

DAMAGE IDENTIFICATION OF RC BEAMS USING FEED-FORWARD BACK PROPAGATION NEURAL NETWORK APPROACH (FFBPN)

KJAROL A. OROZ^{*}, VLADIMIR RAMOS^{*}, JOAN R. CASAS[†] AND RICK M. DELGADILLO^{*}

^{*} Department of Civil Engineering, Peruvian University of Applied Sciences (UPC)
Prolongación Primavera 2390, Monterrico Santiago de Surco, Lima, Perú - web page:
<https://www.upc.edu.pe/>
e-mail: u201714025@upc.edu.pe, u201615839@upc.edu.pe, rick.delgadillo@upc.edu.pe

[†] Department of Civil and Environmental Engineering, Technical University of Catalonia
(BarcelonaTech), Catalonia, Spain.
Campus Nord, C1 building. Jordi Girona, 1-3, 08034, Barcelona, Spain
e-mail: joan.ramon.casas@upc.edu - web page: <https://www.upc.edu/es>

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Abstract. Reinforced concrete (RC) beams are constantly exposed to environmental factors, overloads and aging, which is a challenging problem because they increase the risk of structural failure. In this sense, the detection of structural damage through modal parameters and artificial intelligence (AI) tools makes it possible to establish a comprehensive and precise vibration-based methodology within the field of Structural Health Monitoring (SHM). In this article, the variation of vibration frequencies is used in combination with Machine Learning (ML) techniques through the use of Feed-Forward Back Propagation Network (FFBPN) for structural damage detection in reinforced concrete beams (RC). The proposed methodology considers three steps: (i) the vibration frequencies of the beam are obtained using the ANSYS software for twenty-six structural damage scenarios, of which twenty are for code training and six are test for accuracy validation (ii) the training data and the data set values are used to study the performance of an FFBPN (iii) The FFBPN trained with the natural frequency data is able to detect and assess the severity of transverse cracks in the beam. Finally, the proposed methodology shows an accuracy greater than 91.5% for damage detection and its severity in the reinforced concrete beam for the six test scenarios proposed. Besides, these results serve to evaluate the structural conditions in beams of real constructions such as buildings, hospitals, schools.

1 INTRODUCTION

Buildings are fundamental for the development of society, since they allow the supply of the great demand for housing that currently exists due to the accelerated growth of the population. Therefore, they must be maintained in optimal architectural and, above all, structural condition, which allows them to guarantee the safety of their occupants. However, there are various factors that affect the operability and useful life of buildings such as inadequate structural calculations, landslides, debris flow and environmental conditions, but mainly damage to structural elements due to seismic movements and earthquakes. Currently, modern artificial intelligence techniques are not used for damage detection. Structural inspections are commonly done visually and with images [1]. Furthermore, the presence of damage in a beam produces a change in the stiffness of the structural element, consequently, producing a discontinuity in the mode shape reducing the vibration frequency. Therefore, neural networks are a reliable and practical alternative to model the behavior of reinforced concrete beams and detect their damage through vibration frequencies [2].

In [3] the cracking and deformations of reinforced concrete beams are evaluated using artificial neural networks to evaluate cracking and at the same time maintain a consistent deformation pattern, the results show that ANNs make reliable predictions of crack width and this is supported by laboratory testing and validation. The R statistic value of training, testing and validation is 0.90109, 0.95589 and 0.96000 respectively. The authors [4] present an automated scheme for crack detection based on digital image correlation (DIC) measurement and calibration. This scheme allows an automatic evaluation of opening and sliding along the crack. Also, in [5] a new crack detection approach was proposed that combines DCNN deep convolutional neural networks and local threshold segmentation, 98.26% accuracy was obtained on 25 test images.

In [6] a new method of damage detection and localization using the vibration of the structure is proposed using a long short-term memory (LSTM) network, it is demonstrated that the technique is capable of classifying time series signals into multiple classes and damage levels with high precision. Furthermore, in [7] proposes a methodology for detecting damage in structures where the location of the damage is determined using dynamic data of the damaged structure, a finite element model of damaged beams with various geometries and boundary conditions was used to validate the proposed method performs dynamic tests on a damaged reinforced concrete bridge and the model of a cantilever steel beam. The authors [8] study the detection of damages that present uncertainties such as: modeling errors, measurement errors, variable loading conditions and environmental noises, using finite element modeling (FEM). In this method, only dynamic responses of the healthy real system are used to update the FEM model and minimize errors. An investigation on a cantilever steel beam with multiple cracks is presented in [9] and modeled in ANSYS software. For damage detection, the modal sensitivity method based on Bayesian parameter estimation is used to minimize the difference between the calculated and measured results. Furthermore, the authors of [10] develop a method based on modified damage indices (DI), and use artificial neural network (ANN) in order to locate and quantify damage in steel frames. The results demonstrate the efficiency and competence of the proposed non-destructive method for damage detection. Also, other research studies bridges

