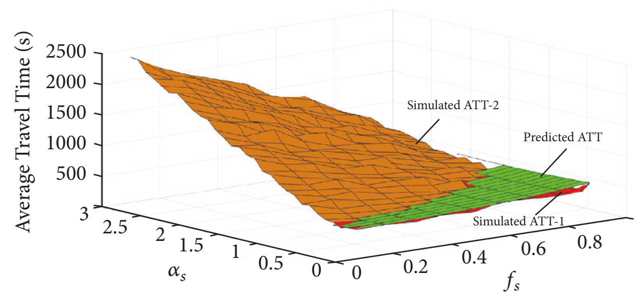


FIGURE 5: Predicted and simulated results of passengers average travel time.

By using the proposed control and optimization model, we can reduce ATT to 912s when we set $\alpha_s = 0.4$ and $f_s = 0.1$. This results in a 27.4% improvement compared to Xuan et al. [23]'s method.

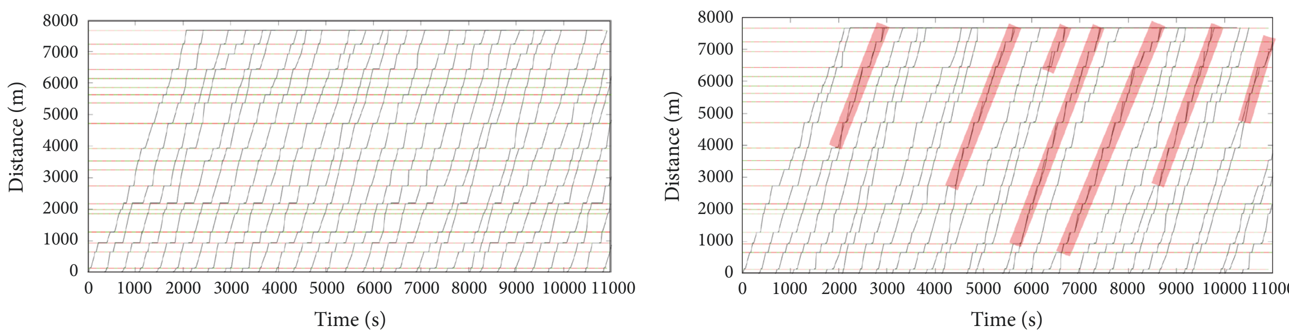
We also compare our strategy with uncontrolled situations. Simulations are performed to find the best bus dispatching headway in uncontrolled cases. The bus operation reaches its best performance when dispatch headway is 345s, and the ATT is 1031s correspondingly. This means that the proposed method also outstrips uncontrolled case in terms of ATT by 11.5%. Figure 6 compares bus trajectories under controlled and uncontrolled cases. It is easy to see that under controlled case, buses travel in more uniform headways, and no bunching occurs in this case. However, the bus line suffers serious bunching problem when no holding control is implemented as shown in Figure 6(b).

Table 5 takes the terminal bus stop as a check point to see how passengers' travel time vary under different control methods. For example, when the proposed control method is implemented, the average travel time for passengers travelling from stop 9 to terminal stop will be 1335 seconds, and the standard deviation of their travel time is 293 seconds. It is irrational at the first glance when seeing that travelling from stop 12 to terminal takes much more time than those passengers who wait at stop 9. The reason is that buses are nearly fully loaded after leaving stop 11, and some of the waiting passengers at stop 12 may suffer an extra waiting time. It is easy to see the proposed method significantly reduce the mean value and SD value of passengers' travel time compared with other two methods, which means it can provide mo re efficient and reliable bus services. Especially for bus stops (like stop 12) with high passenger demand and low capacity supply, the other two methods lead waiting passengers to suffer undesirable extra waiting time, and increase their travel uncertainty.

5.4. Sensitivity Analysis. In this section, we compare control methods under different levels of passenger arriving rate and cruising time deviation. The best uncontrolled case is set as the benchmark to evaluate different control methods. By simulating uncontrolled bus operation under different dispatching headways, the best uncontrolled case can be found when the passenger travel time reaches the minimal value.

Table 5: Comparisons of the Mean Value and Standard Deviation of Passenger Travel Time under Different Control Methods (Unit: seconds).

