MULTISCALE SCATTERING IN NONLINEAR KERR-TYPE MEDIA

Roland Maier*, **Barbara Verfürth**†

* Department of Mathematical Sciences
Chalmers University of Technology and University of Gothenburg
Chalmers Tvärgata 3, 412 96 Göteborg, Sweden
e-mail: roland.maier@chalmers.se

† Institut für Angewandte und Numerische Mathematik
Karlsruher Institut für Technologie
Englerstr. 2, 76131 Karlsruhe
e-mail: barbara.verfuerth@kit.edu

Abstract

Wave propagation in heterogeneous and nonlinear media has arisen growing interest in the last years since corresponding materials can lead to unusual and interesting effects and therefore come with a wide range of applications. An important example for such materials are Kerr-type media, where the intensity of a wave directly influences the refractive index. In the time-harmonic regime, this effect can be modeled with the nonlinear Helmholtz equation

$$-\mathrm{div} A\nabla u - k^2 n(1+\varepsilon \mathbb{1}_{D_\varepsilon}|u|^2)u \;=\; f,$$

where D_ε is the subdomain where the nonlinear Kerr-type medium is active, A, n, and ε are material coefficients and k is the wave number. In this contribution, the coefficients A, n, ε may vary on small spatial scales, such that the numerical approximation of corresponding solutions can be a delicate task.

To deal with microscopic coefficients without the need for global fine-scale computations, multiscale methods can be applied. One such method is the Localized Orthogonal Decomposition method (LOD), which works under minimal structural assumptions. The approach was proposed in [MP14] and refined in [HP13] for an elliptic model problem and constructs appropriate coarse-scale spaces that take into account problem-dependent information. In this talk, which is based on [MV20], an iterative and adaptive construction of approximation spaces based on the LOD is presented. The general idea is to combine ideas of [WZ18] on iterative finite element approximations with the above-mentioned multiscale approach. That is, in each iteration step, a new coarse-scale solution is constructed based on the solution of the previous step. To avoid costly re-computations, an error indicator is used to locally decide in each step whether to update the approximation space. For sufficiently small tolerance employed in this decision, an a priori error estimate can be shown which is of optimal order in the mesh size – independent of the possible low regularity of the exact solution. These results are also illustrated by numerical experiments.

References

[HP13] P. Henning and D. Peterseim. Oversampling for the multiscale finite element method. *Multiscale Model. Simul.*, 11(4):1149–1175, 2013.

[MP14] A. Målqvist and D. Peterseim. Localization of elliptic multiscale problems. *Math. Comp.*, 83(290):2583–2603, 2014.

[MV20] R. Maier and B. Verfürth. Multiscale scattering in nonlinear Kerr-type media. *ArXiv Preprint*, 2011.09168, 2020.

[WZ18] H. Wu and J. Zou. Finite element method and its analysis for a nonlinear Helmholtz equation with high wave numbers. *SIAM J. Numer. Anal.*, 56(3):1338–1359, 2018.