through real case studies and numerical models. For example, [11] proposes a methodology to detect and locate damage to bridges that present environmental variability and traffic considering the non-stationary vehicle-bridge elements. The algorithms efficiently solve the problem when operational and environmental variability in the recorded data is considered. The authors [12] studied images for the evaluation of the conditions of the reinforced concrete road bridge components. The results between the experiments, the simulation and the ANN predictions turned out to be very satisfactory. In [13] identifies structural damage in bridges using the responses of vibrations subjected to environmental and vehicle-induced excitation. As a result, it was proven that the empirical vibration parameters evaluated are suitable for damage identification (detection, localization and quantification). In [14] the non-linear and non-stationary dynamic response of bridges under operational loads is studied, the signals are used which are decomposed into intrinsic mode functions (IMF) using a novel completed set EMD technique enhanced with adaptive noise (ICEEMDAN).). The experimental results demonstrate greater sensitivity and robustness for damage location. In [15] presents a technique to detect damage at the element level of a reinforced concrete building, using the ANN method. As a result, lightweight and robust networks lead to accurate detection at floor and element level quickly.

In this article, artificial neural networks (ANN) will be used to identify the presence and severity of structural damage by varying modal parameters (vibration frequencies) in reinforced concrete (RC) beams. The usefulness of natural vibration frequencies in structures has been proven as a reliable method for detecting and locating damage; in addition, it can be used in various types of real structures in civil engineering [16], [17]. Likewise, the authors of [18], [19] have carried out in-depth investigations on the detection of cracks in beams by analyzing their modal parameters in which it was determined that it is possible to perform the arrest, location of the damage correctly, and The extent of damage can also be estimated. In [20] the Artificial Neural Networks (ANN) technique is used to evaluate the modal parameters of a GFRP polymer beam with cracks. In that study, the modal frequencies of the beam with different cracks are obtained from the MATLAB software and the ANN results are compared with those obtained using the FEM in ANSYS. The article concludes that the ANN can accurately predict taking into account the performance of the neural network to obtain the modal parameters of the beam.

The main objective of this article is to identify the presence and severity of structural damage in a rectangular reinforced concrete cantilever beam using the combination of modal parameters and ML algorithms such as artificial neural networks (ANN). A cantilever beam was chosen since with these support conditions the variations in frequencies and deformations can be better appreciated. To determine the vibration frequencies, the reinforced concrete beam was modeled in the ANSYS software and damage was simulated in 26 scenarios, of which 20 scenarios were used for training the code and 6 test scenarios for validation of operation and determination of the precision. Subsequently, the natural frequencies of the 20 training scenarios were used as input data in an ANN model in order to determine the existence and severity of damage as output data. The neural network was created using MATLAB software [21]. Figure 1 shows the process for damage detection using AI algorithms by evaluating the frequencies in each damage scenario.

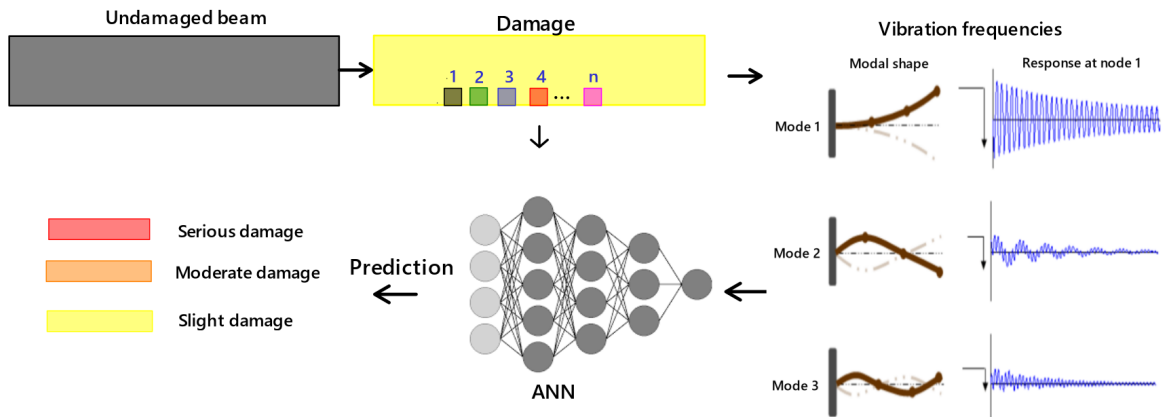


Figure 1: Damage detection process of a reinforced concrete beam.

