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ROBUST DESIGN OPTIMIZATION UNDER UNCERTAIN STRUCTURAL PARAMETERS BY STOCHASTIC SIMULATION-BASED APPROACH

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Abstract. *The inherent uncertainty in the structural parameters directly affects the structural performance, and its variation may lead to improper designs and catastrophic consequences. When subjected to uncertainty, the structure design must be optimized to get an insensitive design using a Robust Design Optimization (RDO) technique. Such design aims to find a system design in which the structural performance is less sensitive (insensitive) to the uncertainty of the inherent structural parameter without eliminating them. This is usually achieved by simultaneously minimizing the mean and variance of the structural performance function. Various RDO approaches, such as those based on Taylor series expansion, simulation-based methods, dimension reduction, and metamodel, can effectively take into account these uncertainties. However, the computational efficiency and accuracy in evaluating the mean and variance of the performance function remain a challenging task. To obliviate this limitation, a novel stochastic simulation-based approach is proposed in the present work. The proposed approach is built on an 'Augmented optimization problem,' in which design variables are artificially considered an uncertain parameters. An unconstraint Genetic algorithm (GA)-based optimization approach is formulated to determine the optimal solution. As the mean and variance frequently conflict with each other, so to obtain the Pareto optimum, a linear scalarized objective function is adopted. To demonstrate the proposed approach, RDO of a four-bar structure is performed. The results obtained are compared with the conventional Monte Carlo simulation approach and confirm that the proposed approach yields accurate results. This paper allows the designers to design the insensitive structure systems by minimizing the variance of the performance function. Moreover, the proposed RDO approach is not only limited to the civil structures but can also be enforced in the design of any realistic linear/nonlinear structures and systems such as machine components (like clutches, gears, etc.), aerospace, etc., having uncertainties in their geometry or material, such as the residual strain, modulus, thickness, density, etc.*

1 INTRODUCTION

In any practical situation, there are various parameters that are unknown at the design stage and affect the performance of a system, such as loadings, structural parameters, geometric parameters, operation conditions, etc. [1–3]. These parameters are classified as uncertain parameters, and their uncertainty is quantified using a joint Probability Density Function (PDF) in a probabilistic framework. During the design process, it is essential to address the uncertainty in these design parameters. In a probabilistic framework, there are two methods for the optimal design of structures under uncertainties: Reliability-Based Design Optimization (RBDO) and Robust Design Optimization (RDO). In RBDO, statistical information of all the uncertain parameters is incorporated to optimize the system by satisfying an acceptable probability of failure [4]. In contrast, RDO studies a design that is less sensitive with respect to the variation of the initial parameters [5]. The mean and variance of the performance function are generally recommended in the literature as a measure of robustness [6]. The present paper focuses on the RDO.

The problem of RDO can be mathematically formulated as the determination of the optimal design $\mathbf{x}^*$ by performing the following optimization problem:

$$\mathbf{x}^* = \arg\min_{\mathbf{x}\in\Phi} \left\{ \mu_g(\mathbf{x}), \sigma_g^2(\mathbf{x}) \right\}, \tag{1}$$

where $\mu_g(\mathbf{x}) : R^{n_x} \to R$ and $\sigma_g^2(\mathbf{x}) : R^{n_x} \to R$ denotes the mean and variance of the performance function $g(\boldsymbol{\theta}, \mathbf{x}) : R^{n_\theta \times n_x} \to R$, respectively. $\mathrm{E}_\theta[\cdot]$ represents expectation with respect to PDF $p(\boldsymbol{\theta} \,|\, \mathbf{x})$ for $\boldsymbol{\theta}$. In the above optimization problem, the two design criteria, namely minimization of mean and minimization of variance, frequently conflict with each other [7], i.e., simultaneously achieving the minima of both is practically infeasible. In such a case, there exist several Pareto optimal solutions or Pareto fronts. The defining characteristic of Pareto fronts is that they cannot be improved in any objective without causing degradation in at least one other objective. In this view, various methods to transform the multiobjective problem into a single objective problem, such as the compromise programming method [8], physical programming method [9], and weighted sum method [10], are employed to deal with the trade-offs between conflicting objectives. In the present study, a linear scalarized performance function is adopted to find the Pareto optimum.

