XVI International Conference on Computational Plasticity. Fundamentals and Applications
COMPLAS 2021
E. Oñate, D. Peric, M. Chiumenti and Eduardo de Souza Neto (Eds)

SOME COMPUTATIONAL ISSUES IN THE ELASTO-PLASTIC MODELLING OF SNOW

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Key words: Snow mechanics, Snow models, Elasto-plasticity, Computational methods.

Abstract. Snow is a high porosity and multi-phase material whose behaviour is strongly influenced by peculiar features, such as time-driven metamorphisms of its microstructure and sintering processes between ice grains. Furthermore, from a mechanical point of view, the response of snow to external actions (anthropic, atmospheric, etc.) is characterized by both material and geometric non-linearities. This latter item is also accompanied by a clear strain-rate dependence induced by volumetric viscous effects acting on the ice microstructure.

These factors influence the choice of the best constitutive model capable of reproducing this complex behaviour especially in the framework of Finite Element (FE) analyses. In the scientific literature, many authors suggested the use of elasto-plastic constitutive models, in many cases derived from soil mechanics applications, to perform reliable FE analyses of snow behaviour with reference to both laboratory and on-site experiments. Nevertheless, the available models often show some issues both in their initial hypotheses and in the following FE implementations. For instance, problems may arise in: i) choosing the proper deformation field (e.g., small or large deformations), ii) selecting the most appropriate shape of the yield function, iii) defining the hardening and plastic-flow rules, etc.

In this work, some of the still open and unsolved questions related to the constitutive modelling of snow and suggestions on possible computational solutions through FE tools are highlighted. The goal is twofold: first, we try to summarize the current state-of-the-art of constitutive modelling and FE analysis on snow; and second, we suggest some possible research directions and computational solutions to improve the existing mechanical models.

1 INTRODUCTION

Within the wide set of natural materials, snow represents a peculiar case. It is indeed a complex, multi-phase and high-porosity medium whose mechanical behaviour at the macroscopic scale of observation is strongly influenced by the peculiar characteristics of its microstructure [1]. Moreover, snow can be considered as a “hot material” because, in many terrestrial environments, exists at temperatures that are very close to its melting point. Therefore, as soon as snowflakes fall on the ground, they are first subjected to mechanical-induced shape modifications and then to thermal-induced changes (i.e., snow metamorphisms) that transform the single sharpened precipitation particles in more complicated interconnected structures [2]. The physical and mechanical properties of metamorphosed snow are thus determined by the shape of the crystals (or grains) and the connections between them (sintering bonds). All these issues, coupled with the fact that layers of snow with different properties are usually present in a snowpack, are at the basis of the comprehension of the mechanical response of snow to external actions and should be taken into account in several different application fields. Snow, indeed, is a natural resource which is essential for the alpine ecosystem as well as for winter tourism and sports, but on the other hand is a critical element to be considered in the design of structures and infrastructures in alpine and cold environments. Moreover, snow avalanches represent one of the main hazards in mountain areas, that can potentially endanger human lives, economical activities, structures, and also historical and natural heritages [3].

To investigate the multifaceted aspects of the mechanical behaviour of snow and to deal with such a wide range of engineering applications, a more and more crucial role was assumed by constitutive relationships and computational methods. Generally, snow constitutive models can be divided in two groups, namely phenomenological and micro-mechanical models [4]. Those in the first group reproduce the macroscopic behaviour of snow neglecting all the processes that occur at the microscopic level and are usually implemented in the field of Continuum Mechanics. Instead, the ones in the second group specifically take into account the role of microstructure (i.e., intergranular glide, inelastic bond deformations, etc.) in deformational processes and can be adopted both in the framework of Continuum and Discrete Mechanics.

