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TOWARD CONCURRENT MULTISCALE TOPOLOGY OPTIMIZATION FOR HIGH HEAT CONDUCTIVE AND LIGHT WEIGHT STRUCTURE

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Abstract. *Solid structures that are light and heat conductive are significant in a variety of engineering applications. We investigated multiscale topology optimization for excessive lightweight heat-conductive porous structures and introduced a mathematical optimization model formulation for concurrently optimizing the macrostructure and the constitutive pores (microstructure) to maximize the design performance. The microscale is constructed utilizing the asymptotic homogenization approach as a representative volume element. During the optimization process, the effective heat conductivity tensor of the microstructure is assessed and utilized as the heat conductivity of the macrostructure for each iteration. To address the macro and microstructure connection, a sensitivity analysis of this concurrent optimization approach was developed. Moreover, the method of introducing initial predetermined design domain was investigated to attain fin-like design in order to despite heat efficiently. Results showed very good results for attaining excessive weight reduction with attaining high heat conductivity. Moreover, the method of predetermined design domains increased the performance significantly.*

1 INTRODUCTION

Thermal conductive solid structures, such as the thermal solid-state passive component, are widely employed in industrial applications, particularly as the demand for high-frequency transmission to boost data bandwidth grows (As in 5G cellphone communication and self-driving cars). The standard design of the thermal dissipation component takes up a lot of room and adds unnecessary weight to the printed circuit board (PCB), which might cause the printed wires on the PCB to deteriorate over time. Add to that the change in high-frequency impedance characteristics caused by bending under weight or resonance interference with the fins. As a result, using a lightweight heat dissipater to solve the concerns listed above is extremely desirable. Topology optimization has been utilized successfully for designing multipurpose structures [1–8]. Topology optimization is described as one of the most constantly evolving approaches for developing creative conceptual designs. Topologically optimized structures, which are associated with additive manufacturing, are gradually finding their way into industrial applications to manufacture lightweight structures with excellent utility. As a result, the purpose of structural topology optimization is to find the best and most reliable material distribution in order to maximize structural performance to weight ratio while satisfying various design requirements. In order to utilize topology optimization to maximize of heat conductivity of light weigh structures, minimizing thermal compliance was used, such that by minimizing the thermal compliance, the the stored thermal energy will decrease in the favour of heat conduction. Although, heat compliance is utilized successfully as an objective function in designing high conductivity solid problems, careful addressing of the design domain discretization [9], and optimization process are important in order to achieve an optimal design. For example, Iga et al [10] discussed the heat compliance problem and suggested introducing boundary influence to achieve a robust and extremum thermal diffusivity. Their argument was the non-sufficiency of depending on heat transfer coefficient. Such argument was also discussed in various intuitive formations in [11,12].Furthermore, for heat compliance minimization, the tree-like design was reported by researchers, yet, with increasing the design domain finite element resolution beyond million elements, a spike-like design was reported by Wadbro et al [13]. A detailed discussion of the optimality of heat compliance problem and tree like designs are presented in the paper of Yan et al [14]. In their work, several cases were studied and benchmarked to show how a good design variable interpolation scheme and a good piremetrization conditions as well as careful choice of initial design domain can be utilized to achieve optimal solution. They showed that the tree like design is not the optimum design for attaining high thermal conductivity. In this work we are suggestion that by addressing the optimization of the heat conductivity of microstructure as well as the macrostructure, a better performance to weigh ratio can be achieved under the relevant design specifications. In other words, optimizing the material layout on the representative volume equivalent (RVE) as a secondary design domain within the macroscale will lead to extremum the materials to have a much higher performance to weight ratio. The early work of Bendsoe [15] was the successful establishment of designing microstructures with the inverse approach of the homogenization method. Additional work that tried to approach structural design by addressing the mesoscale aspect, investigated by many researchers such as Zhou et al, [16] which addressed truss-like structure unit cell as the building block of the complex truss structure. Also, Hierarchal based topology optimization was a solution for scaling topology optimization to both micro and macrostructure [17][18]. Concurrent micro and macro design method were investigated for maximizing structural stiffness [3,19–21]. There are few researches that addressed the multiscale concurrent optimization for maximizing thermal conductivity [22][23]. Moreover, the address of design methodology to attain fin-like structures (and not the tree-like thermal structures) using concurrent multiscale topology optimization is not yet introduced, therefore; this work aims to establish and examine the methods and the formulation of concurrent multiscale optimization for thermal conductivity problem, and the interpolation schemes between the macro and micro scale in the design system and introduced the predetermine initial design domain scheme to obtain a robust and optimum porous structure with high thermal conductivity. This paper is organized as the following: Section 2 is dedicated to the mathematical modeling of the multiscale problem. Section 3 is dedicated to presenting and discussing the numerical examples and finally, section 4 is dedicated to the conclusions.