The initial methods to solve the RDO problem in structural engineering entail explicit calculation of these measures of robustness. The analytical calculation of these measures is only possible in a limited number of cases. Therefore, several approximation techniques such as those based on Taylor expansion of the objective and constraint functions, have been proposed. In these cases, the resulting optimization problem is a deterministic problem that can be solved using standard nonlinear programming techniques. The application of this class of methods can be found in [11–17]. Other methods for solving RDO include metamodel-based methods. Metamodels allow a mathematical approximation of the objective response. The application of these metamodel-based methods can be found in [18–20]. Furthermore, other methods, including direct search methods in the presence of uncertainty, such as stochastic approximation methods and stochastic quasi-gradients methods, are also widely adopted by the researchers. The application of these methods is limited due to the high computational requirements of evaluating the robustness measure. The application of these approaches can be found in [21, 22].

The approach presented in this paper is based on a direct search method proposed by [23], termed Stochastic Subset Simulation (SSO). SSO is an effective and efficient method for designing optimal stochastic systems through stochastic simulation. The method is based on an augmented formulation proposed by Au [24] to investigate reliability-based design sensitivity, wherein design variables are considered as uncertain variables with a predefined PDF over the design space. This paper adopts the augmented formulation to minimize the weighted function of a mean and variance. The proposed approach is validated by means of an example, including the RDO of a four-bar structure. The results attained by the proposed approach are validated by a Monte Carlo simulation-based optimization approach known as Sample Average Approximation (SAA).

2 PROBLEM FORMULATION

Consider a system with some adjustable parameters that define the system design, which are referred to as design parameters $\mathbf{x} = [x_1, \cdots, x_{n_x}]^T \in \Phi \subset R^{n_x}$ and where Φ represents the bounded admissible design space. Consider a PDF $p(\boldsymbol{\theta} \mid \mathbf{x})$ that specifies a collection of uncertain variables $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_{n_\theta}]^T \in \Theta \subset R^{n_\theta}$, where Θ denotes the set of possible values for the uncertain variables. It is assumed that $p(\boldsymbol{\theta} \mid \mathbf{x}) = p(\boldsymbol{\theta})$ without compromising generality. The resulting RDO is defined as the determination of:

$$\mathbf{x}^* = \arg\min_{\mathbf{x}\in\Phi} \left\{ \mu_g(\mathbf{x}), \sigma_g^2(\mathbf{x}) \right\}, \quad \text{subjected to} \quad \mathrm{f}_c(\mathbf{x}) \le 0, \tag{2}$$

where

$$\mu_g(\mathbf{x}) = \mathrm{E}_\theta \left[g(\boldsymbol{\theta}, \mathbf{x}) \right] = \int_\Theta g(\boldsymbol{\theta}, \mathbf{x}) p(\boldsymbol{\theta}) d\boldsymbol{\theta}, \tag{3}$$

and

$$\sigma_g^2(\mathbf{x}) = \mathrm{E}_\theta \left[\left(g(\boldsymbol{\theta}, \mathbf{x}) - \mu_g(\mathbf{x}) \right)^2 \right] = \int_\Theta \left(g(\boldsymbol{\theta}, \mathbf{x}) - \mu_g(\mathbf{x}) \right)^2 p(\boldsymbol{\theta}) d\boldsymbol{\theta}, \tag{4}$$

denote the mean and the variance of the structural performance function $g(\boldsymbol{\theta}, \mathbf{x}) : R^{n_\theta \times n_x} \to R$, respectively, and $\mathrm{E}_\theta[\cdot]$ denotes expectation with respect to the PDF for $\boldsymbol{\theta}$. $\mathbf{f}_c(\mathbf{x})$ correspond to a vector of constraints that can be deterministic or stochastic (like the objective function). Such optimization problems arising in decision-making under uncertainty are typically referred to as stochastic optimization. Dealing with a large uncertainty space is a major challenge in these problems, and it typically leads to a challenging evaluation of the multi-dimensional integral. Design constraints, which are also expressed as stochastic integrals, and/or integer design variables that model logical and other discrete design options can make optimization even more difficult.