Historically, the modelling of snow as a continuum medium is performed in the framework of pressure-dependent elasto-plastic (EP) constitutive laws, that allows to properly describe large inelastic deformations and the observed macroscopic behaviour with reference to shear stress and hydrostatic pressure [5]. Furthermore, this type of EP models (Modified Cam Clay, for instance) was already widely exploited in soil mechanics and, taking advantage of the numerous similarities between soil and snow, they were adapted to the case of snow. From a purely computational point of view, the Finite Element (FE) method was largely adopted in snow applications starting from the beginning of 1970s [6]. FE methods are generally used, under the hypothesis of infinitesimal strains, to reproduce laboratory tests and to calibrate constitutive models for snow [7] but can also be employed for on-site applications such avalanche release or ski-snowpack interaction [8]. Unfortunately, considerations of finite strain are necessary for the proper description of snow and the traditional mesh-based numerical methods (e.g., FE method) generally suffer from mesh-distortion issues associated with large deformation of snow. Alternative to FE methods are the continuum point-based methods, such as the Lagrangian Smoothed Particle Hydrodynamics (SPH) and the hybrid Eulerian-Lagrangian Material Point Method (MPM), that allow to solve field-scale applications involving large deformation and post-failure of geomaterials [9]. For instance, with the aim of studying the processes causing the release of snow slab avalanches, Gaume et al. (2018) [10] apply MPM and finite strain EP model to reproduce the onset and dynamic propagation of fracture in a weak interface layer, which generates between two stiffer snow slabs. Therefore, MPM and SPH, as well as their further developments, could be able to face more complex processes involving snow, such as heat flux or water flow developing into the snow microstructure, melting and re-crystallization, temperature-deformation relations, localization and bifurcation, etc.

In this work, the focus will be on Continuum Mechanics applications and FE analyses in particular. Here, we summarize the current state-of-the-art of the constitutive models specifically devoted to snow that are currently available in literature and, at the same time, we highlight some of the still unsolved questions in the EP theory applied to snow mechanics. To achieve this goal the paper is organized in three parts (Sect. 2 to 4) following the main ingredients of a classical EP model: elastic stress-strain law, yield criterion and hardening law and flow rule. A final section (Sect. 5) is devoted to the formulation of FE problems involving snow.

2 ELASTIC STRESS-STRAIN LAW

The vast majority of snow constitutive models currently available are based on the theory of small strains and on the additive decomposition of the strain rate tensor $\dot{\boldsymbol{\varepsilon}}$:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{e} + \dot{\boldsymbol{\varepsilon}}^{irr} \tag{1}$$

where $\dot{\boldsymbol{\varepsilon}}^{\mathrm{e}}$ and $\dot{\boldsymbol{\varepsilon}}^{irr}$ are the recoverable (elastic) and the irrecoverable (plastic) part of the strain rate tensor, respectively.

In the framework of small strain EP, the $\dot{\boldsymbol{\varepsilon}}^{\mathrm{e}}$ component is typically defined via the well-known elastic law of the Cam Clay type, wherefrom it is possible to obtain the volumetric ($\dot{\varepsilon}^{e}{}_{\mathrm{vol}}$) and deviatoric ($\dot{\varepsilon}^{e}{}_{\mathrm{dev}}$) components as follows [7]:

$$\dot{\varepsilon}^{e}{}_{\mathrm{vol}} = -\frac{k}{v}\frac{\dot{p}}{p} \tag{2$_\mathrm{i}$}$$

$$\dot{\varepsilon}^{e}{}_{\mathrm{dev}} = \frac{1}{3G}\dot{q} \tag{2$_\mathrm{ii}$}$$

where k is a dimensionless material constant (related to the bulk modulus), v is the specific volume, G is the shear modulus, and p and q are two stress invariants named isotropic stress and equivalent shear stress, respectively. This formulation highlights the intimate relation existing between the currently available snow mechanical models and the constitutive laws originally developed for soil mechanics purposes. In fact, most of the currently used models for snow derives from models originally developed for soils, conveniently adapted. However, Eqn. (2_i) shows some problems in case of finite strain models, due to the inverse proportionality existing between the specific volume and the compressibility [11].

The irreversible component is classically linked to the viscous behaviour of snow microstructure and has a paramount relevance in the whole amount of deformation experienced by snow under load. Some authors quantified the irreversible viscous deformation of snow starting from creep, relaxation and triaxial compression tests [12], and implemented linear [1,13] or non-linear [14] rheological models by combining elastic springs and viscous dashpots.

