Probabilistic FEM for Nonlinear Concrete Structures. II: Applications

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Abstract: This is the second part of an investigation on probabilistic finite element methods for nonlinear concrete structures. The study is concentrated on two reinforced concrete application examples including a simply supported beam and a portal frame. Nonlinearity in material and geometry, and randomness in loading, material, and geometry are considered. Extensive computations using a probabilistic finite element method imbedded in the computer code PFRAME developed at the University of Colorado, Boulder, are carried out to verify analytical results. The effect of Taylor series expansions about different values of various random variables on structural response is demonstrated in both application examples. Furthermore, using the simply supported beam example, it is shown that the probabilistic finite element method proposed is applicable to the assessment of structural safety of material and geometric nonlinear concrete structures. Further investigations are necessary in order to develop this probabilistic finite element approach into a mature structural safety assessment method.

Introduction

This is the second part of an investigation on probabilistic finite element methods for analysis of nonlinear concrete structures. The formulation of a method that accounts for both randomness in loading, material and geometry, and nonlinearity in material and geometry is stated in part 1 of this paper (Teigen et al. 1991). Also, the assumptions, analytical results, and description of a computer code for probabilistic finite element analysis of nonlinear concrete structures are given in part 1. Using the concepts and methodologies described in part 1, this study is concentrated on reinforced concrete application examples, including a simply supported beam and a portal frame. The emphasis is on the consideration of randomness and nonlinearity, accuracy of the computed response quantities, and application of the probabilistic finite element proposed in part 1 to safety assessment.

Application Examples

Example 1: Simply Supported Beam

The problem definition for this example is summarized in Fig. 1. As shown, the random fields for material and load are discretized into only one random

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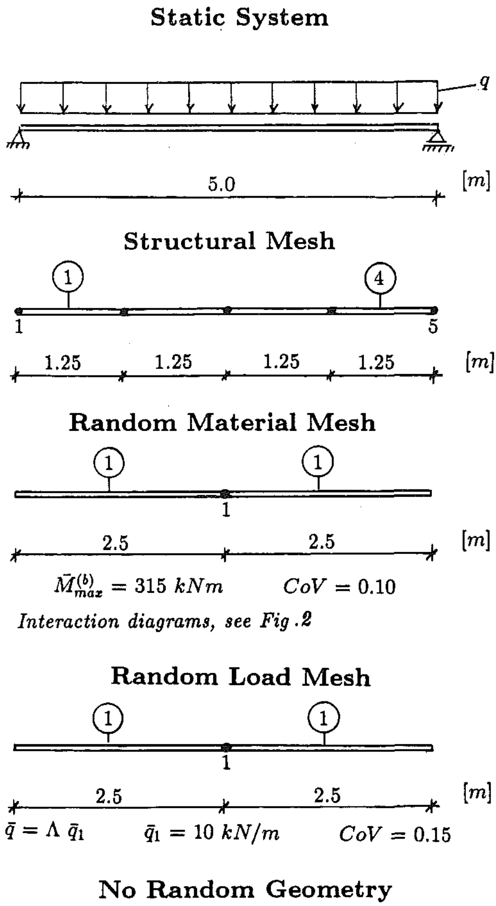

FIG. 1. Simply Supported Beam: Problem Definition

variable each. No random geometry is considered. Thus the problem is statically determinate with two independent random variables. The reason for choosing this example is that the results from the computer code PFRAME (Teigen et al. 1991) are easy to verify.

The trilinear interaction diagrams used in the analysis are shown as broken lines in Fig. 2. The correspondingly calculated values based on the material and section properties given in Fig. 3 are shown as solid lines. While the simplified M_{max}-N diagram is in good agreement with the one that is calculated, a trilinear approximation for the κ_{max}-N diagram turns out to be more crude, especially for the tensile force region. The material characteristics shown in Fig. 3 are meant to represent the Norwegian grades C 25 and K 400 TS for concrete and reinforcing steel, respectively. Extrapolation to mean values are based on data given by Mirza and MacGregor (1982). Also, the assumed coefficient of variation of 0.10 for the maximum balanced moment $M^{(b)}_{max}$ is based on results reported by Mirza and MacGregor (1982). In the

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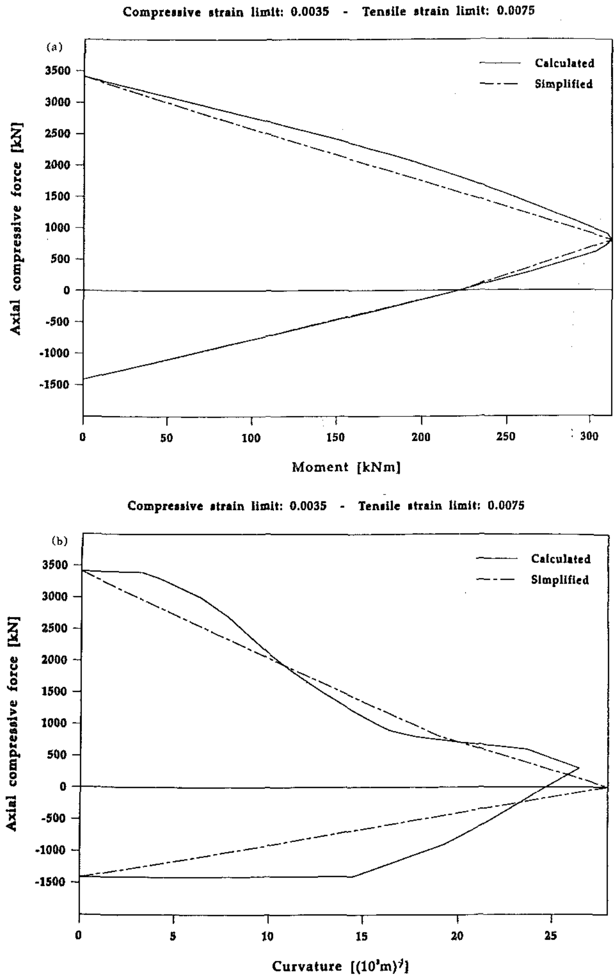

FIG. 2. Beam Interaction Diagrams: (*a*) Maximum Moment-Axial Force; (*b*) Maximum Curvature-Axial Force

moment-curvature relationship [see (22) in Teigen et al. (1991)] a parameter $\gamma = 0.0$ is chosen. Thus the moment becomes constant in the plastic region.