2 MATHEMATICAL MODELLING OF TOPOLOGY OPTIMIZATION FOR THERMAL CONDUCTION PROBLEM

Concurrent topology optimization was performed inasmuch as macro and microsystems are simultaneously optimized for minimizing the heat compliance on both, the micro $\mathbf{x}_{\mathrm{M}}$ and macroscale $\mathbf{x}_{\mathrm{m}}$. Macro and microscale design domains are discretized using two distinctive finite element systems. In this paper, we used bilinear structured mesh for both systems. When $\mathbf{x}$ is equal to 1, this means that the corresponding element is a solid while if it is zero, it means that the element is representing a void, as shown in Eq. (1).

$$\mathbf{x}_{\mathrm{M,m}} = \begin{cases} 1 & \textit{Design material} \\ 0 & \textit{Void} \end{cases} \tag{1}$$

Concurrent design of multiscale problem requires adopting homogenization method for two purposes; the first purpose is the calculation of effective properties to use for the macrostructure. The second purpose is to use inverse homogenization as a tool to design the microstructure concurrently with the macrostructure.

2.1 Homogenization approach

Homogenization has been performed numerically to evaluate the effective properties, which is the homogenized conductivity tensor Λ^{H} of the micro design domain. By definition, the micro design domain is an RVE that is statistically homogenous in comparison to the macroscale domain. The material distribution, the behaviour of the property to be estimated inside the material, and the material's interface layer behaviour all influence the effective properties. The thermal conductivity is assumed to be equally distributed throughout the material in this case. Furthermore, the interface layer effect has been neglected such that the RVE is considered to be a single-phase infinitesimal solid material and void. By utilizing the numerical homogenization method, the effective properties are calculated for the microstructure and used to construct the finite element of the macroscale domain. Considering the two-phase microstructure with overall thermal conductivity as of Λ_k , k=1,2 which denote to the material's thermal conductivity tensors associated with RVE phases. The general steady state conduction heat flow is formulated by Eq. (2).

$$\nabla.\mathrm{q(T)} = -\frac{\partial}{\partial\zeta}\left(\Lambda\frac{\partial \mathrm{T}}{\partial\zeta}\right) \quad , \quad \forall x \in \Omega \tag{2}$$

Where q is the heat transfer rate and $\frac{\partial \mathrm{T}}{\partial\zeta}$ is the temperature gradient. Λ represents the material's thermal conductivity constant. For multiscale problem, heat conductivity of the macroscale is the homogenized heat conductivity tensor that is obtained from the microscale.

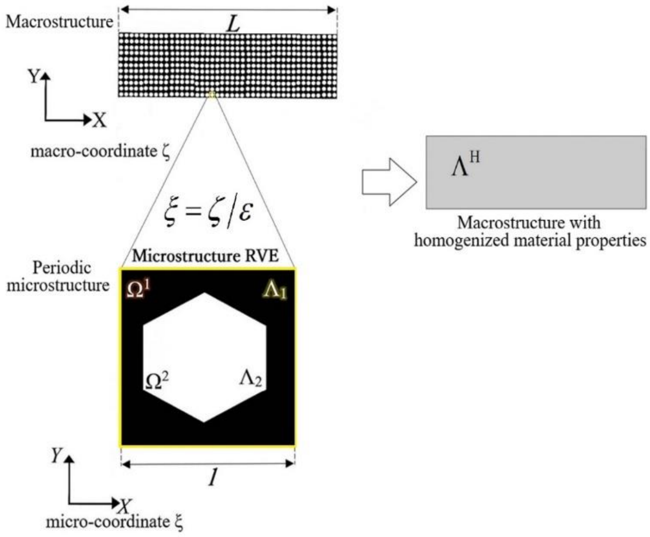

Figure 1: Homogenized thermal properties representation of the micro for the macro scale.