To obtain a Pareto optimum, a widely used approach is adopted that substitute the vector of objective functions with a scalarized objective function. In this view, a straightforward scalarization approach is the linear combination method that constitutes a weighted linear combination of the individual objectives. The weighting factors can be adjusted to determine the relative weights assigned to the multiple objective functions, allowing the user to easily analyze the trade-offs between them. Therefore, the problem of RDO is formulated as:

$$\mathbf{x}^* = \arg\min_{\mathbf{x}\in\Phi} \left\{ \alpha \frac{\mu_g(\mathbf{x})}{\tilde{\mu}_g} + (1-\alpha)\frac{\sigma_g^2(\mathbf{x})}{\tilde{\sigma}_g^2} \right\}, \tag{5}$$

where $\tilde{\mu}_g$ and $\tilde{\sigma}_g^2$ are the normalization factors, and the weighting factor $\alpha \in [0,1]$ denotes the relative importance of the two objective functions. With $\alpha = 1$ and $\alpha = 0$, the problem can be transformed into a pure mean value minimization problem and pure variance minimization problem, respectively.

2.1 Augmented formulation for optimization

Consider a single objective stochastic optimization problem that is, minimization of the mean of the performance function. In this case, the performance function in the augmented formulation is given by:

$$h_1(\boldsymbol{\theta}, \mathbf{x}) = g(\boldsymbol{\theta}, \mathbf{x}). \tag{6}$$

Similarly, for minimization of the variance, the performance function in the augmented problem can be represented as:

$$h_2(\boldsymbol{\theta}, \mathbf{x}) = \left(g(\boldsymbol{\theta}, \mathbf{x}) - \mu_g(\mathbf{x}) \right)^2. \tag{7}$$

In this framework, for minimization of the weighted problem as presented in Eq. (5), the performance function in the augmented problem is formulated as:

$$h_3(\boldsymbol{\theta}, \mathbf{x}) = \alpha \frac{g(\boldsymbol{\theta}, \mathbf{x})}{\tilde{\mu}_g} + (1-\alpha)\frac{\left(g(\boldsymbol{\theta}, \mathbf{x}) - \mu_g(\mathbf{x}) \right)^2}{\tilde{\sigma}_g^2}, \tag{8}$$

where estimates of $\tilde{\mu}_g$ and $\tilde{\sigma}_g^2$ are obtained from solving the optimization problem for minimization of the mean and minimization of variance of the structural response, respectively.

Therefore, upon successfully formulating the augmented objective function, consider any general performance measure of the system denoted by $h(\boldsymbol{\theta}, \tilde{\mathbf{x}}) : R^{n_\theta} \times R^{n_{\tilde{x}}} \to R$ where $h(\boldsymbol{\theta}, \tilde{\mathbf{x}}) = h_1(\boldsymbol{\theta}, \mathbf{x})$ for pure mean minimization and $h(\boldsymbol{\theta}, \tilde{\mathbf{x}}) = h_3(\boldsymbol{\theta}, \mathbf{x})$ otherwise. $\tilde{\mathbf{x}}^*$ denotes the optimal design solution obtained by optimizing the stochastic design problem, formulated as:

$$\tilde{\mathbf{x}}^* = \arg\min_{\tilde{\mathbf{x}}\in\tilde{\Phi}} \mathrm{E}_\theta \left[h(\boldsymbol{\theta}, \tilde{\mathbf{x}}) \right]. \tag{9}$$