The mean load is applied incrementally for prescribed values of the load parameter Λ until collapse of the system takes place. Since the problem is statically determinate, the structure should fail as soon as M_{max} is attained at one integration point.

By expanding the solution about the mean values of both basic random variables the structure failed immediately after exceeding the load parameter $\Lambda = 7.2$. This corresponds to $M_{max} = 225$ kNm, which is in close agreement with the value that can be read from Fig. 2(*a*) corresponding to a zero axial force. For this solution Fig. 4(*a*) shows the evolution of the mean and mean

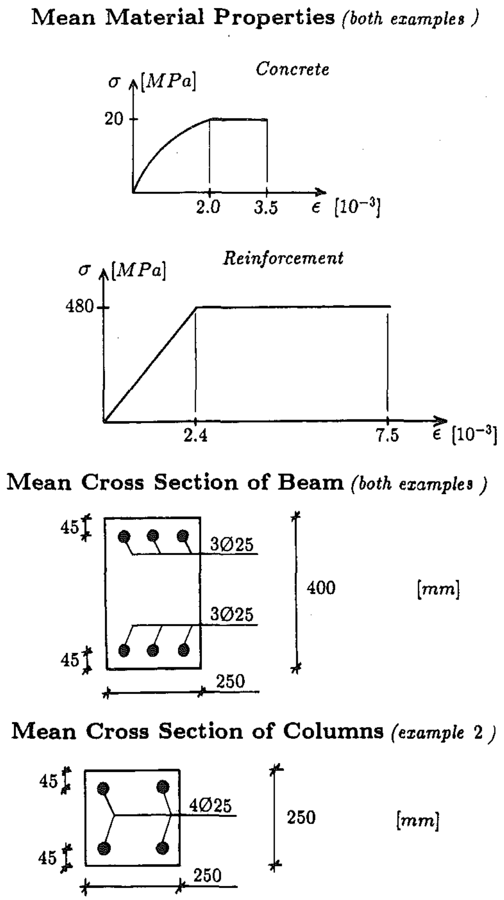

FIG. 3. Mean Material and Cross-Section Properties

± one standard deviation of the midspan displacement with the load parameter Λ. As shown, the standard deviation of the midspan displacement increases rapidly, when the structure approaches collapse. This is due to a corresponding increase in the displacement gradients. Although the structure is statically determinate, the displacements are still a function of the stiffness properties as well as the applied load. The member moments will, on the other hand, be a function of the load only. Fig. 4 (*b*) shows the corresponding evolution of the midspan member moment (or, strictly, the member moment pertaining to the integration point nearest the midspan) versus the load parameter Λ. For a statically determinate system, these curves are straight lines as shown. No "blow up" effect is experienced for the standard deviation as in the case of the displacement. In fact, the results show that the coefficients of variation for all member moments are exactly the same and equal to the coefficient of variation of the applied load (0.15) during the entire load history. This is in agreement with the exact solution, and it is

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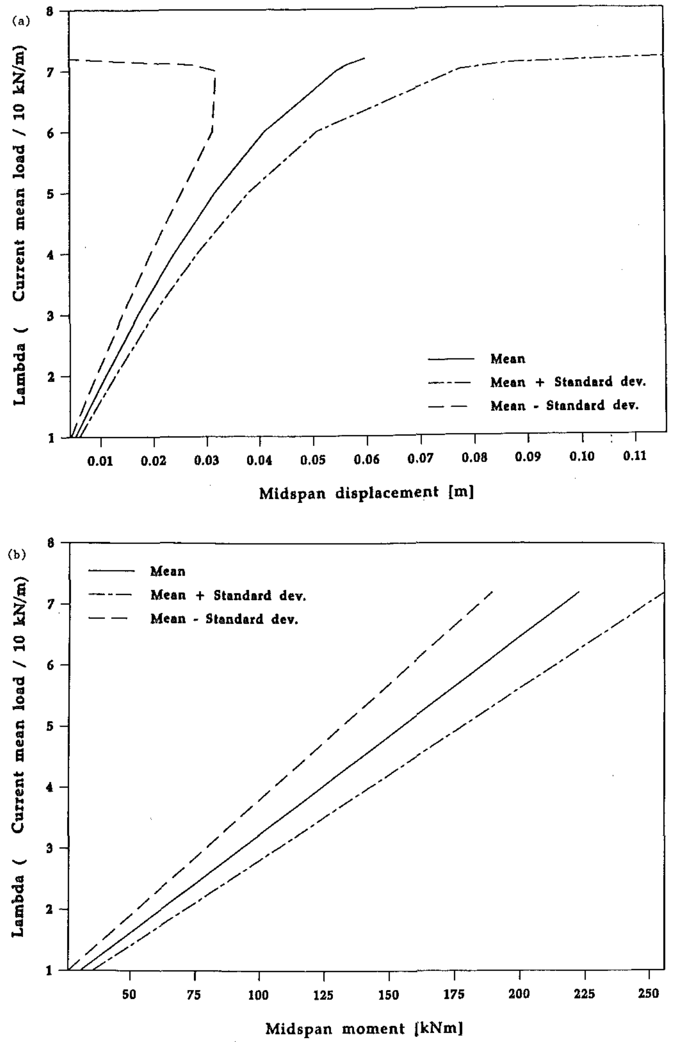

FIG. 4. Simply Supported Beam: Effect of Expansion about Mean Values on: (*a*) Midspan Displacement; (*b*) Midspan Moment

an especially promising result considering that the gradients of member moments were computed based on the displacement gradients, which again become unstable when the structure approaches collapse.