Other types of snow constitutive models are extended to finite strains in order to better describe the large inelastic deformations suffered by snow under the action of shear and hydrostatic pressures. In these cases, the usual approach is to adopt a multiplicative decomposition of the deformation gradient tensor $\mathbf{F}$ into elastic ($\mathbf{F}^{e}$) and plastic ($\mathbf{F}^{p}$) parts:

$$\mathbf{F} = \mathbf{F}^{e}\mathbf{F}^{p} \tag{3}$$

Then, the elastic stress-strain law can be derived in different ways. For instance, following an hyperelastic approach, the Kirchhoff stress tensor $\boldsymbol{\tau}$ can be obtained from the strain energy density function ψ with the following general relation [15]:

$$\boldsymbol{\tau} = 2\frac{\partial\psi(\mathbf{F}^e,\boldsymbol{\alpha})}{\partial\mathbf{b}^e}\mathbf{b}^e \tag{4}$$

where $\mathbf{b}^e = \mathbf{F}^e\mathbf{F}^{eT}$ is the left Cauchy-Green strain tensor and $\boldsymbol{\alpha}$ is a vector of (strain-like) internal material variables. Other types of elastic constitutive laws can be used. Gaume et al. (2018) [10], for example, adopt the Hencky strain tensor ($\boldsymbol{\epsilon} = \frac{1}{2}\log\mathbf{b}^e$) that provides a convenient description of elastic deformation of snow. By referring to the principal components of Kirchhoff stress and Hencky strain tensors (i.e., $\hat{\boldsymbol{\tau}}$ and $\hat{\boldsymbol{\epsilon}}$ vectors, respectively), they propose the following constitutive relation:

$$\hat{\boldsymbol{\tau}} = \mathbf{C}\hat{\boldsymbol{\epsilon}}. \tag{5}$$

The tensor **C** is a matrix given by $\mathbf{C} = 2\mu\mathbf{I} + \lambda\mathbf{1}\mathbf{1}^{\boldsymbol{T}}$, where **I** is the identity tensor, **1** is the all ones vector, and μ and λ are the Lamé parameters.

3 YIELD CRITERION AND HARDENING LAW

The choice of yield criteria for snow is a crucial point in the constitutive modelling of this natural material. Depending on the application field and on the purpose of the study, different choices can be done. Generally, with reference to volumetric and tangential load conditions, the pressure dependency of snow leads to pressure-dependent EP constitutive laws with strain hardening. Moreover, many authors suggested that snow modelling could be faced in a way similar to soils. This is due to some patterns of behaviour that snow and soils have in common, such as [16]: i) immediate response to external action, ii) linear response in the semi-logarithmic plane volume-volumetric strain, iii) increasing volumetric stiffness with density, iv) dilation by shearing and creep deformation, and v) wide inelastic deformation. For these reasons, Lang and Harrison (1995) [16] suggested to apply to snow the critical state theory conceived for soils. In 1996, Meschke et al. [5] introduced a large strains EP model similar to Modified Cam Clay [17] with a yield convex surface, symmetric around the hydrostatic axis and properly adapted for snow (Figure 1a). This approach had a remarkable success and in the following decades was implemented in many scientific works. As a brief overlook, some examples are given below.

Meschke (1995) [18] proposed a visco-EP model characterized by two coexisting yield functions (Figure 1b), i.e., f_t and f_c, that define the tension cut-off and the closed smooth shape of the surface, respectively. With reference to I_1 and J_2 Kirchhoff stress tensor invariants, the yielding criteria can be expressed with the following relationships:

$$f_t(I_1, q_t) = \frac{I_1}{3} - q_t \leq 0, \tag{6$_\text{i}$}$$

$$f_c(I_1, J_2, q_c) = \sqrt{J_2 - \frac{c_c}{q_c}(\bar{I}_1 + q_c)^4} - k_c\bar{I}_1 - \sqrt{c_c q_c^{\frac{3}{2}}} \leq 0 \tag{6$_\text{ii}$}$$

where $\bar{I}_1 = I_1 - 3t$, being t the intercept of the yield surface with the p-axis on the traction side, and c_c and k_c are two material parameters. q_t and q_c, are two parameters describing the hardening/softening mechanisms of snow. Experimental findings show that the tensile strength is reduced when inelastic volumetric tensile strains are accumulated (tensile volumetric softening), thus q_t shifts the tension cut-off towards the origin. On the contrary, q_c describes the evolution of the shape of the yield surface in the meridian plane. According to isotropic compression and direct shear test data, the shape of f_c changes continuously in the stress space, passing from the closed smooth shape with compression cap and tension cut-off to a shape similar to the Drucker-Prager failure criterion as ultimate condition.

