A METHODOLOGY FOR INCLUDING SUSPENSION DYNAMICS IN A SIMPLE CONTEXT OF RAIL VEHICLE SIMULATIONS

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Abstract. The running behaviour of rail vehicles is highly influenced by suspension components. Dealing with ride comfort, secondary suspensions are adopted to reduce the vibrations transmitted to the carbody. In this context, the dynamics of the suspension elements themselves has to be properly included in multibody system. This paper proposes a strategy for modelling the passive vertical secondary suspension in the frequency domain. To this aim, a mathematical model is defined and its parameters are tuned to be representative of a real system. Then, a sensitivity analysis over the model parameters is proposed to discuss the suspension performances in terms of dynamic stiffness. Finally, a finite element model of the carbody is considered and coupled to the rear and front suspensions. The model is adopted to simulate the vehicle running on a rail track irregularity in the frequency domain, in the 0-30 Hz frequency range.

1 INTRODUCTION

To guarantee a high level of ride comfort, passenger trains are equipped with two stages of suspension systems. In this respect, the secondary suspension stage plays a major role and it is traditionally designed as a parallel of a coil spring and a hydraulic damper. Mathematical models of the secondary suspensions are typically adopted to investigate the vibration transmissibility and to improve passenger comfort [1].

When it comes to designing a suspension system, various types of solutions have been investigated. At first, passive suspensions have been widely adopted and still nowadays represent a valuable solution. Examples of attempts to identify the damping of the secondary suspension system to isolate the vibrations transmitted to the carbody have been provided in [2, 3].
To account for the operating conditions of an in-service vehicle, the possibility to adopt active suspensions has been considered. The attention has been focused on the design and control of the systems, that allow to make the suspension performance vary along with running dynamics [4, 5, 6]. Air spring represents the most successful example of these technology, that are nowadays more and more spread across the railway industry [7]. Finally, as a compromise between energy consumption and solution effectiveness, semi-active suspension systems have been proposed [8, 9].

Other than the suspension systems, carbody flexibility represents a key parameter that affects ride comfort [10, 11, 12]. For instance, vertical accelerations measured on the carbody floor along a high-speed line clearly highlight relevant excitation phenomena in the 7-15 Hz frequency range, which is associated with the first carbody flexible modes [13].

In this paper, a methodology to include the dynamics of the secondary suspension system at the simulation stage is proposed. The system is modelled in the frequency domain and a sensitivity analysis over the system parameters is carried out to assess its performances. Secondly, a finite element model of the carbody is considered to account for its dynamic contribution. Finally, the two subsystems are integrated into a simplified in-plane vehicle model running on an irregular track. Simulations are performed to assess the influence of carbody vibration modes and suspension dynamics.

2 MODELLING OF THE SECONDARY SUSPENSION SYSTEM

To design the secondary suspension system, reference is made to a specific railway vehicle. Each bogie is equipped with two dampers and four coil springs acting in parallel, that result in a device with a static stiffness value of $1.1 \times 10^6$ N/m. In order to reproduce the vertical dynamics of a secondary suspension in the frequency range of 0-30 Hz, the lumped parameter models shown in Figure 1 are adopted, where a) and b) respectively represent a single hydraulic damper and a single coil spring subsystem that make up the suspension.

![Figure 1: Subsystems composing the model of the vertical secondary suspension. a) Hydraulic damper, b) coil spring.](https://www.sciencedirect.com/science/article/pii/S0001457515304138)

The strategy adopted to model the dynamics of both spring and damper elements is addressed here. On the one hand, the hydraulic damper is modelled in the form of an ideal viscous damper, where the springs $k_b$ represent the stiffness of the mountings. On the other hand, the coil spring is modelled as two pairs of ideal spring-dashpot, together with a mass $m_s$ included and tuned to reproduce the first mode of vibration of the subsystem (38 Hz). The described modelling choice was driven by the typical arrangement of these components, and is based on available information and experimental data. The top ends of both
the hydraulic damper and the coil spring represent the rigid connections to the carbody ($z_c$), whereas the lower ends are connected to the bogie frame ($z_b$). A brief description of the symbols of the mechanical parameters is provided in Table 1 and 2.

In this paper, major attention is paid to the effect of the damper characteristics over the dynamic response of the secondary suspension system. Therefore, a sensitivity analysis over the $k_b$ stiffness of the mount bushing is first addressed in Figure 2a). The dynamic stiffness of the single damper element is shown in the 0-30 Hz frequency range. The simulation results show that reducing the stiffness of the bushings leads to the reduction of the overall dynamic stiffness of the component.

![Figure 2: Sensitivity analysis over the stiffness of the mount bushing $k_b$ of the hydraulic damper. a) Dynamic stiffness of a single damper element; b) dynamic stiffness of the whole secondary suspension (per bogie).](image)

Figure 2: Sensitivity analysis over the stiffness of the mount bushing $k_b$ of the hydraulic damper. a) Dynamic stiffness of a single damper element; b) dynamic stiffness of the whole secondary suspension (per bogie).