For evaluating the homogenized (effective) heat conductivity tensor, Eq. (3) is used:

$$\mathbf{\Lambda}^{\mathrm{H}} = \frac{1}{|V|}\int_V \mathbf{\Lambda}_{ijqp}\left(\mathbf{G}_{qp}^{0(kl)} - \mathbf{G}_{qp}^{*(kl)}\right)dV \tag{3}$$

Where $\mathbf{G}_{qp}^{0(kl)}$ represents the unit temperature gradient test, and $\mathbf{G}_{qp}^{*(kl)}$ periodic characteristic gradient. Moreover, the finite element representation of the steady state heat conduction problem is taking the form:

$$\mathbf{P} = \mathbf{KT} \tag{4}$$

Where $\mathbf{P}$ is the nodal thermal load, $\mathbf{T}$ is the nodal temperature and $\mathbf{K}$ is the global heat conductivity matrix. Topology optimization of design high thermal conductive structure to weight ratio is achieved by minimizing heat compliance C_{H}.

$$C_{\mathrm{H}} = \mathbf{P}^T\mathbf{T} \tag{5}$$

Here, the heat compliance in terms of the macro and micro design variables is given by Eq. (6).

$$C_{\mathrm{H}}\left(\mathbf{x}_{\mathrm{M}},\mathbf{x}_{\mathrm{m}}\right)=\sum_{i=1}^{N}\mathbf{T}_i^T\mathbf{K}_i\left(\mathbf{x}_{\mathrm{M}},\mathbf{x}_{\mathrm{m}}\right)\mathbf{T}_i \tag{6}$$

Where $\mathbf{T}_{\mathrm{i}}$ and $\mathbf{K}_{\mathrm{i}}$ represents the nodal temperature vector, and the thermal conductivity matrix of the i^{th} element with respect to the macrostructure of the total number of the element equal to N. The general form of the thermal conductivity matrix is taking the form:

$$\mathbf{K} = \int_V \mathbf{D}^T \mathbf{\Lambda}\, \mathbf{D}\, dV \tag{7}$$

Where $\mathbf{D}$ is the conversion matrix of temperature gradient to nodal temperature. For microstructure domain, the material's thermal conductivity tensor $\mathbf{\Lambda}^{\mathrm{H}}$ is associated with heat conductivity tensor of the based material $\mathbf{\Lambda}_0$ such that:

$$\mathbf{\Lambda}^{\mathrm{H}} = \mathbf{x}_{\mathrm{m}}^p\mathbf{\Lambda}_0 \tag{8}$$

$\mathbf{\Lambda}^{\mathrm{H}}$ is calculated using the homogenization method and used to establish the elemental heat conductivity tensor of the macroscale $\mathbf{\Lambda}_{macro}$ with a similar material interpolation scheme as for the microstructure system.

$$\mathbf{\Lambda}_{macro} = \mathbf{x}_{\mathrm{M}}^p\mathbf{\Lambda}^{\mathrm{H}} \tag{9}$$

As such, the formulation of concurrent topology optimization of minimizing heat compliance is taking the form:

$$
\begin{aligned}
&\textit{find} \quad \mathbf{x}_{\mathrm{M}}, \mathbf{x}_{\mathrm{m}} \quad (\mathrm{M}=1,2,..,N_{\mathrm{M}}\,; \mathrm{m}=1,2,..,N_{\mathrm{m}}) \\
&\min: C_{\mathrm{H}}(\mathbf{x}_{\mathrm{M}}, \mathbf{x}_{\mathrm{m}}) \\
&\quad \mathrm{s.t.} \left\{ \mathbf{K}\left(\mathbf{x}_{\mathrm{M}}, \mathbf{x}_{\mathrm{m}}\right)\mathbf{T} = \mathbf{P} \right. \\
&\quad \int_{\Omega_{dM}} \mathbf{x}_{\mathrm{M}} d\mathbf{x}_{\mathrm{M}} \le v_{\mathrm{M}} \;,\; 0 < \mathbf{x}_{\mathrm{M}} < 1 \quad \forall \mathbf{x}_{\mathrm{M}} \in \Omega_{dM} \\
&\quad \int_{\Omega_{dm}} \mathbf{x}_{\mathrm{m}} d\mathbf{x}_{\mathrm{m}} \le v_{\mathrm{m}} \;,\; 0 < \mathbf{x}_{\mathrm{m}} < 1 \quad \forall \mathbf{x}_{\mathrm{m}} \in \Omega_{dm}
\end{aligned}
\tag{10}
$$