Fig. 5(*a*) shows a comparison between standard deviations of midspan displacements for solutions expanded about different values of the material random variable $M_{max}^{(b)}$. The trend is the same for all cases, namely that the standard deviations "blow up," when the structure approaches its load carrying capacity. Fig. 5(*b*) shows that the corresponding relationships between the mean load and midspan moment are still linear and have identical values for all three cases considered.

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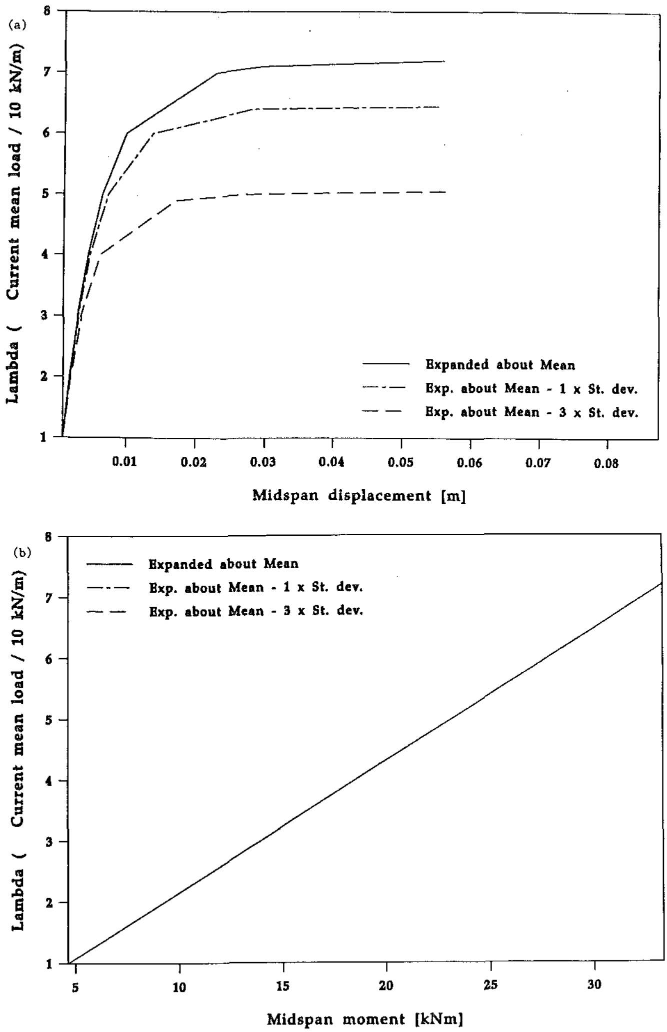

FIG. 5. Simply Supported Beam: Effect of Expansion about Different Values of Material Random Variable on Standard Deviations of: (*a*) Midspan Displacement; (*b*) Midspan Moment

Also, the mean values of the same response quantities are computed for these cases. The results are presented in Figs. 6(*a*) and 6(*b*). Due to the "blow up" effect for the displacement gradients, the predicted mean midspan displacement becomes heavily unreliable for expansions about lower values of the material random variable. This indicates clearly that the computation of mean responses in nonlinear problems should be based on solutions expanded about mean values of the basic random variables. However, for the linear behavior of the midspan moment, the mean values become the same for all three cases.

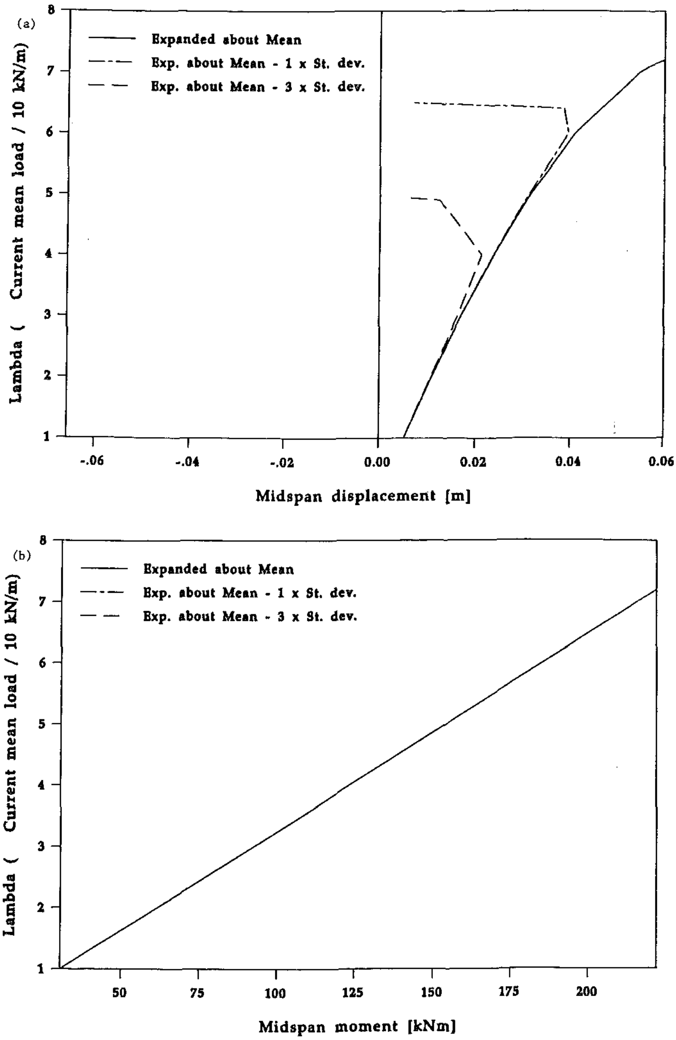

FIG. 6. Simply Supported Beam: Effect of Expansion about Different Values of Material Random Variable on Mean Values of: (*a*) Midspan Displacement; (*b*) Midspan Moment

Fig. 7 shows the calculated correlation coefficients along the beam among displacements and member moments, respectively. Both response quantities are fully correlated. This is in accordance with the exact solution.