In the formulation of an augmented problem, the design variables are artificially considered uncertain with PDF $p(\tilde{\mathbf{x}})$. In the setting of this augmented design problem, an auxiliary PDF is defined as:

$$\pi(\boldsymbol{\theta},\tilde{\mathbf{x}})=\frac{h(\boldsymbol{\theta},\tilde{\mathbf{x}})\,p(\boldsymbol{\theta},\tilde{\mathbf{x}})}{\mathrm{E}_{\theta,\tilde{x}}[h(\boldsymbol{\theta},\tilde{\mathbf{x}})]},\tag{10}$$

where $p(\boldsymbol{\theta},\tilde{\mathbf{x}})=p(\boldsymbol{\theta}\,|\,\tilde{\mathbf{x}})\,p(\tilde{\mathbf{x}})$ and the normalizing constant in the denominator is defined as:

$$\mathrm{E}_{\theta,\tilde{x}}[h(\boldsymbol{\theta},\tilde{\mathbf{x}})]=\int_{\Phi}\int_{\Theta}h(\boldsymbol{\theta},\tilde{\mathbf{x}})\,p(\boldsymbol{\theta},\tilde{\mathbf{x}})\,d\boldsymbol{\theta}\,d\tilde{\mathbf{x}}.\tag{11}$$

Note if $h(\boldsymbol{\theta},\tilde{\mathbf{x}})\le 0$, it must be suitably transformed to ensure that $\pi(\boldsymbol{\theta},\tilde{\mathbf{x}})\ge 0$. One way to do this is to define $h_s(\boldsymbol{\theta},\tilde{\mathbf{x}})=h(\boldsymbol{\theta},\tilde{\mathbf{x}})-s$, since $\mathrm{E}_{\theta}\left[h_s(\boldsymbol{\theta},\tilde{\mathbf{x}})\right]=\mathrm{E}_{\theta}\left[h(\boldsymbol{\theta},\tilde{\mathbf{x}})\right]-s$, that is the two expected values differ only by a constant, optimization of the expected value of $h(\boldsymbol{\theta},\tilde{\mathbf{x}})$ is equivalent, in terms of the optimal design to optimization for the expected value for $h_s(\boldsymbol{\theta},\tilde{\mathbf{x}})$. In terms of the auxiliary PDF, the objective function $\mathrm{E}_{\theta}\left[h(\boldsymbol{\theta},\tilde{\mathbf{x}})\right]$, is expressed as

$$\mathrm{E}_{\theta}\left[h(\boldsymbol{\theta},\tilde{\mathbf{x}})\right]=\frac{\pi(\tilde{\mathbf{x}})}{p(\tilde{\mathbf{x}})}\mathrm{E}_{\theta,\tilde{x}}[h(\boldsymbol{\theta},\tilde{\mathbf{x}})],\tag{12}$$

where the marginal PDF $\pi(\tilde{\mathbf{x}})$ is equal to

$$\pi(\tilde{\mathbf{x}})=\int_{\Theta}\pi(\boldsymbol{\theta},\tilde{\mathbf{x}})\,d\boldsymbol{\theta}.\tag{13}$$

In Eq. (10) since $\mathrm{E}_{\theta,\tilde{x}}[h(\boldsymbol{\theta},\tilde{\mathbf{x}})]$ is a normalizing constant, minimization of $\mathrm{E}_{\theta}\left[h(\boldsymbol{\theta},\tilde{\mathbf{x}})\right]$ is equivalent to minimization of $J(\tilde{\mathbf{x}})$ which is equal to:

$$J(\tilde{\mathbf{x}})=\frac{\pi(\tilde{\mathbf{x}})}{p(\tilde{\mathbf{x}})},\tag{14}$$

where for simplicity, uniform distribution can be chosen for $p(\tilde{\mathbf{x}})$. For minimization, the PDF $\pi(\tilde{\mathbf{x}})$ in the numerator of $J(\tilde{\mathbf{x}})$ must be evaluated. Stochastic subset optimization (SSO) a stochastic simulation-based approach was proposed for the minimization of $J(\tilde{\mathbf{x}})$ for reliability based design optimization [23]. A brief overview of the SSO algorithm is presented in the following section. For a detailed explanation of SSO, the reader is referred to the original publication.

