Goal-Oriented *hp*-Adaptive Finite Element Methods: A Painless Multi-level Automatic Coarsening Strategy

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ABSTRACT

In many engineering applications, the global energy of the problem may be a quantity of limited relevance. This motivated the design of goal-oriented adaptive finite element methods (see e.g. [1,4-6]). They intend to approximate a particular Quantity of Interest (QoI) using an error estimator based on the solution of the adjoint problem to guide the refinements. In this framework, Darrigrand et al. [3] presented an alternative dual operator for the representations of the error in the QoI.

In the context of *hp*-adaptivity, Darrigrand et al. [2] proposed an algorithm for symmetric and positive definite (SPD) problems based on performing global refinements followed by optimal unrefinements. This algorithm marks the basis functions with the lowest contributions to the energy of the solution and remove them. The resulting automatic *hp*-adaptive strategy employs a multi-level hierarchical data-structure proposed by Zander et al. [7].

In this presentation, we extended Darrigrand et al. [2] algorithm to non-SPD problems in the framework of goal-oriented adaptivity. To do so, we evaluate the contribution to the energy in the alternative dual operator proposed by Darrigrand et al. [3], thus, considering both the direct and the adjoint problem.

As a result, we obtain an *hp*-adaptive algorithm for SPD and non-SPD problems in the context of goal-oriented adaptivity. We test and analyze our algorithm on two-dimensional (2D) Laplace, Helmholtz, and convection-dominated problems. We also describe the main features and limitations of the proposed method. In particular, our algorithm is robust and simple to implement; therefore, it can be used for industrial applications.

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