Example 2: Portal Frame

The problem definition for this example is summarized in Fig. 8. Since the girder is the same as in the previous example, the main difference now is that the simple supports have been replaced by columns, which are fixed at the ends. The interaction diagrams for the columns are shown in Figs.

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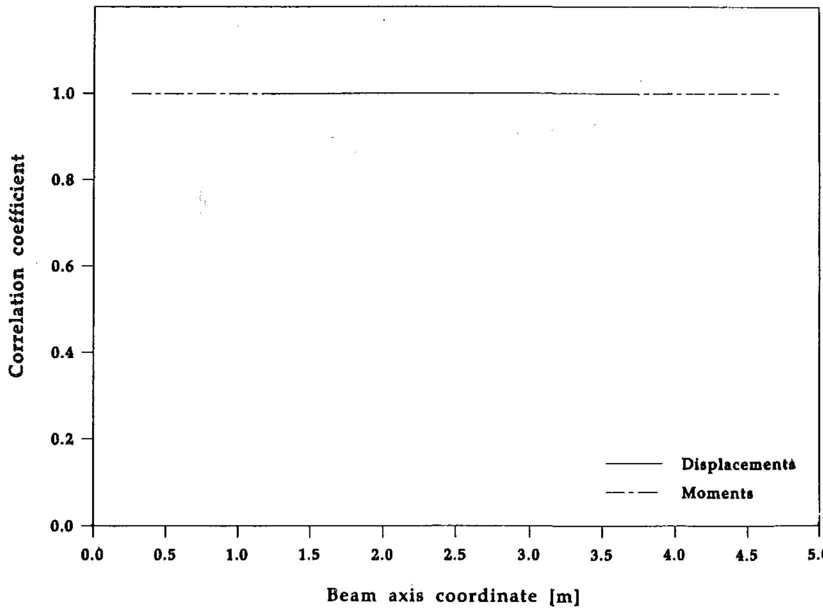

FIG. 7. Simply Supported Beam: Correlation of Displacements and Correlation of Moments along Beam for Expansion about Mean Values

9(*a*) and 9(*b*), both the simplified ones that are applied as input to PFRAME (Teigen et al. 1991) and those calculated based on the material and section properties given in Fig. 3. In the moment-curvature relationships a parameter $\gamma = 0.0$ is again chosen. Therefore, moments are constant in the plastic region.

As can be seen from Fig. 8, all three basic random fields are now present. The discretizations of material properties and loading into random variables are more refined than in the beam example. Two random material variables are applied to each structural unit (a total of six), while the girder has also two random load variables compared to one in the previous example. The random geometry field (i.e., deviation from ideal geometry normal to the beam axis) is only applied to the columns by assuming zero mean and a linearly increasing standard deviation. The random geometry nodes coincide with the structural nodes. Thus there are a total of ten discretized random geometry variables, although the two at the fixed ends of the columns have no effect on the results. The correlation lengths for the three fields are all assumed to be $\lambda = 5.0$ m. As mentioned in the companion paper (Teigen et al. 1991), correlation is only considered within the same structural unit and only among variables originating from the same field. Otherwise the basic random variables are taken as independent.

As in the previous example, the solution is first expanded about the mean values of the basic random variables. The portal frame failed immediately after exceeding the load parameter $\Lambda = 10.7$. The failure mode for this case was a three plastic hinge mechanism, namely initial yielding in the midspan of the girder and collapse after plastification of the top regions of the two

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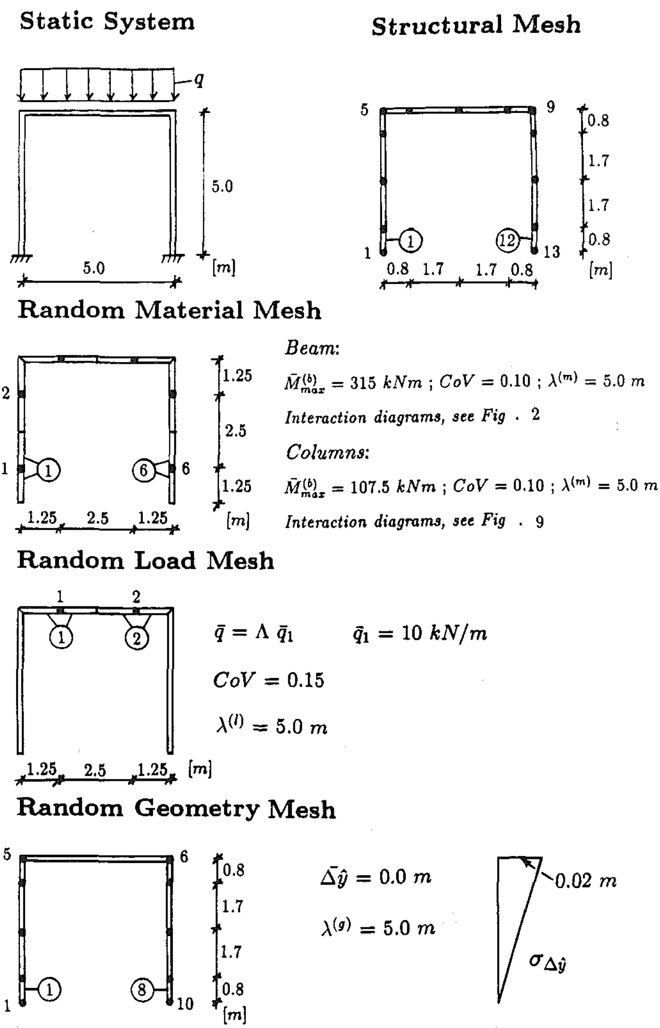