2.2 Stochastic subset optimization

The basic idea of the SSO is iteratively identifying subregions (subsets) for the optimal design variables within the original design space. In the SSO algorithm, the average value (or equivalently the volume density) of $J(\tilde{\mathbf{x}})$ over any subset of the design space is determined by using the $\pi(\tilde{\mathbf{x}})$ samples obtained by any sampling techniques. The average value is given as:

$$H(I_k)=\frac{1}{V_{I_k}}\int_{I_k}J(\tilde{\mathbf{x}})\,d\tilde{\mathbf{x}}=\frac{V_{\hat{I}_{k-1}}}{V_{I_k}}\int_{I_k}\pi(\tilde{\mathbf{x}})\,d\tilde{\mathbf{x}}.\tag{15}$$

where $\hat{I}_{k-1}$ is the optimal subset identified at the (k-1)th iteration. $H(I_k)$ above expresses the average relative sensitivity of $\mathrm{E}_{\theta}\left[h(\boldsymbol{\theta},\tilde{\mathbf{x}})\right]$ to $\tilde{\mathbf{x}}$ within the set $I_k \subset \hat{I}_{k-1}$. Based on the samples distributed according to from $\pi(\tilde{\mathbf{x}})$ belonging to the set $\hat{I}_{k-1}$ obtained using Markov Chain Monte Carlo (MCMC) procedure, an estimate of $H(I)$ is provided by

$$\bar{H}(I_k) = \frac{N_{I_k} / V_{I_k}}{N_{\hat{I}_{k-1}} / V_{\hat{I}_{k-1}}}, \tag{16}$$

where N_{I_k} and $N_{\hat{I}_{k-1}}$ denote the number of samples from $\pi(\tilde{\boldsymbol{\varphi}})$ belonging to the sets I_k and $\hat{I}_{k-1}$, respectively, and V_{I_k} and $V_{\hat{I}_{k-1}}$ denote the volume of the set I_k and $\hat{I}_{k-1}$, respectively. A deterministic subset optimization (based on the estimate $\bar{H}(I_k)$ of $H(I_k)$) is performed to identify a set $\hat{I}_k$ that contains the smallest volume density N_{I_k} / V_{I_k} of samples, such that:

$$\hat{I} = \arg\min_{I \in A} \bar{H}(I), \tag{17}$$

where A denotes a set of admissible subsets $\hat{I}_{k-1}$ that have some predetermined shape and some size constraint. In this way, SSO adaptively converges to a relatively small sub-region for the optimal design variables $\tilde{\mathbf{x}}^*$ within the original design space. Following this, any standard direct search method can be employed to determine the optimal design solution within the identified optimal design region. Specifically, an unconstraint Genetic Algorithm (GA)-based optimization approach NSGA-II is employed.

3. ILLUSTRATIVE EXAMPLE

An exemplary example is considered in order to demonstrate the efficacy of the proposed approach. The example is a four-bar truss structure adapted from [17]. The shape for set *I* is selected as a hyper-rectangle, and Metropolis-Hastings is used to simulate samples at each level, with proposal PDF equal to uniform PDF for design variables and equal to initial PDF for uncertain parameters.

3.1 Four-bar truss structure

Consider a simple four-bar truss structure shown in Figure 1. The free node of the truss is subjected to a random horizontal load *P* normally distributed with a mean value of 100kN and standard deviations of 20 kN. The Young's modulus (*E*) for the truss is uncertain and normally distributed with mean and standard deviation of 200N/m^2 & 80 N/m^2 and 30 N/m^2 &10 N/m^2, respectively, for the two groups. The cross-sectional areas of the two groups, A_1 & A_2, are considered design variables. The nodal displacement of the free node is considered the performance function. A volume constraint $V \leq 500$ is considered. In this setting, uncertain parameters $\boldsymbol{\theta} = [E_1, E_2, P]$ and $\mathbf{x} = [A_1, A_2]$.

