MULTISCALE SCATTERING IN NONLINEAR KERR-TYPE MEDIA

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ABSTRACT

Wave propagation in heterogeneous and nonlinear media has arisen growing interest in the last years since corresponding materials can lead to unusual and interesting effects and therefore come with a wide range of applications. An important example for such materials are Kerr-type media, where the intensity of a wave directly influences the refractive index. In the time-harmonic regime, this effect can be modeled with the nonlinear Helmholtz equation

$$-\operatorname{div} A \nabla u - k^2 n(1 + \varepsilon \mathbb{1}_{D_\varepsilon} |u|^2) u = f,$$

where D_ε is the subdomain where the nonlinear Kerr-type medium is active, A , n , and ε are material coefficients and k is the wave number. In this contribution, the coefficients A , n , ε may vary on small spatial scales, such that the numerical approximation of corresponding solutions can be a delicate task.

To deal with microscopic coefficients without the need for global fine-scale computations, multiscale methods can be applied. One such method is the Localized Orthogonal Decomposition method (LOD), which works under minimal structural assumptions. The approach was proposed in [MP14] and refined in [HP13] for an elliptic model problem and constructs appropriate coarse-scale spaces that take into account problem-dependent information. In this talk, which is based on [MV20], an iterative and adaptive construction of approximation spaces based on the LOD is presented. The general idea is to combine ideas of [WZ18] on iterative finite element approximations with the above-mentioned multiscale approach. That is, in each iteration step, a new coarse-scale solution is constructed based on the solution of the previous step. To avoid costly re-computations, an error indicator is used to locally decide in each step whether to update the approximation space. For sufficiently small tolerance employed in this decision, an a priori error estimate can be shown which is of optimal order in the mesh size – independent of the possible low regularity of the exact solution. These results are also illustrated by numerical experiments.

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