

A modified Constitutive Relation Error (mCRE) framework to learn nonlinear constitutive models from strain measurements with thermodynamics-consistent Neural Networks

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ABSTRACT

The damage of mechanical structures is a permanent concern in engineering, related to issues of durability and safety. The theme is currently the subject of various research activities; a typical example is the ERC project DREAM-ON (2021-2026), in which this work is involved, which focuses on complex mechanical structures in composite materials and aims to address the numerical challenges related to integrated health monitoring of large-scale structures, in order to move from smart materials to smart structures, able to monitor their condition autonomously and operate safely even in degraded mode. More specifically, the work addresses a particular challenge of the ERC project; it aims at building an efficient numerical procedure for the assimilation of data from distributed fiber-based sensors. The idea is to create a hybrid numerical twin, combining physical models (which represent a rich history of engineering sciences, and which provide a strong a priori knowledge) and learning techniques from AI (here, neural networks). To get rid of model bias, neural networks, known as universal approximators, can be used to represent the constitutive relation. Yet, classical neural networks (in the sense that they are not informed by physics), have the disadvantages of requiring very large volumes of data to be trained, as well as decreasing accuracy when generalizing to new data. Coupling techniques between machine learning and physical models help to alleviate frequent concerns in neural network such as the lack of physical consistency, lack of generalization and difficulty to train (quantity of data, hyperparameters tuning).

Here, a method using neural networks for learning behavior laws in the form of thermodynamic potentials is proposed. In this approach, the architecture of the network satisfies thermodynamic principles thanks to computation of some quantities by automatic differentiation as well as convexity properties imposed in the neural network structure. Coupling physical knowledge with neural network for constitutive modelling is now an emerging field and two trends can be distinguished.

A first community aims to train neural networks in a supervised learning procedure with strain-stress database (or strain-free energy) generated from a known constitutive model [1, 2, 3, 4]. As the forward pass of a neural network can be easily parallelized, the use of neural network after training can achieve high gain in computational time when the initial model is costly to compute, for example in the case of microscale modelling. Even though the goal of this paper is not part of this trend, inspiration can be found in the way to include physical knowledge.

One other goal - the one of this work - is to discover constitutive relation from observations. Measuring strain-stress couple with complex loading is today a challenge, so the supervised training procedure cannot be used to train neural network. In [5] and [6] methods tackled the issue of unsupervised training of neural network for constitutive modelling but full-field displacement observations are needed. The presented work proposes an unsupervised method based on the minimization of the modified Constitutive Relation Error [7] to train a thermodynamics-consistent Neural Network with partial strain observations. The methodology will be illustrated and analyzed on different test cases.

REFERENCES

- [1] F. As'ad, P. Avery, C. Farhat, A mechanics-informed artificial neural network approach in data-driven constitutive modeling, *International Journal for Numerical Methods in Engineering* 123 (03 2022). doi:10.1002/nme.6957.
- [2] J. N. Fuhg, C. M. Hamel, K. Johnson, R. Jones, N. Bouklas, Modular machine learning-based elastoplasticity: generalization in the context of limited data (2022). doi:10.48550/ARXIV.2210.08343
- [3] F. Masi, I. Stefanou, Multiscale modeling of inelastic materials with thermodynamics-based artificial neural networks (tann), *Computer Methods in Applied Mechanics and Engineering* 398 (2022) 115190. doi:https://doi.org/10.1016/j.cma.2022.115190.
- [4] C. Bonatti, D. Mohr, One for all: Universal material model based on minimal state-space neural networks, *Science Advances* 7 (26) (2021) eabf3658.
- [5] P. Thakolkaran, A. Joshi, Y. Zheng, M. Flaschel, L. De Lorenzis, S. Kumar, Nn-euclid: Deep-learning hyperelasticity without stress data, *Journal of the Mechanics and Physics of Solids* 169 (2022) 105076. doi:10.1016/j.jmps.2022.105076.
- [6] D. Z. Huang, K. Xu, C. Farhat, E. Darve, Learning constitutive relations from indirect observations using deep neural networks, *Journal of Computational Physics* 416 (2020) 109491.
- [7] A. Chouaki, P. Ladevèze, L. Proslie, An updating of structural dynamic model with damping, *Inverse Problems in Engineering: Theory and Practice* (1996) 335 – 342 Cited by: 9.

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