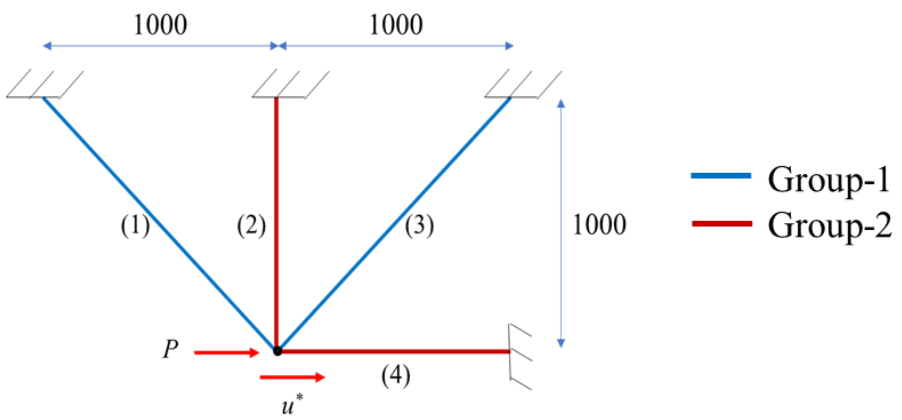

Figure 1: Four-bar truss structure

Figure 2 (a, b) shows the mean and variance minimization results evaluated from the MCS approach using 100,000 samples. It can be observed that the definite minima are observed for both mean and variance minimization, located as $A_1^* = 176.78$ and $A_1^* = 105.0$, respectively.

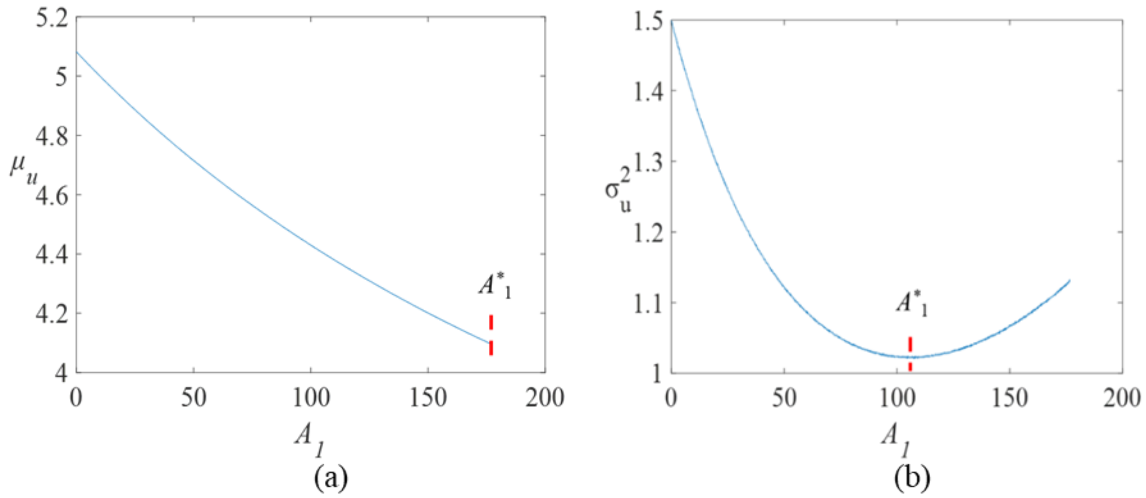

Figure 2: Variation of (a) Mean minimization and (b) Variance minimization versus design variable A_1