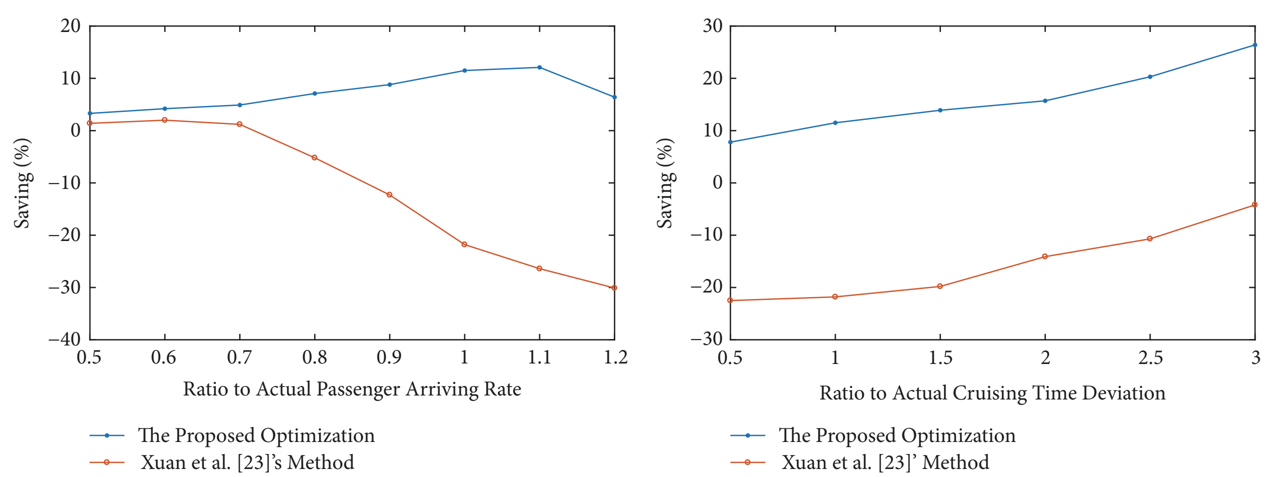
Origin Stop	Control Methods					
	The Proposed Optimization Model		Xuan et al. [23]		Non Control	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
Stop 9	1335	293	1400	304	1490	426
Stop 10	1199	289	1443	367	1420	459
Stop 11	973	330	1060	313	1082	420
Stop 12	1032	384	3960	858	1378	537
Stop 13	707	348	733	304	893	445



(a) Controlled case with $\alpha_s = 0.4$, $f_s = 0.1$

(b) Uncontrolled case with dispatch headway 345s

Figure 6: Bus trajectories under the proposed holding control and uncontrolled situation.



(a) Sensitivity to passenger demand

(b) Sensitivity to cruising time deviation

Figure 7: Saving of passenger travel time under different passenger demand and cruising time deviation.

Figure 7(a) presents the performance of different holding control methods when the ratio of simulated passenger arriving rate to actual passenger arriving rate ranges from 0.5 to 1.2. The vertical axis indicates the saving of passenger travel time compared with uncontrolled case. As shown, the proposed optimization outstrips uncontrolled case under all situations. When the ratio is in the range of 0.5 to 1.1, the saved passenger travel time increases with the increment of passenger demand. The reason is that higher passenger demand leads to larger headway variance and more serious uneven bus load problem in uncontrolled case, which subsequently increases the ordinary waiting time and extra waiting time for passengers. In contrast, the proposed optimization helps to maintain bus headways to the target value, and therefore ensures smaller passenger travel time. However, there is a performance drop when the value of demand ratio reaches 1.2. This result clearly shows the tradeoff between reliability and efficiency when implementing holding control. Even though bus holding helps to improve the reliability of bus operation and alleviate bus bunching problem, the added slack time will increase the time cost per round of bus operation, which means less amount passengers can be served in a unit of time compared with uncontrolled case. When passenger demand is too high, oversaturated loading may even occur in controlled case, and such situation will counteract the benefits of the high reliability provided by holding control. It should be noted that route 56 is already a very heavy-loaded bus route, so that the ratio value 1.2 can rarely occurs in field operation. Due to large slack time required, Xuan et al. (23)'s holding strategy leads to oversaturated situation when the passenger demand ratio is higher than 0.7, and its control performance will drop quickly afterwards. Figure 7(b) evaluate control performances under different levels of cruising time deviation. Not surprisingly, the control performance of both methods will increase when cruising time deviation increase. That's because holding control results in much more stable and reliable bus operations compared with uncontrolled case under high randomness of traffic condition.

6. Conclusions and Future Work

Holding control strategy is an effective way to alleviate bus bunching phenomenon and improve the reliability of bus service, however too much slack time may significantly reduce the operational efficiency and lead to undesirable passenger travel cost and bus overloading. This paper proposes an integrated modeling of schedule planning and dynamic holding control. The proposed approach considers both planning factors (e.g., fleet size, bus capacity, dispatching frequency and layover time) and control factors (control coefficients, slack time, number and location of control stops) when modeling. An optimization model is presented to solve best control strategies. The proposed methods are tested by simulations. All parameters in the simulation environment are calibrated with field data. Simulation shows that the proposed optimization model can precisely predict the control effects of dynamic holding under different control parameters, and therefore can be used to optimize bus operation. Result shows that the proposed optimization achieves a 27.4% improvement compared with Xuan et al. [23]'s method under actual case. Sensitivity analysis further validate the proposed model under different levels of passenger demands and cruising time deviations. A summary of findings is listed as follows:

(1) We relaxed Xuan et al. [23]'s "simple control strategy" by reducing slack time required at control stops. Slack time is correlated with buses schedule deviation by introducing equivalent holding control parameters. We prove that buses can adhere to schedule even with quite small slack time.