Here, N_{M} , and N_{m} are the element number of the macro- and the microscale structure respectively. v_{M} and v_{m} are the volume fraction of the design variable $\mathbf{x}_{\mathrm{M}}$ and $\mathbf{x}_{\mathrm{m}}$ within the macro and micro design domains (Ω_{dM} and Ω_{dm} respectively). Figure 1 is showing the modelling of multiscale topology optimization for stiffness and heat conductivity problems.

2.2 Sensitivity analysis and optimization method

Sensitivity analysis plays a major role in achieving the global extremum solution. First order sensitivity analysis (Eq. 11) is adopted to be performed for each iteration.

$$\dot{C}_{\mathrm{H}} = \dot{\mathbf{P}}^{T}\mathbf{T} + \mathbf{P}^{T}\dot{\mathbf{T}} \tag{11}$$

$\dot{\mathbf{T}}$ can be obtained by differentiating Eq. (4) and arranging and separating the variables which gives:

$$\dot{\mathbf{T}} = \mathbf{K}^{-1}\dot{\mathbf{P}} - \mathbf{K}^{-1}(\dot{\mathbf{K}}\mathbf{T}) \tag{12}$$

Now, substituting Eq. (12) into (11) gives:

$$\dot{C}_{\mathrm{H}} = 2\dot{\mathbf{P}}^{T}\mathbf{T} - \mathbf{T}^{T}\dot{\mathbf{K}}\mathbf{T} \tag{13}$$

The current problem has a fixed load condition one with the design domain, therefore; the problem is heat load independent. This will lead to eliminating the first term of Eq. (13) which reduce the first order sensitivity of the heat compliance to:

$$\dot{C}_{\mathrm{H}} = -\mathbf{T}^{T}\dot{\mathbf{K}}\mathbf{T} \tag{14}$$

Now writing Eq. (14) in terms of both, macro and micro design variables give:

$$\dot{C}_{\mathrm{H}}\left(\mathbf{x}_{\mathrm{M}},\mathbf{x}_{\mathrm{m}}\right)=-\mathbf{T}_{\mathrm{M}}{}^{T}\dot{\mathbf{K}}(\mathbf{x}_{\mathrm{M}},\mathbf{x}_{\mathrm{m}})\mathbf{T}_{\mathrm{M}} \tag{15}$$

Eq. (15) has two parts, first is the derivative with respect $\mathbf{x}_{\mathrm{M}}$ which is used to update the macroscale design domain. This is given in Eq. (16).

$$\dot{C}_{\mathrm{H}}\left(\mathbf{x}_{\mathrm{M}}\right)=\frac{\partial C_{\mathrm{H}}}{\partial \mathbf{x}_{\mathrm{M}}}=-p\left(\mathbf{x}_{\mathrm{M}}^{p-1}\right)\mathbf{T}^{T}\int_{|\Omega_{\mathrm{M}}|}\mathbf{D}^{T}\mathbf{\Lambda}^{\mathrm{H}}\mathbf{D}\ d\Omega_{\mathrm{M}}\,\mathbf{T} \tag{16}$$

The second part is the derivative with respect to $\mathbf{x}_{\mathrm{m}}$ which is controlling the micro design domain heuristic process which is given in Eq. (17).

$$\dot{C}_{\mathrm{H}}\left(\mathbf{x}_{\mathrm{m}}\right)=\frac{\partial C_{\mathrm{H}}}{\partial \mathbf{x}_{\mathrm{m}}}=-\mathbf{T}^{T}\int_{|\Omega_{\mathrm{m}}|}\mathbf{D}^{T}\frac{d\mathbf{\Lambda}^{\mathrm{H}}}{d\mathbf{x}_{\mathrm{m}}}\mathbf{D}\ d\Omega_{\mathrm{m}}\,\mathbf{T} \tag{17}$$