FIG. 8. Portal Frame: Problem Definition

columns had taken place. Fig. 10(a) shows the evolution of the mean and mean $\pm$ 1 standard deviation of the midspan girder displacement. As can be seen, we experience the same "blow up" effect for the standard deviation of displacement when the structure approaches collapse as in the previous example. Fig. 10(b) shows the corresponding evolution of the midspan girder moment (or the moment pertaining to the integration point nearest to the midspan). The yielding shortly before collapse can clearly be identified. Also, a "dent" in the evolution of the midspan girder moment when yielding takes place can be experienced. In the final state, the coefficient of variation of the moment becomes exactly equal to the coefficient of variation of the corresponding random material variable $M^{(b)}_{\max}$. Thus the statistics of the plasti-

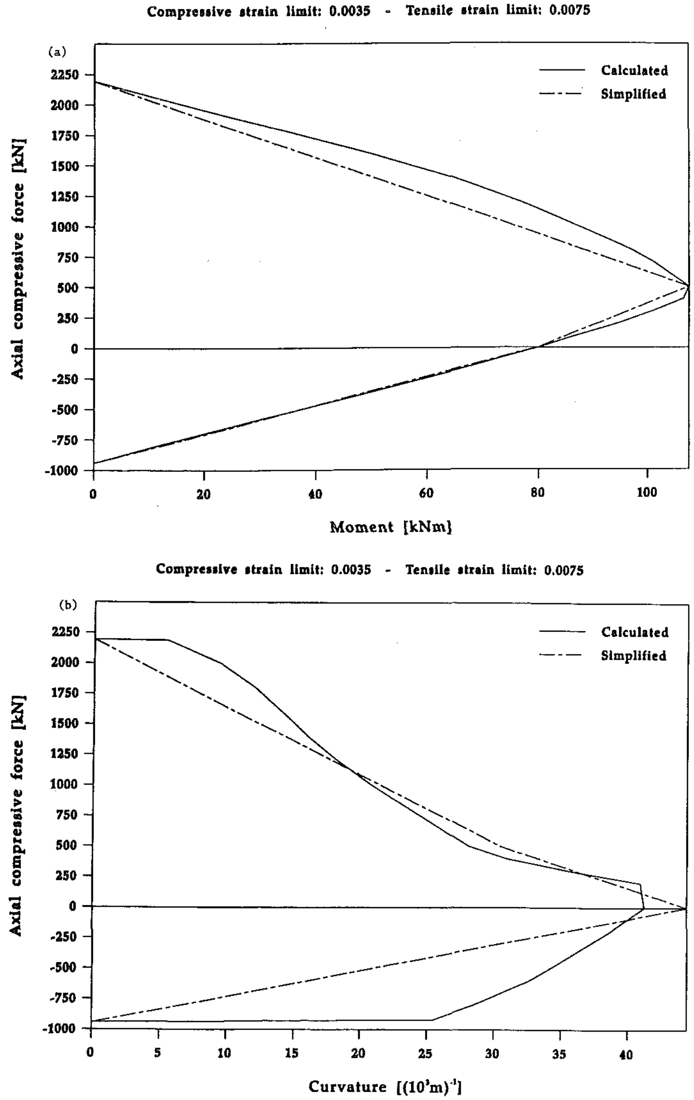

FIG. 9. Column Interaction Diagrams: (*a*) Maximum Moment—Axial Force; (*b*) Maximum Curvature—Axial Force

fied moment depend no more on other properties of the system.

Fig. 10(*c*) shows the corresponding values for the moment at the top of one of the columns (or the moment pertaining to the integration point nearest to the top). As for the displacements, the moment also "blows up" when yielding takes place in the midspan of the girder. However, the standard deviation in this case reduces rapidly again (can be seen on the figure), because the moment is also approaching its plastic limit. Since this failure

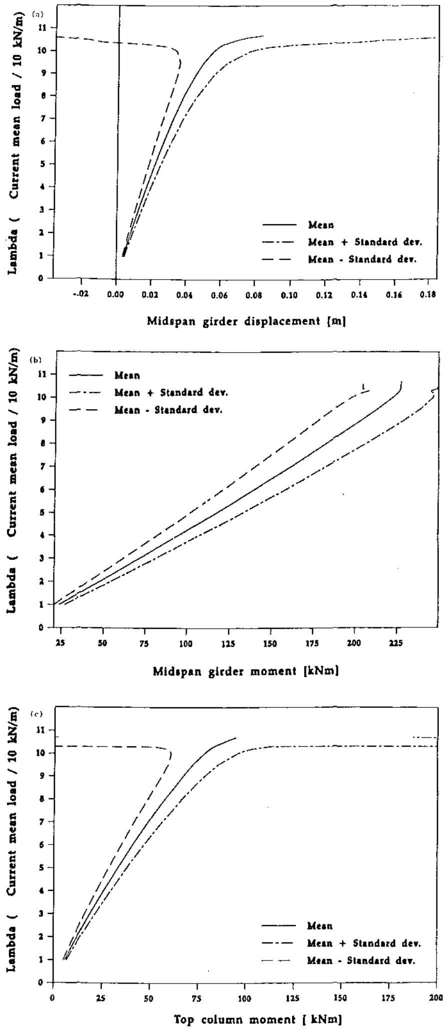

FIG. 10. Portal Frame: Effect of Expansion about Mean Values on: (*a*) Midspan Girder Displacement; (*b*) Midspan Girder Moment; (*c*) Top Column Moment

mode is perfectly symmetric, the plastic limit is reached simultaneously in the two columns, representing the load carrying capacity of the system. For an asymmetric case (not shown here), the plastic moment had an evolution similar to that in Fig. 10(*c*), but in that case the coefficient of variation returned to the value of the corresponding $M^{(b)}_{\max}$ before the structure failed.