However, the suspension system installed on each bogie of the considered vehicle is composed by two dampers and four coil springs in parallel. Therefore, the full suspension model was realized to evaluate its dynamic response, as shown in Figure 2b) according to the same sensitivity analysis for different values of the $k_b$ parameter. All results are achieved considering constant properties of the coil spring and a fixed end of the suspension.

**Table 1**: Nominal parameters of a single hydraulic damper of the secondary suspension (Figure 1a).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$ Damping coefficient of the viscous damper</td>
<td>$3.8 \cdot 10^4$ Ns/m</td>
</tr>
<tr>
<td>$k_b$ Stiffness of the damper’s bushings</td>
<td>$9.5 \cdot 10^6$ N/m</td>
</tr>
</tbody>
</table>

**Table 2**: Nominal parameters of a single coil spring of the secondary suspension (Figure 1b).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_s$ Stiffness of the coil spring</td>
<td>$5.5 \cdot 10^5$ N/m</td>
</tr>
<tr>
<td>$c_s$ Damping coefficient of the coil spring</td>
<td>$1.0 \cdot 10^2$ Ns/m</td>
</tr>
<tr>
<td>$m_s$ Equivalent mass of the coil spring</td>
<td>$19.4$ kg</td>
</tr>
</tbody>
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In order to better clarify the importance of the design parameters over the whole railway vehicle system dynamics, in the next section a carbody flexible model is introduced and later coupled to the secondary suspension system to carry out simulations in the frequency domain.

3 RAILWAY VEHICLE MODEL

Previous research works showed that ride comfort of railway vehicles can be strongly influenced by resonance excitation of the carbody’s flexible modes [10]. Therefore, a carbody Finite Element Model (FEM) was considered to properly account for the dynamics of the component. Out of several vibration modes populating the 0-30 Hz frequency range of interest, only those contributing to the vertical response of the system have been considered. Thus, modeshapes driven by the flexibility of the tube have been considered (first bending mode). In addition, attention has been paid to the local deformation of the vehicle floor, including in the analysis the vibration modes presenting one single antinode along the lateral direction and $n$ antinodes along the longitudinal coordinate, with wavelengths decreasing from 16 to 3 m.

Once the carbody has been considered, the numerical model of the complete railway vehicle has been realized. As shown in Figure 3, the carbody model is connected to the rear and front bogies (not modelled and reproduced as simple points to follow the track irregularity profile) via the secondary suspension model described in Section 2.

![Figure 3](https://www.scipedia.com)

**Figure 3:** In plane model of the railway vehicle, composed of the carbody and two pairs of secondary suspension systems.

Considering the whole system dynamics, the modal coordinates of the carbody must be coupled with the dynamics of the suspension system. To this end, the equations of motion of the system have been written considering all the degrees of freedom (dofs) simultaneously, for a total of 20 dofs. Notice that to evaluate the displacements of the bogie frame connections, whose expression is required to define the mutual interaction between the suspension system and the carbody, the amplitude of the $i^{th}$ mode shape can be numerically computed. Moreover, the set of dofs can be split into two subsets: the free dofs and the constrained ones, whose amplitude is set to be equal to the track irregularity profile. As this is a random process, track irregularity is described by means of a Power Spectral Density (PSD) as specified by the reference standard [14]. It is worth mentioning that the vertical displacement of the rear bogie $z_{b,r}$ is the same as that of the front bogie $z_{b,f}$ with a time delay related to the pivot pitch $d$ and the train speed
Thus, the imposed displacements exciting the system can be written in the frequency domain as follows:

\[ z_{b,r}(\omega) = z_{b,f}(\omega) \cdot e^{-j\omega T} \]  (1)

Once the full railway vehicle model has been realized, simulations of the dynamic response of the vehicle have been carried out. The results of the numerical simulations are presented in the next section.

4 SIMULATION RESULTS

The rail vehicle model was considered for carrying out running dynamics simulations in the frequency domain. In Figure 4, the frequency response of the carbody centre and the bogie frame connections of the vehicle is computed as a function of the track irregularity. The results are shown in terms of PSD for a vehicle running at a speed of 300 km/h, restricting the analysis to the 0-30 Hz frequency range. Different suspension configurations were considered in accordance with the sensitivity analysis presented in Section 2 to provide a critical assessment of the dynamic behaviour of the coupled system, accounting for the flexibility of the carbody and the dynamic response of the suspension to track irregularity excitation.

At first, the discussion considers the sensitivity analysis over the \( c \) parameter, referring to Figure 4a-c). In the considered frequency range, several resonance excitations can be identified, related to the carbody bounce and pitch (nearby 1 Hz) and various vibration modes of the carbody floor, that all result to be excited by the track irregularity. Decreasing the \( c \) parameter, a worsening effect is observed in the low frequency range due to the amplification of the rigid motion contribution. Specifically, this effect is amplified in correspondence of the front and rear suspensions connections (Figure 4a,c)) with respect to the carbody centre (Figure 4b)).