The results attained by the proposed approach are shown in Figure 3. The iteratively identified reduced design space for variance minimization case obtained at each SSO iteration level is shown in Figure 3 (a). One can observe that at the 11th iteration, the optimal solution of the design variable is accurately obtained. This affirms the accuracy and the effectiveness of the proposed approach. Furthermore, the samples in the design space simulated at each SSO iteration level are shown in Figure 3 (b). It can be observed that despite the lower samples realization, 2,000, it is challenging to locate the location of optimal design visually. Furthermore, at mean minimization, $(\alpha = 1)$ a higher variance is observed. This highlights the necessity of the RDO, as, at mean optimal design, a minimum value of the objective function is observed, but it is highly susceptible to the variation due to the structural parameter uncertainties. In contrast, for the variance minimization case $(\alpha = 0)$, although the insensitive performance is attained but the performance degrades as compared to the mean minimization case $(\alpha = 1)$. A good trade-off between mean and variance minimizations can be obtained by considering the intermediate values of α in the range of 0 and 1. In this view, the proposed approach is found to be very effective in locating the optimal solution. This effectiveness is expected to further enhance for higher-dimensional problems.

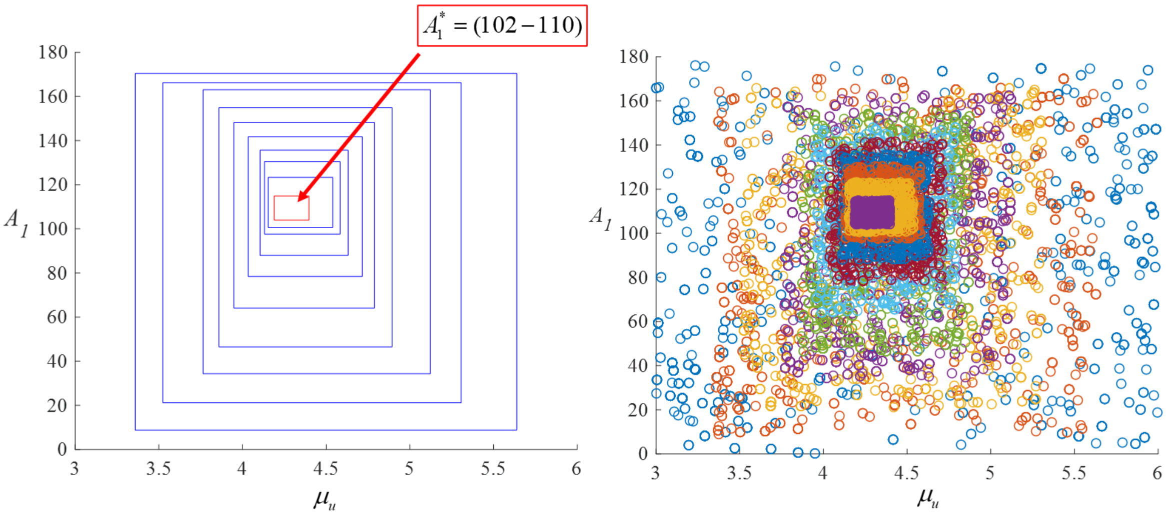

Figure 3: (a) Identified reduced design space and (b) the corresponding samples of the design variable for variance minimization at various SSO iterations.

4. CONCLUSION

This study aims to provide a novel stochastic simulation-based optimization approach for performing structural RDO. The proposed approach is based on the augmented formulation concept. To achieve the desired accuracy while effectively optimizing, a two-stage optimization strategy has been proposed. Initially, the size of the design space is reduced using the stochastic subset optimization concept, and then direct search optimization is used to determine the best design in the reduced design space. The effectiveness of the proposed approach is illustrated with the help of well-known optimization problems, including four bar truss structure. Comparisons are made between the proposed approach and the conventional Monte Carlo Simulation approach. The obtained results are well-matched, affirming the accuracy of the proposed approach. This study allows the designers to design insensitive structure systems. Moreover, the proposed RDO approach is general and not limited to the civil structures only but can also be enforced in the design of any realistic linear/nonlinear systems. It should be noted that this study focuses on unconstrained optimization and could be extended to constrained optimization. Further, research efforts will focus on the issues and applications of engineering design in practice.

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