Fig. 11 shows the correlation coefficient between the midspan girder and the top of column moment. Initially they are almost fully correlated (negative value means that the two moments have opposite sign) because of the strong correlation through the applied load. In the final stage, however, they approach independence since the moments tend to adopt the same statistics as

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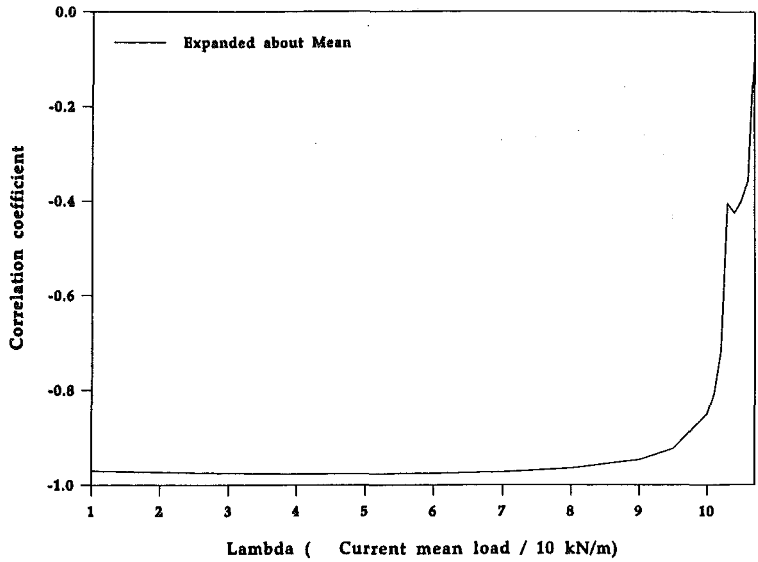

FIG. 11. Portal Frame: Correlation between Midspan Girder Moment and Top Column Moment

their corresponding random material variables, which in turn have been assumed independent in the analysis.

Also, solutions based on various expansion levels of the "resistance governing" basic random variables have been performed. By reading the signs and values of the gradients of certain response quantities, it is possible to assess the influence of the individual basic random variables on the response. However, since this information affects only one response quantity at a time, it can be difficult for more complex structural systems to predict the influence of a change in a basic random variable on the behavior of the system. To improve this situation, future work should also include the gradient computation of an overall stiffness estimator with respect to the basic random variables.

Based on the aforementioned judgments, the following two configurations, named symmetric imperfections and asymmetric imperfections, have been considered.

1. Symmetric imperfections are associated with the two columns that are initially given out-of-alignment positions that are symmetric with respect to the vertical center axis of the system. The lower part of the columns have an inward position, while the position is outward for the upper part. Furthermore, all random material variables are expanded about values that have the same relative reduction compared to their mean. The two random load variables are expanded about values on opposite side of their means such that the total load on the system is retained (i.e., right load increased, left load decreased).

2. Asymmetric imperfections are associated with the two columns that are initially given out-of-alignment positions that are asymmetric with respect to the vertical center axis of the system. The columns are positioned to the left of their

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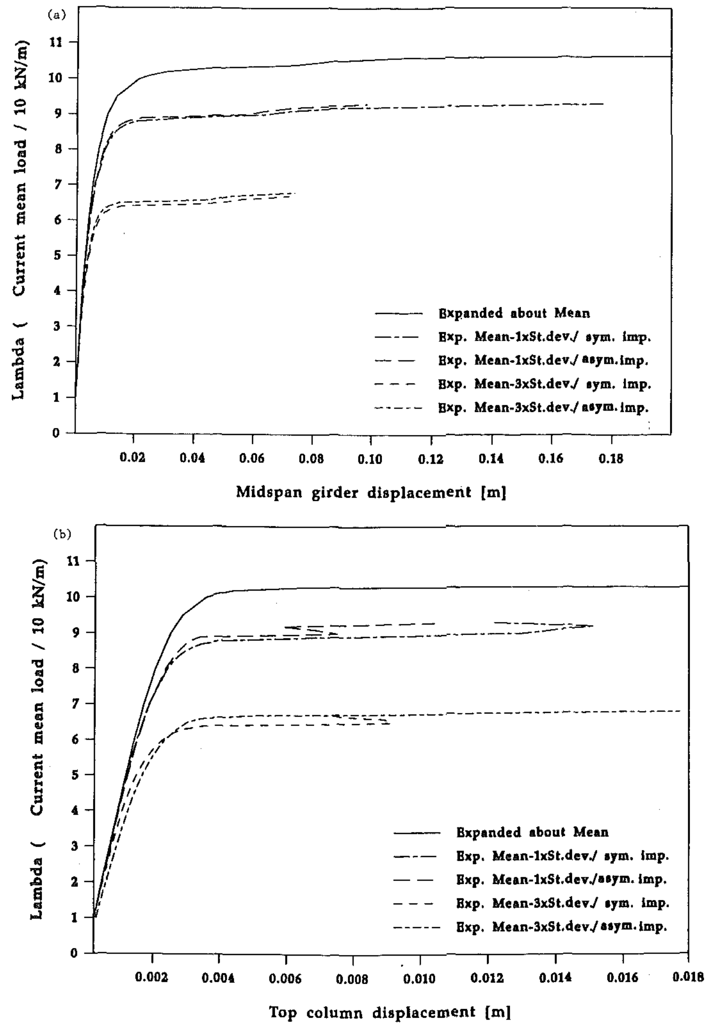

FIG. 12. Portal Frame: Effect of Expansion about Different Values of "Resistance" Random Variables on Standard Deviations of: (*a*) Midspan Girder Displacement; (*b*) Top Column Displacement

ideal geometry line in their lower part and to the right in their upper part. Random material and load variables are expanded about the same values as for the symmetric case.

Thus the two configurations deviate only with respect to the form of the geometric imperfections.

Fig. 12(*a*) shows the evolution of the midspan girder displacement, when the solutions are expanded about different levels of the basic random variables for the two configurations. While the results differ significantly for the various expansion levels due to different load carrying capacities, the differences between the two configurations at each level are only minor. In Fig. 12(b) similar results are shown for the horizontal displacement at the top of the left column. The "dents" here occur when the initial yielding in the midspan of the girder takes place. In addition, the trend in the results is the same as that shown in Fig. 12(a).