Other models followed the path traced by Meschke, also including some other snow features. For instance, Cresseri et al. (2010) [7] introduced a visco-EP model based on Modified Cam Clay with some modifications in order to account for the effects of sintering (i.e., the formation of bonds between snow particles), both in compression and in tension. Three internal material variables were taken into account: p_0, p_m, and p_t. The first one (p_0) is linked to snow density and represents the intersection of the yield surface with the hydrostatic axis in case of non-sintered state. Its evolution is associated with the amount of volumetric plastic deformation, which reflects the macroscopic effects of an irrecoverable change of snow density. The two further parameters (p_m and p_t) represent the additional strength, in compression and tension, respectively, conferred by intergranular bonding to the material. The variation of p_m in time is supposed to follow a linear dependence on the degree of bonding. p_m and p_t are proportional to each other: $p_t = \chi p_m$, with $\chi = 0.1$. A representation of the Cresseri et al. (2010) yield surface can be observed in Figure 2a.

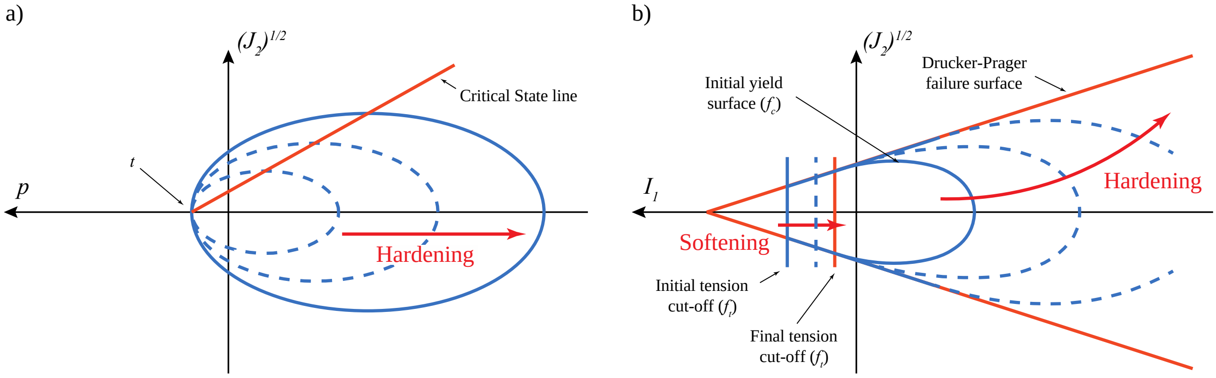

Figure 1: a) Sketch of the meridian section of the Meschke et al. (1996) yield criterion at different values of the hardening parameters (adapted from [5]). Blue dashed lines show the progressive hardening of the surface; b) Sketch of the meridian section of the Meschke (1995) yield criterion. Blue solid lines highlight the initial yield surface and tension cut-off, while blue dashed lines and solid orange ones represent the ongoing evolution of the surface and the ultimate conditions, respectively (adapted from [18]).

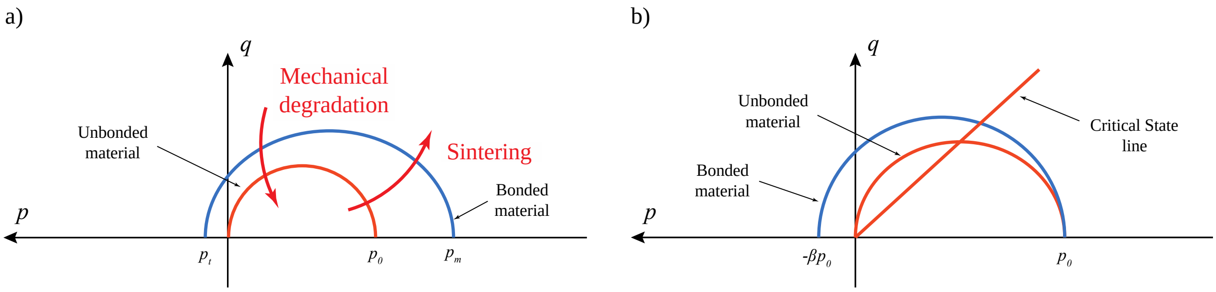

Figure 2: a) Sketch of the meridian section of the Cresseri et al. (2010) yield criterion. Orange line shows the surface for the unbonded material while the blue one is that for the bonded snow (adapted from [7]); b) Sketch of the meridian section of the Gaume et al. (2018) yield criterion. Bonded (blue line) and unbonded (orange line) Modified Cam Clay yield surfaces are reported in the *p*-*q* plane snow (adapted from [10]).