2 FEM BEAM SIMULATION

In the present investigation, a cantilevered concrete beam was modeled, where the mechanical properties of the beam are Young's modulus 3200 N/m^2 , Poisson's modulus 0.2, density 2460 kg/m^3 , shear modulus 17 N/m^2 , tensile strength 12 N/m^2 , elastic limit 82 N/m^2 , compressive strength (f_c) 210 kg/cm^2 . Figure 2 shows the geometric characteristics of the beam; 25 cases of damage were simulated with cracks located in different positions.

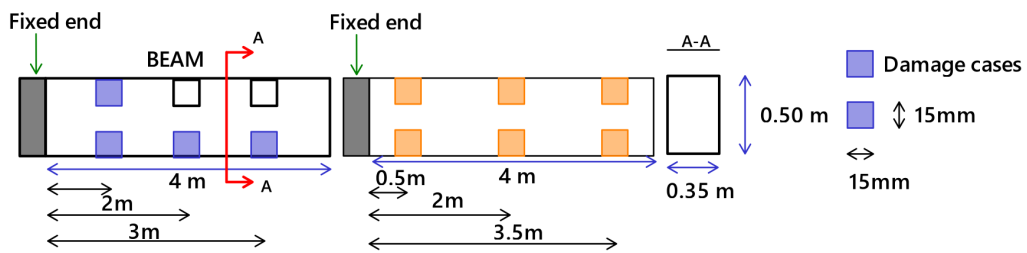


Figure 2: Geometric properties of the beam.

Figure 3 shows the damage scenarios located at 1 m, 2 m and 3 m from the fixed support on the upper and lower face of the beam. Also, Figure 4 shows the damage scenarios located at 0.5 m, 2 m and 3.5 m from the fixed support, in which the damage states are shown and 6 cracks with a square section of $15 \times 15 \text{ mm}$ were considered that run along the transverse axis.

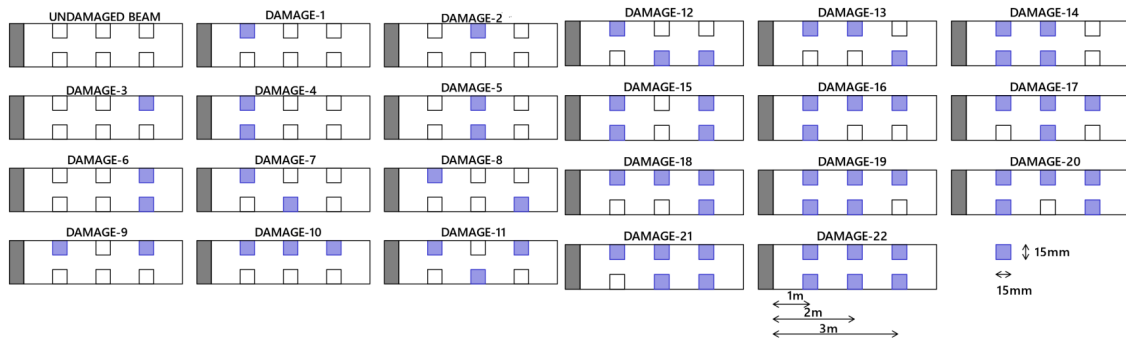


Figure 3: Damage scenarios in reinforced concrete beam.

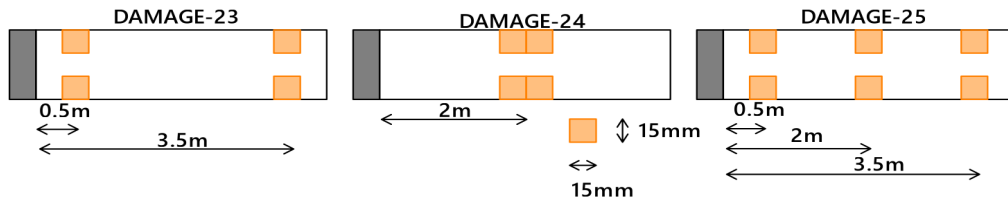


Figure 4: Test damage scenarios on reinforced concrete beam.