Safety Assessment Using Probabilistic Finite Element Method

How can results from a probabilistic finite element method (PFEM) be applied to structural safety assessment? This section will deal with an approach to this problem where the simply supported beam analyzed in the previous section serves as a reference example. The reason for this is that the PFEM results can then be compared to an analytical solution.

Suppose that a reliability-based code requirement is imposed with regard to a minimum reliability index $\tilde{\beta}$ that the structure should possess in order to be considered safe, i.e., $\beta \geq \tilde{\beta}$. For the simply supported beam example (see Fig. 1) with only two independent random variables (i.e., material $M_{\max}^{(b)}$ and load $q = \Lambda q_1$), we can derive an explicit expression between the load parameter Λ and the corresponding reliability index β. By introducing the following relationships

$$\text{load effect: } Q = \frac{\Lambda q_1 l^2}{8} \quad (1)$$

$$\text{resistance (zero axial force): } R = M_{max}^{(0)} \quad (2)$$

$$\text{safety margin: } S = R - Q \quad (3)$$

$$\text{reliability index: } \beta = \frac{\bar{S}}{\sigma_S} \quad (4)$$

and substituting the statistical quantities for the material and load random variables in (4), we finally arrive at the following expression:

$$\Lambda = \frac{\bar{M}_{\max}^{(0)}}{\bar{Q}_1(1 - \delta_q^2\beta^2)} [1 - \sqrt{1 - (1 - \delta_M^2\beta^2)(1 - \delta_q^2\beta^2)}] \quad (5)$$

where $\bar{Q}_1 = \bar{q}_1 l^2/8$ = the mean load effect corresponding to q_1, and δ_M and δ_q = the coefficients of variation (CoV) of material and load random variables, respectively. If the code requires a minimum reliability index, say $\tilde{\beta} = 3.0$, the corresponding load parameter $\tilde{\Lambda}$ can be found from (5). For the actual set of data (see Fig. 1) this value then becomes $\tilde{\Lambda} = 4.27$. Thus for the design to be safe enough, the mean value of the applied load $\bar{q}_a$ needs to satisfy: $\bar{q}_a \leq \tilde{\Lambda}\bar{q}_1 = 42.7$ kN/m.

The results obtained by using the computer code PFRAME (Teigen 1990; Teigen et al. 1991) for the simply supported beam are shown in Fig. 13. Fig. 13(a) shows different ($\mu + 3\sigma$) scenarios of the midspan displacement. The standard deviations are based on different expansion levels of the material random variable, while all curves are based on the same mean value, namely when expanded about the mean of the material random variable. The mean response is also shown [the same curve as that presented in Fig. 4(a)]

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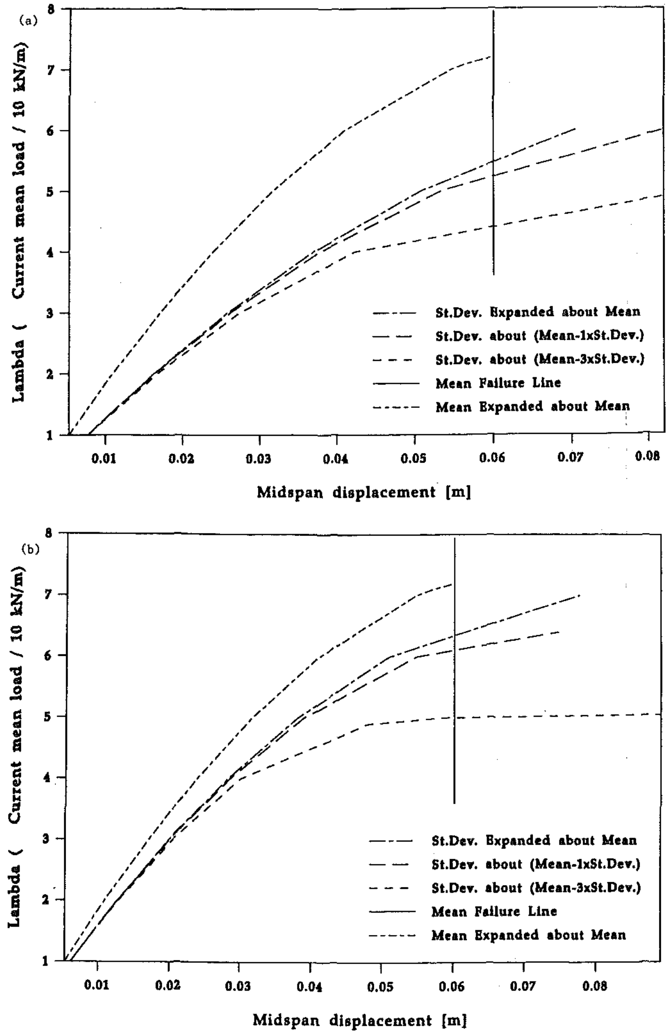

FIG. 13. Simply Supported Beam: Effect of Expansion about Different Values of Material Random Variable on: (*a*) Mean + 3 Standard Deviations of Midspan Displacement; (*b*) Mean + Standard Deviation of Midspan Displacement

together with a vertical line through the maximum mean displacement, called mean failure line. The horizontal distance between the mean response and the mean failure line can be interpreted as the mean safety margin for various values of the load parameter Λ. Furthermore, the calculated standard deviations account for all uncertainties in the system (displacements are dependent on both stiffness and load). The Λ values corresponding to intersections between the different ($\mu + 3\sigma$) curves and the mean failure line can then be taken directly as estimates for $\tilde{\Lambda}$ corresponding to $\tilde{\beta} = 3.0$. As shown, the three ($\mu + 3\sigma$) curves give quite different estimates for $\tilde{\Lambda}$. Since the correct