Finally, Gaume et al. (2018) [10] proposed a new cohesive Cam Clay model with the following ellipsoidal and symmetric yield function in p-q plane (Figure 2b):

$$y(p,q) = q^2(1+2\beta) + M^2(p+\beta p_0)(p-p_0) \tag{7}$$

where p_0 is the consolidation pressure, M is the slope of the cohesionless critical state line, and $\beta \geq 0$ is the ratio between the tensile and compressive strength that controls the amount of bonding between grain (cohesion). The hardening/softening behaviour of snow is introduced through the variation of the p_0 parameter, that follows a hyperbolic relationship depending on the volumetric plastic deformation. When the plastic deformation is compressive, p_0 increases and the yield surface expands. Otherwise, in case of tensile deformation, p_0 decreases and yield surface shrinks, allowing the snow to fracture in tension. This model allows to reproduce the collapse of snow under compression (anticrack) and the onset and propagation of fracture in weak snow layers.

From these simple examples, the need for a variable, shrinkable and expandable yield surface for snow is clear. Different responses in compression and tension have to be considered in snow modelling and the role of hardening due to both volumetric plastic deformation and bonding assumes a crucial and not negligible role.

4 FLOW RULE

In the framework of snow constitutive models, flow rules generally follow the same laws governing the evolution of plastic deformation in case of other materials with EP behaviour. Simple and classical associative flow rules were adopted by some authors [5,10], while non-associative flow rule is used, for instance, by Cresseri et al. (2010) [7]. This latter example is interesting because the plastic deformation is supposed to be linked to the viscous properties of snow through the following relation:

$$\dot{\boldsymbol{\varepsilon}}^{irr} = \bar{\psi}\phi(f)\frac{\partial g}{\partial \boldsymbol{\sigma}} = \psi\frac{\sqrt{q^2+p^2}}{\sqrt{3}p_0}e^{\alpha f}\frac{\partial g}{\partial \boldsymbol{\sigma}}\frac{1}{|\boldsymbol{\nabla} g|} \tag{8}$$

where g is the plastic potential, $\bar{\psi}$ is the so-called fluidity parameter and $\phi(f)$ is the viscous nucleus. The $\bar{\psi}$ parameter defines the rate at which irrecoverable strains take place and depends on the actual stress level, the viscous coefficient ψ, and the density of snow (through p_0). The viscous nucleus is strictly positive ($0 < \alpha f \leq 50$) and rules the dependence of the amount of deformation on a measure of the distance between the stress state and the yield surface f.

5 FINITE ELEMENT FORMULATION

Since snow is a highly non-linear, porous and visco-EP material, an analytical solution to the partial differential equations (PDEs) governing the stress-strain constitutive relation is not available. Therefore, the FE method was historically identified as the simplest tool to compute the approximate solution of the set of constitutive PDEs. Through the discretization of a given continuous domain into a finite number of sub-domains, or elements, the FE method describes the mechanical behavior of these elements and, therefore, of the entire system.

The implementation of FE methods for snow mechanics purposes began in early 1970s and rapidly increased. In 2013, Podolskiy et al. [6] summarized the application of FE analysis in snow mechanics. They identified nine major categories of physical and engineering problems in snow mechanics that has been studied with FE method: i) on-site state of strain and stress in snowpack, ii) influence of snow weak layer on the mechanical state of snowpack and slab avalanche release, iii) fracture propagation in snow, iv) skier loadings on inclined snowpack, v) shock loading and explosive loading on snowpack, vi) reproduction of mechanical experiments for validating snow mechanical models, vii) assessment of actions exerted by snow cover on avalanche defense structures, viii) interaction between tires and snow, and ix) microstructure studies of snow volume obtained from X-ray microtomography.