3 VIBRATION FREQUENCIES

Table 1 shows the results in ANSYS Workbench for the first 5 scenarios, the three-dimensional (3D) finite element model was used, where a 50 mm mesh was used and the boundary conditions were established to obtain the 8 modes of vibration damage cases.

Table 1: Vibration frequencies

Mode	1	2	3	4	5	6	7	8
Undamaged Beam	4.017E-03	5.703E-03	2.437E-02	3.356E-02	4.001E-02	6.513E-02	7.138E-02	8.646E-02
Damage 1	4.015E-03	5.695E-03	2.437E-02	3.356E-02	3.999E-02	6.512E-02	7.134E-02	8.639E-02
Damage 2	4.017E-03	5.701E-03	2.436E-02	3.351E-02	4.001E-02	6.513E-02	7.136E-02	8.646E-02
Damage 3	4.018E-03	5.703E-03	2.437E-02	3.354E-02	4.002E-02	6.511E-02	7.138E-02	8.634E-02
Damage 4	4.013E-03	5.687E-03	2.437E-02	3.356E-02	3.998E-02	6.511E-02	7.131E-02	8.632E-02
Damage 5	4.016E-03	5.699E-03	2.435E-02	3.347E-02	4.000E-02	6.513E-02	7.134E-02	8.646E-02

4 IMPLEMENTATIONS OF THE ANN

ANNs are computational information processing systems that imitate the way human neurons operate; upon receiving information, these systems analyze and retain it for future application. They commonly present an architecture of three main components: an input layer, one or more hidden layers, and an output layer. Figure 5 shows the architecture of an ANN in which W_{ij} represents the weight of the neuronal connection between an input and the neurons in the hidden layer, the bias is represented by B_j and W_{j1} defines the weight of the neuronal connection between the neuron in the hidden and output layers. The B_1 is the bias associated with the neurons in the output layer. The indices $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$ represent the amount of data collected and the number of hidden layer neurons respectively.

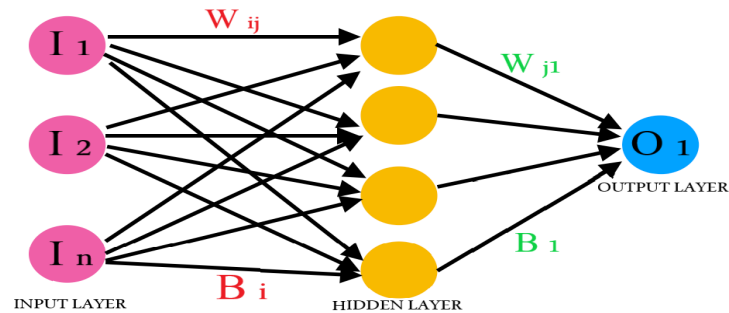


Figure 5: ANN's architecture.

Equation (1) defines the total number of parameters (bias and weight) used in this neural network.

$$n \times (m + 2) + 1 \quad (1)$$

Two formulas are required to go from the input layer to the output layer. Equation (2) presents a summation function that is related to the training parameters and the results of the previous layers

$$\varphi^j = \varphi \left(\sum_{i=1}^n W_{ij} f_i + b_j \right), \quad j = (1 - m) \quad (2)$$

Where W and B represent the weight and bias respectively of the training parameters. The variable n is the number of extracted data and m is the number of neurons selected in the hidden layer. Finally, φ^j and f_i are the input and output data of the neural network respectively. On the other hand, in equation (3) the output of the hidden layer towards the outputs is determined as presented in the following formulation:

$$\varphi_j^1 = \frac{1}{1 + e^{-\varphi_j}} \quad (3)$$

The MatLab software was used to train a feedforward backpropagation ANN. For this, the Levenberg–Marquardt algorithm was used as the training function, the hyperbolic tangent as the activation function, and the mean square error (MSE) as performance validation. Likewise, the method shown in [24] was applied to determine the number of neurons required for the ANN. For the development of the ANN, it was determined to use 2 hidden layers of 25 neurons each. The complete information of the neural network architecture (metadata) is shown in Table 2.