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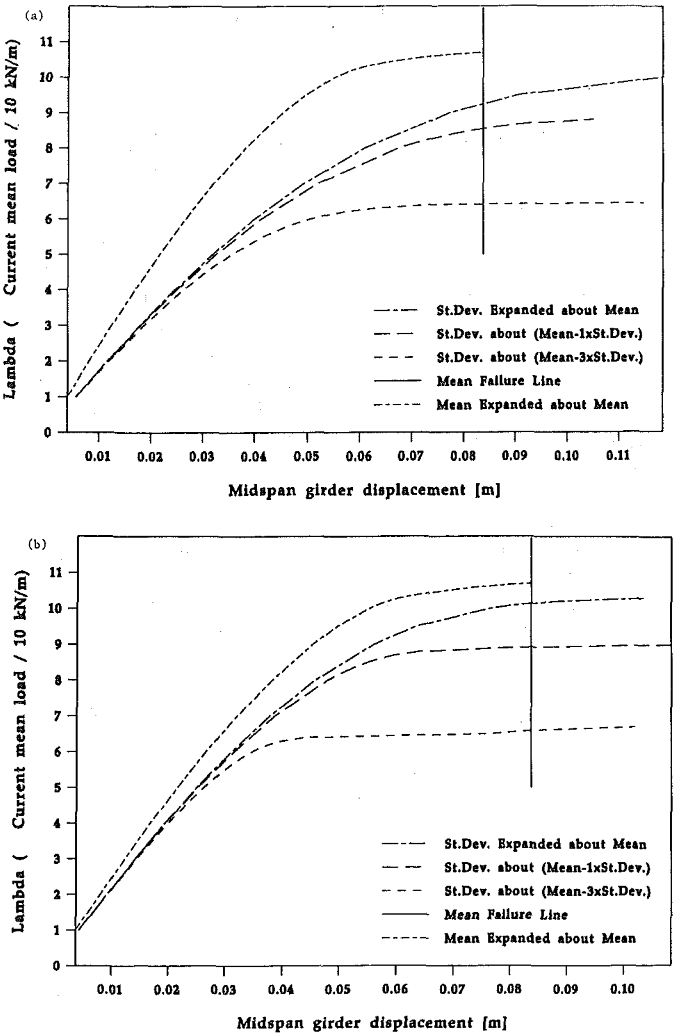

FIG. 14. Portal Frame: Effect of Expansion about Different Values of "Resistance" Random Variables on: (*a*) Mean + 3 Standard Deviations of Midspan Girder Displacement; (*b*) Mean + Standard Deviation of Midspan Girder Displacement

answer is $\tilde{\Lambda} = 4.27$, it is readily seen that, among the various curves, the one where the standard deviation of the displacement is based on expansion about $(\mu - 3\sigma)$ of the material random variable, gives the best estimate for $\tilde{\Lambda}$. For this curve we can read a value of $\tilde{\Lambda} \approx 4.40$ from the graph in Fig. 13(*a*), which is in good agreement with the correct value.

To see how the results change with $\tilde{\beta}$, the same procedure has been repeated for $\tilde{\beta} = 1.0$. The corresponding $(\mu + \sigma)$ scenarios of the midspan displacement are presented in Fig. 13(*b*). From (5) we get the correct value of the load parameter: $\tilde{\Lambda} = 6.00$. For this case the best estimate of $\tilde{\Lambda}$ is

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obtained from the curve where the standard deviation of the displacement is based on an expansion about ($\mu - \sigma$) of the material random variable. For this curve the value from the graph in Fig. 13(*b*) is $\tilde{\Lambda} \approx 6.10$, which is also in close agreement with the exact result.

This procedure for safety assessment has finally been applied to the portal frame structure shown in Fig. 8. The midspan girder displacement has been selected as the governing displacement quantity of the system. Since, as shown in Fig. 12(*a*), the expansions based on symmetric and asymmetric imperfections have almost the same effect on the midspan displacement, only the symmetric configuration has been considered in the following examples. Figs. 14(*a*) and 14(*b*) show different scenarios corresponding to a required minimum reliability index $\tilde{\beta} = 3.0$ and $\tilde{\beta} = 1.0$, respectively. The curve of interest in Fig. 14(*a*) is the one where the standard deviation of displacement is based on an expansion taken about ($\mu - 3\sigma$) of the "resistance governing" basic random variables, while expansion about ($\mu - \sigma$) is the curve of interest in Fig. 14(*b*). The load parameters corresponding to the intersections between these curves and the mean failure lines are $\tilde{\Lambda} \approx 6.40$ and $\tilde{\Lambda} \approx 8.90$ for $\tilde{\beta} = 3.0$ and $\tilde{\beta} = 1.0$, respectively. In the simply supported beam example the corresponding results were $\tilde{\Lambda} \approx 4.40$ and $\tilde{\Lambda} \approx 6.10$. Since the girders are the same in the two examples, the columns have improved the saftey of the structure substantially.

These promising results indicate that for general cases reasonably accurate safety assessments with PFEM might be obtained following a three-step procedure:

1. Expand about mean values of the basic random variables to obtain the mean response and the mean failure line.
2. Expand about ($\mu - \tilde{\beta}\sigma$) of the "resistance governing" basic random variables to obtain a standard deviation that is representative for the required reliability index $\tilde{\beta}$.
3. Obtain $\tilde{\Lambda}$ either by means of a graphic solution or, more efficiently, using a computer program.

Summary and Conclusions

In this second part of the investigation on probabilistic finite element methods for nonlinear structures two reinforced concrete application examples including a simply supported beam and a portal frame are presented. Nonlinearity in material and geometry, and randomness in load, material, and geometry are taken into account. It is shown that the probabilistic finite element method proposed in the companion paper (Teigen et al. 1991) is able to predict the behavior of material and geometric nonlinear concrete structures accounting for the uncertain nature of these structures and their environments. Furthermore, it is shown that the method is applicable to the assessment of structural safety of nonlinear concrete structures. A simple approach to this problem that matches a reliability index design code format is developed. Although this approach, which does not require an expression of the limit state function or the solution of a time-consuming optimization problem, is still in its infancy, the results are so far encouraging. Further investigations are necessary in order to develop this probabilistic finite element approach into a mature structural safety assessment method.

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