Despite this wide number of applications, FE models of snow present a certain number of issues, especially for slope stability and avalanche purposes. For instance, Cresseri et al. (2010) [7] implemented their small-strain visco-EP 14-parameter model in the FE code ABAQUS [19]. During the implementation of Eqn. (2_i) to the case of isotropic compression, they observed that the hydrostatic pressure increment (Δp) calculated by the effect of a prescribed volumetric strain increment ($\Delta\dot{\varepsilon}^{e}{}_{\mathrm{vol}}$) had an incorrect sign. To avoid this problem, i.e. ensuring that the predicted Δp has the same sign as the imposed $\Delta\dot{\varepsilon}^{e}{}_{\mathrm{vol}}$, the authors introduced a restriction on the magnitude of the strain increments. This goal could be achieved through two possible strategies: i) sub-incrementation, by which the current strain increment is divided into smaller intervals, or ii) time step cutting, by which the current time step is reduced. This example highlights one of the most important issues in FE analysis of snow, that is linked to the proper choice of the deformation or time increment.

Another type of problems involving FE analyses in snow is related to the proper mechanical description of the weak interface layer that can be potentially buried between two stiff snow slabs. Typically, this part of the snowpack is the preferential zone in which fracture develops. In 2006, Stoffel [8], following Bader et al. (1989) [13], suggested to introduce a special finite element to model the crack in a snowpack. This element, called weak layer element (WLE), has special features. By referring to a weak layer parallel to the slope surface, the WLE has an infinite stiffness in the direction transversal to the slope, thus any forces acting in this direction do not deform the element and are simply transmitted through the element. Otherwise, along the direction parallel to the slope the WLE has lower stiffness and viscosity, and the slide can occur.

Another solution for the modelling of weak layer in FE methods is to adopt a fracture mechanics approach. From this point of view, the snow layer can be seen as a component with a crack. Under certain hypotheses (i.e., limited size of the plastic zone close to the crack spike), the general and computationally intensive theory of elastic plastic fracture mechanics (EPFM) can be simplified with the linear elastic fracture mechanics (LEFM). This method allows to simulate the evolution of the crack within a snowpack in very simple conditions. More complex situations need for more detailed numerical methods, even different from the FE method (e.g. MPM, SPH, etc.) (Gaume et al. 2018) [10].

6 DISCUSSION AND CONCLUSIONS

In this work a brief overview of EP models for snow is presented with particular attention to constitutive modelling and FE analysis. Some literature works are briefly introduced in order to summarize the current state-of-the-art with reference to the topic. Starting from the analysis of the available literature works, and to the best of our knowledge, some conclusions can be drawn:

- EP models allow to reproduce some of the main features of the mechanical behaviour of snow, such as wide inelastic deformations, hardening and softening, etc.
- In the framework of continuum EP models for snow, the modified Cam Clay, originally developed for soil mechanics applications, was largely used as a starting point for deriving snow-specific constitutive models, usually under the assumption of isotropy. The modified Cam Clay allows to account for a closed, smooth and convex yield surface with finite extent of the elastic range both in compression and in tension. Moreover, this surface is able to considerably shrink and expand to reproduce the gain in cohesion due to sintering or to the increasing compressive stiffness.
- Generally, the available snow constitutive models do not consider several micro-structural parameters that in reality strongly influence the snow response, such as the change in shape of snow crystals (due to snow metamorphism).
- Infinitesimal strain theory was typically introduced in FE codes to validate the reliability of constitutive relationships with reference to laboratory tests, even though it shows some limits in describing the typically large deformation of snow in natural conditions.
- Finite strain theory was implemented since long time for snow, both for validating theoretical models and for on-site applications. Referring to the latter topic, constitutive models implemented with large deformation are able to properly reproduce, for instance, the growth of fracture within weak interface layers and the avalanche onset.

Despite the considerable developments of both theoretical and numerical models in recent decades, further improvements in snow constitutive models are required in order to account for many other issues, that are in part highlighted in this work. The research should be pointed to the following key points:

- A new class of EP constitutive models based on large deformation theory and multiplicative decomposition of the deformation gradient are needed.
- These models should be capable of accounting for: i) the hardening/softening behaviour of snow linked to the sintering process (formation, growth and breakage of bonds between grains), ii) the viscous effects of snow ice skeleton, iii) the implementation of some micro-structural parameters that can describe the variation in time of the snow particles and their effects on the mechanical response of snow.
- The yield surfaces should be able to vary their shapes, to considerably shrink and stretch on both the meridian and deviatoric sections in order to follow the different stress paths observed in laboratory and on-site conditions.

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