The derivative of the homogenized material's thermal conductivity tensor with respect to micro design variable $\frac{d\mathbf{\Lambda}^{\mathrm{H}}}{d\mathbf{x}_{\mathrm{m}}}$ can be determined as:

$$\frac{d\mathbf{\Lambda}^{\mathrm{H}}}{d\mathbf{x}_{\mathrm{m}}}=\frac{p}{\left|\Omega_{\mathrm{m}}\right|}\int_{\Omega_{\mathrm{m}}}\left(\mathbf{I}-\mathbf{DT}\right)^{T}\left(\mathbf{x}_{\mathrm{m}}^{p-1}\right)\Lambda\left(\mathbf{I}-\mathbf{DT}\right)d\Omega_{\mathrm{m}} \tag{18}$$

Optimization method that used in this work is Solid Isotropic Material with Penalization (SIMP) method. Furthermore, optimality criteria method is used to update the design variables. To guarantee that solutions to the topology optimization problem exist and that checkerboard problem do not arise, a sensitivity filter is introduced to modify the sensitivities $\dot{C}_{H}\left(\mathbf{x}_{\mathrm{M}}\right)$ and $\dot{C}_{H}\left(\mathbf{x}_{\mathrm{m}}\right)$ as follows:

$$\hat{\dot{C}}=\frac{\partial C}{\partial \mathbf{x}_{e}}=\frac{1}{\mathbf{x}_{e}\sum_{f=1}^{N}H_{f}}\sum_{f=1}^{N}H_{f}\mathbf{x}_{f}\frac{\partial C}{\partial \mathbf{x}_{f}} \tag{19}$$

Where H_f is the convolution operator to perform the modification, $\mathbf{x}_e$ is the design variable at which the sensitivity is calculated, and $\mathbf{x}_f$ **.** The H_f is defined as

$$H_f = r - dist(e, f), \{f \in N | dist(e, f) \le r\} \tag{20}$$

After modifying the sensitivity, the following is a heuristic updating technique that is identical to the one employed in this paper:

$$\mathbf{x}_e^{updated} = \begin{cases} \max(0, \mathbf{x}_e - \varepsilon) & if \ \ \mathbf{x}_e B_e^{\omega} \le \max(0, \mathbf{x}_e - \varepsilon) \\ \max(0, \mathbf{x}_e + \varepsilon) & if \ \ \mathbf{x}_e B_e^{\omega} \ge \max(1, \mathbf{x}_e - \varepsilon) \\ \mathbf{x}_e B_e^{\omega} & Otherwise \end{cases} \tag{21}$$

where ε denotes a positive search step. Moreover, ω which is equal to 1/2 denotes a numerical damping coefficient, and B_e denotes the optimality condition ($-{}^{\partial C}\!/_{\partial \mathbf{x}_e} \Big/ L\, {}^{\partial V}\!/_{\partial \mathbf{x}_e}$), where L here is a Lagrangian multiplier, and ${}^{\partial V}\!/_{\partial \mathbf{x}_e}$ is the volumetric topological derivative. Finally, the flowchart of concurrent multiscale topology optimization for maximizing heat conductivity is shown in figure 2.

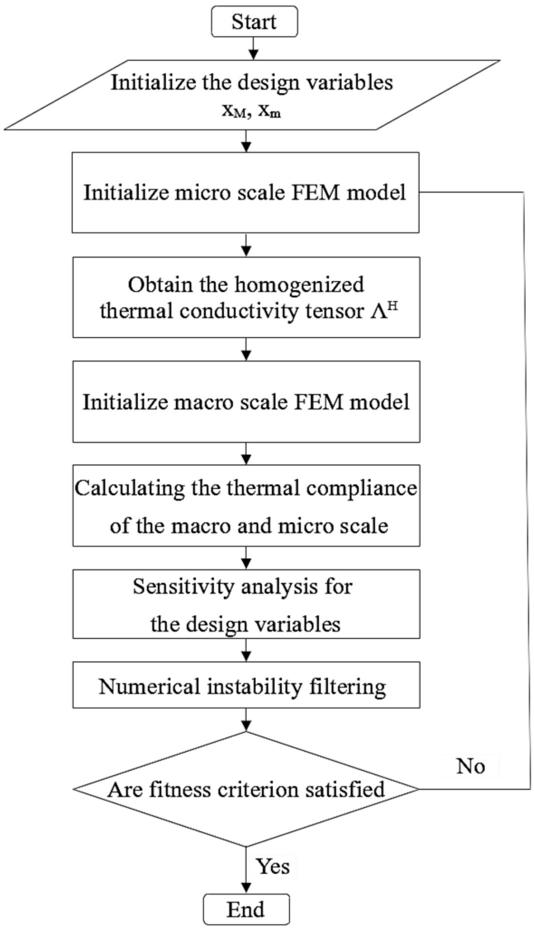

Figure 2: Flowchart for concurrent multiscale topology optimization for maximizing heat conductivity.

