Multiscale Finite Element approaches: error estimations and adaptivity for an enriched variant

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ABSTRACT

The Multiscale Finite Element Method (MsFEM) is a Finite Element type approximation method for multiscale problems, where the basis functions used to generate the approximation space are precomputed as solutions to problems posed on local elements and ressembling the global problem of interest. These basis functions are thus specifically adapted to the problem at hand. Once these local basis functions have been computed, a standard Galerkin approximation of the global problem is performed. Many ways to define these basis functions have been proposed in the literature over the past years. While a priori error estimates have been established for all these variants, a posteriori estimates are much less frequent and we refer e.g. to [1, 2] for some contribution in that direction.

In this work, we introduce and analyze a specific MsFEM variant, the construction of which is inspired by component mode synthesis techniques. In particular, we enrich the standard MsFEM basis set by highly oscillatory basis functions that are solutions to local equilibrium problems and satisfy Dirichlet boundary conditions (on the boundary of the local elements) given by (possibly high order) polynomials.

After having discussed the performance of this new approach, we present a posteriori error estimates that are useful to appropriately choose the degrees of the polynomial functions used as boundary conditions on each edge of the coarse mesh.

This work [3] is joint with U. Hetmaniuk (University of Washington), C. Le Bris and P.-L. Rothé (ENPC and Inria).

REFERENCES

[1] L. Chamoin and F. Legoll, A posteriori error estimation and adaptive strategy for the control of MsFEM computations, *Computer Methods in Applied Mechanics and Engineering*, Vol. **336**, pp. 1-38, (2018).

[2] L. Chamoin and F. Legoll, Goal-oriented error estimation and adaptivity in MsFEM computations, *Computational Mechanics*, Vol. **67**, pp. 1201-1228, (2021).

[3] F. Legoll, P.-L. Rothé, C. Le Bris and U. Hetmaniuk, An MsFEM approach enriched using Legendre polynomials, *SIAM Multiscale Modeling and Simulation*, Vol. **20**, pp. 798-834, (2022).