Considering the 5-30 Hz frequency range, a beneficial effect is instead observed when reducing the damping contribution: reducing the force transmitted to the carbody floor, its vibration modes tend to contribute less to the overall system response. These results demonstrate that the reduction of the \( c \) damping coefficient alone is not adequate to optimize the dynamic behaviour of the vehicle, due to the amplification of low frequency motions that would be amplified, with consequent worsening of passenger perception.

If attention is paid to the effect of the \( k_b \) parameter, that is the stiffness of the mount bushings, Figure 4d-e) demonstrate its low influence to the rigid motions, while significant benefits can be observed in the 20-30 Hz frequency range, independently on the measuring position. These results prove that reducing the bushing stiffness does not compromise the damping effectiveness at low frequency (rigid carbody modes). On the other hand, reducing the transmissibility at higher frequency leads to lower carbody acceleration. In the end, the reduction of the dynamic stiffness of the whole secondary suspension to be an effective strategy towards the mitigation of carbody floor vibration, that is an enhancement of ride comfort.
Figure 4: Frequency domain simulation: PSD of the acceleration measured at V=300 km/h on the floor of the carbody. Sensitivity analysis considering the \( c \) parameter: a) carbody floor in correspondence of the front suspension, b) carbody centre, c) carbody floor in correspondence of the rear suspension. Sensitivity analysis considering the \( k_b \) parameter: d) carbody floor in correspondence of the front suspension, e) carbody centre, f) carbody floor in correspondence of the rear suspension.

As a last step of the analysis, the effect of the dynamics of the coil spring subsystem is considered. Up to now, the spring first natural frequency was set to be outside of the frequency range of interest. However, the typical arrangement of the component is such that it is likely to show resonance excitation in the 20-30 Hz frequency range. Therefore, the coil spring parameters (\( m_s \) and \( k_s \), see Table 2) have
been tuned so as to set the first natural frequency at 27 Hz. The effect of the modification is presented in terms of dynamic stiffness of the whole secondary suspension in Figure 5. A strong amplification of the dynamic stiffness is observed in the 25-30 Hz frequency range (dashed line) if compared to the standard suspension arrangement previously considered (solid line, with natural frequency set to 38 Hz), driven by the resonance excitation of the coil spring subsystem.

**Figure 5:** Effect of the resonance of the coil spring on the dynamic stiffness of the whole secondary suspension (per bogie).

The new secondary suspension configuration is considered to perform vehicle dynamics simulations at the same constant speed. For the sake of brevity, only acceleration in correspondence of the front suspension connection is considered in Figure 6.

**Figure 6:** Effect of the resonance of the coil spring over the vehicle response: carbody front acceleration a) without the contribution of the spring resonance in the considered frequency range; b) with spring resonance at 27 Hz.
In Figure 6a) the very same results of Figure 4d) are reported, while Figure 6b) shows the effect of the coil spring resonance on the vehicle acceleration response. As expected, no differences are observed below 20 Hz, while a significant amplification of the vehicle response is registered nearby the spring resonance (22-28 Hz).

Interestingly, the sensitivity analysis demonstrates that even worse results would be achieved in case soft bushings are installed, being the blue curve significantly higher than the others in the 26-28 Hz frequency range. Due to the transmissibility increase caused by the coil spring resonance (see Figure 5), in this frequency range a higher damping contribution provides a beneficial effect, being the system response damping controlled. However, outside this narrow bandwidth lower damping contributions would be preferable. Thus, a trade-off between these two effects is required to chose the most suitable model parameters, considering the overall secondary suspension system (parallel of hydraulic dampers and coils springs).

To conclude, the design of the secondary suspension system proves to strongly affect the response of the vehicle in running operations. Therefore, the choice of model parameters plays a major role. In this respect, the designed model may provide useful guidelines to include the dynamics of the secondary suspensions at the simulation stage, allowing to investigate the system response for different combinations of model parameters.

5 CONCLUSIONS

In this paper, a strategy to model the dynamics of the secondary suspensions system is proposed. The performances of the whole suspensions system has been evaluated in the frequency domain in terms of dynamic stiffness, performing a sensitivity analysis over the damping and stiffness of the mount bushings of the damper component, for fixed coil spring characteristics. The simulation results show a significant effect of the damping coefficient $c$, whose value can be optimized to achieve enhanced vibration transmission.

To deepen the discussion, a FEM model of the carbody was considered to evaluate the typical carbody response, considering the vibration modes that affect the dynamic response of the system in the 0-30 Hz frequency range. Considering the peak values of the PSD of the acceleration measured on the carbody floor at its centre and in correspondence of the suspensions connection were considered, for different damper parameters.

Simulations prove that reducing the bushing stiffness does not compromise the damping effectiveness at low frequency, providing benefits at higher frequency due to transmissibility reduction. Moreover the dynamic amplification resulting from the resonance excitation of the coil spring subsystem has been observed.

REFERENCES