3 NUMERICAL INVESTIGATIONS

3.1 Concurrent multiscale versus macroscale optimization

The numerical investigation in this section is dedicated to investigating the macroscale optimization versus the multiscale optimization. First, consider a macro design domain of 200 by 200 elements in the x and the y-directions, with a volume fraction of 0.5 (i.e., 50% weight reduction). Also, a uniform distributed heat load of 0.1 for the whole area is applied while the boundary of an upper left side has a heat sink in the middle of the left side (As shown in Figure 3 a), and the remaining outer boundaries are adiabatic. Minimizing the heat compliance of only the macroscale has the design shown in Figure. 4 a. Now reducing the volume fraction to 0.25 (i.e., 75% weight reduction) for the same design problem is raising the final heat compliance due to excessive materials' reduction (as shown in Figure 4 b). By applying the concurrent optimization for the macro and micro scales of the initial design domain, volume reduction can be achieved on both, the micro (that is shown in Figure 3 (b)) and macro scales.

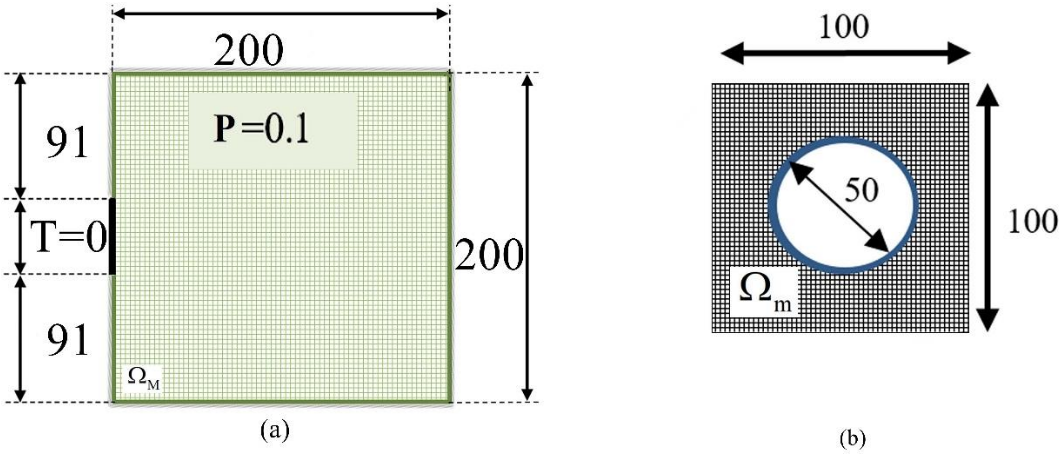

Figure 3: Design domain in (a) macro and (b) microscale

Therefore; by assuming that each element will have a volume equal to the volume of microscale, the multiscale optimization of 50% weight reduction on both macro and microscale will match the 75% total volume reduction. The results of 75% total volume reduction of the multiscale optimization (Figure 4 c) showed better performance in terms of minimizing heat compliance compared to the similar total 75% volume reduction on the macroscale alone.

3.2 Concurrent multiscale optimization with predetermined initial design domain

By introducing an initial predetermine branched design domains for the same problem of section (3.1), a fin-like structure is attained for the final design (as shown in figure 5). The performance of designing is improved significantly by optimizing the multiscale problem for the initial design domain by increasing the number of the introduced initial branches. As shown in figures 5 (b)-(d), the heat compliance minimization is improved compared to the multiscale design problem of section (3.1).