(2) We propose performance measures from passenger perspective to precisely predict the control impact. The passenger travel cost consists of the waiting time, extra waiting time and in-vehicle travel time. We advance Welding [2]'s method by taking the holding procedure into consideration and eliminated the overestimation by assuming bus only collects passengers who arrive during the headway. Passenger loads on different buses are not even due to bus deviations from schedules. Some passengers will not be able to board the first bus they meet when that bus is fully loaded, and will therefore suffer extra waiting time. We enumerate all possible cases, and propose a performance measure which can theoretically predict passengers extra waiting time for any specific holding control strategies.

(3) As shown in case study, even though holding control may provide more reliable bus services than uncontrolled operations, it may increase passenger travel cost due to a large slack time. Benefiting from performance measures proposed in this paper, we formulate an optimization model by combining the schedule planning and holding control into an integrated procedure. The resulting model allows us to minimize passenger travel time with limited operating resources and layover time.

Although the research has reached its aims, there are still limitations that need to be solved in future studies. First, passenger arriving rate at each bus stop is treated as a fixed value in this work, which does not satisfy the real situation in some bus lines of high arrival randomness. Second, the proposed optimization is a non-convex integer programming, and it is time-consuming to solve optimal control strategy for some large-scale bus lines. Algorithms need to be proposed in future works which can solve the optimization more efficiently and accurately.

Appendix

Data Inputs for Case Study

Data inputs shown in Table 6 are calibrated by AVL data, smartcard data, and signal timing information. AVL data and smartcard data are collected during morning rush hour (7:30-8:30) from 17th to 21st (Monday to Friday) October 2016. More than 10,000 pieces of AVL data and more than 5,000 pieces of smartcard data can be collected each day during the rush hour. Signal timing information is collected on 18th October 2016.

Data in the second and third column of Table 6 indicates the average travel time and STD of travel time between nodes (stop/intersection). For example, the travel time from stop 1 to intersection 1 follows distribution of mean value 18 seconds and STD 9.47 seconds. Data in the fourth column represents passenger arriving rate at each stop. For instance, the passenger arriving rate at stop 1 is 0.045 person/second. Data in the fifth and sixth column is signal timing information. For example, the cycle length of the first intersection is 187 seconds, and the phase length for bus movement is 63 seconds.

Table 6: Data Inputs for Case Study.

(a)

Fleet Size	Bus Capacity	Layover Time	Passenger Boarding Rate
13 veh	80 prs/veh	40 min	1 s/prs

(b)

Node	Average Travel Time (s)	STD of Travel Time (s)	Passenger Arrival Rate (prs/s)	Green Time (s)	Cycle Length (s)
Stop 1			0.045		
Int 1	18 *(from Stop 1 to Int 1)*	9.47		63	187
Stop 2	19	6.29	0.059		
Int 2	56	26.62		63	179
Int 3	39	17.99		33	192
Stop 3	14	4.14	0.056		
Int 4	27	13.92		80	186
Stop 4	17	6.74	0.029		
Int 5	61	24.83		85	120
Stop 5	5	1.15	0.038		
Int 6	19	4.21		136	182
Int 7	26	6.01		48	182
Stop 6	12	3.88	0.024		
Int 8	69	25.52		36	192
Stop 7	19	7.18	0.021		
Int 9	53	23.88		88	194
Int 10	40	6.49		105	194
Stop 8	7	2.04	0.050		
Int 11	50	10.76		94	194
Stop 9	21	6.24	0.081		
Int 12	92	35.08		70	194
Stop 10	12	2.81	0.063		
Int 13	82	28.19		84	192
Int 14	37	12.58		81	182
Int 15	32	11.29		136	182
Stop 11	11	4.58	0.042		
Int 16	30	16.61		85	120
Int 17	41	13.29		48	186
Stop 12	15	4.42	0.113		
Int 18	55	20.28		68	192
Int 19	45	10.21		63	179
Stop 13	13	4.02	0.065		
Int 20	48	20.14		63	187
Stop 14 (Terminal)	12	3.22	0		

Note: Int stands for Intersection.

Based on smartcard data analysis, passengers average travel distance is approximately 3 bus stops. Therefore, we set passenger alighting rate as [0.1, 0.15, 0.5, 0.15, 0.1], where the vector indicates the percentage of passengers of different travel distance. For example, 10% passengers' travel distance is 1 bus stop; 15% passenger travel distance is 2 stops. In addition, when bus arrives at terminal, all boarding passengers have to alight. Therefore, all passengers who board at Stop 13 have to alight at the terminal.

Data Availability

The data used to support the findings of this study are included within the article.

Disclosure

Permission has been obtained for use of copyrighted material from other sources.

Conflicts of Interest

The authors declare that the received fund did not lead to any conflicts of interest regarding the publication of this paper.

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