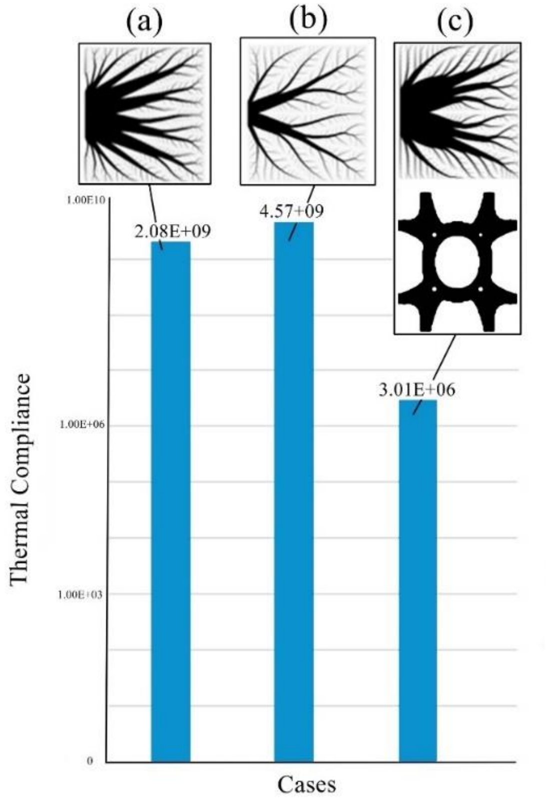

Figure 4: Optimization of (a) the case of 50% volume fraction on the macroscale, (b) the 25% volume reduction case on the macroscale, and (c) the case of total 25% volume reduction on both macro and microscale.

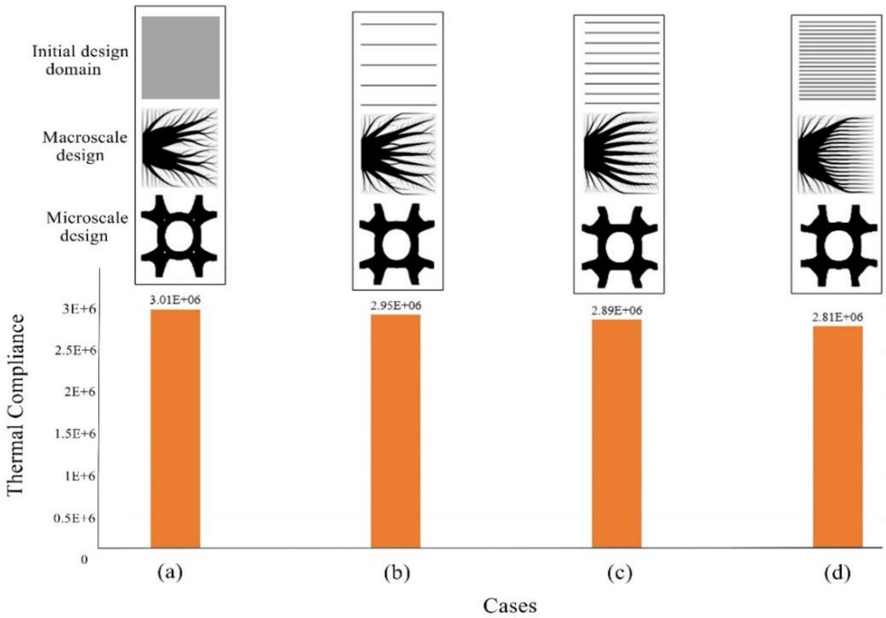

Figure 5: Optimization of (a) the case of dully distributed initial design domain (b) prescribed initial design domain of 5 branches, (c) prescribed initial design domain of 9 branches, and (d) prescribed initial design domain of 20 branches.

4 CONCLUSIONS

Concurrent multiscale topology optimization of high heat conductivity for macro and periodic microstructure is presented in this paper. The goal is to show how to use a multiscale formulation and to look at the design requirements. Minimizing heat compliance was chosen as a simple example of the concurrent multiscale optimization's efficiency in attaining goal function reduction. On both macro and periodic microstructure, formulations were devised for the multiscale issue. The asymptotic homogenization approach was utilized to determine the macrostructure's effective heat conductivity tensor as well as to produce the microstructure. Optimizing the multiscale showed better results than for optimizing the macroscale alone, Furthermore, a high weight reduction was attained with superior heat compliance minimization. Moreover, From the results introduced in section 3.2, we conclude that our method of introducing predetermine branches to the design macroscale of the multiscale problem is promoting the final design to obtain the desirable fin-like structures with minimal mathematical complexity.

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