Table 2: ANN'S Metada

ANN Type	Feed Forward Backpropagation
Number of hidden layers	2
N. ° of neurons in the input layer	8
N. ° of neurons in hidden layers	25
N. ° of neurons in the output layer	4
Activation function	Hyperbolic Tangent
Performance validation	Mean Square Error
Training algorithm	Levenberg–Marquardt

The structure of the ANN and its respective metadata are shown in Figure 5. You can see the 8 neurons as inputs that correspond to the 8 modes of vibration of the beam, the two hidden layers of 25 neurons and the 4 output neurons, which correspond to the four possible states of damage severity that the neural network can detect. The green squares belong to the input layer and output layer. Likewise, the two hidden layers are observed with blue squares.

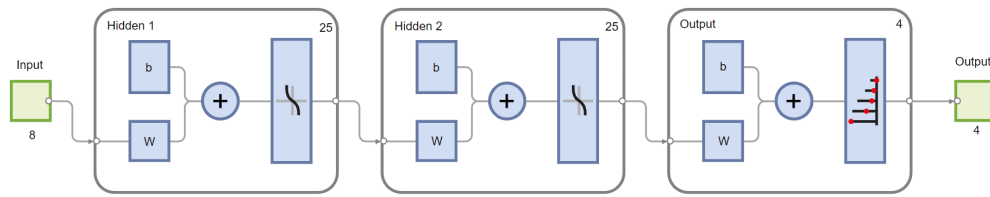


Figure 5: ANN's structure.

To train the network, the frequencies of the first eight modes of the 20 training scenarios obtained through finite element simulations (FEM) of the beam were used as input data, as explained in the previous chapter. It was determined to use 8 modes since the sum of the accumulated modal energies was greater than 85% of the total energy of the beam and good precision was obtained. 70% of the input data was used to train the network, while 15% was used for error validation and another 15% as tests to validate the performance of the neural network. Likewise, the operation of the ANN was validated with 6 additional test scenarios to obtain the accuracy of the neural network. Regarding the output data, four variables were established. The first variable describes the presence of damage, with two possible options: 0 if there is no presence of cracks, 1 if there is a presence of cracks. The other three output variables indicate the magnitude of the beam damage. The damage scenarios considered during the training phase are the same as those detailed in the previous chapter, which implies that the crack parameters remain constant, with an opening of 0.015m and the positions at $x=0.50\text{m}$, $x=2.00\text{m}$ and $x=3.50\text{m}$ considering $x=0$ as the left end of the beam.

5 ANALYSIS OF THE RESULTS

Once the training stage was completed, the evaluation of the performance of the ANN was carried out using the MatLab Neural Network Training interface in which the performance and regression of the network could be observed by drawing lines and curves. neuronal. Figure 6 shows the validation graph of the most accurate performance obtained from ANN training evaluated using the mean square error (MSE), which is a metric used to measure the performance of a neural network. The lines on the graph represent the MSE for each of these training stages. The blue line represents the MSE for the training data set, the red line for the testing data set, and the green line for the validation data set. The goal is to minimize the MSE in all these stages to obtain an accurate and well-fitted model. The lines follow a trend towards the x-axis since with each cycle (Epoch), which is the number of times the complete model sees the training data set during the training process, the error decreases. The lowest MSE value was 1.9476×10^{-23} which was obtained in cycle number 60 for the data validation set. Which indicates that although the code is designed for 1000 training cycles, only 60 were necessary to validate them. In this sense, depending on the required precision, the number of targets and especially the number of neurons in the hidden layers, the training may require fewer cycles to validate a minimum MSE that guarantees considerable precision of the ANN.

Likewise, the bar graph of the neural network's errors when estimating the severity of the damage is also shown. The inaccuracy histogram is the representation of the discrepancies between the desired and estimated values after training the feedforward ANN. These erroneous

values show the difference between the estimates and the objectives set for the neural network, which were identification of the presence and severity of the damage. Each of the bars represents a range of error values and the height of each bar indicates the number of predictions that fell within that range, blue bar for the training data set, red bar for the test and the green bar for the validation data set. The graph shows 15 predictions (11 training, 3 test and 1 validation) that fell within the error range of approximately 0.00412, 1 validation prediction in error range -0.2187, 3 training predictions in error range 0.01734 and 1 validation prediction in error range 0.189. In general, a symmetric distribution of errors is observed around zero, which indicates that the ANN is making predictions with high precision for both high and low values.

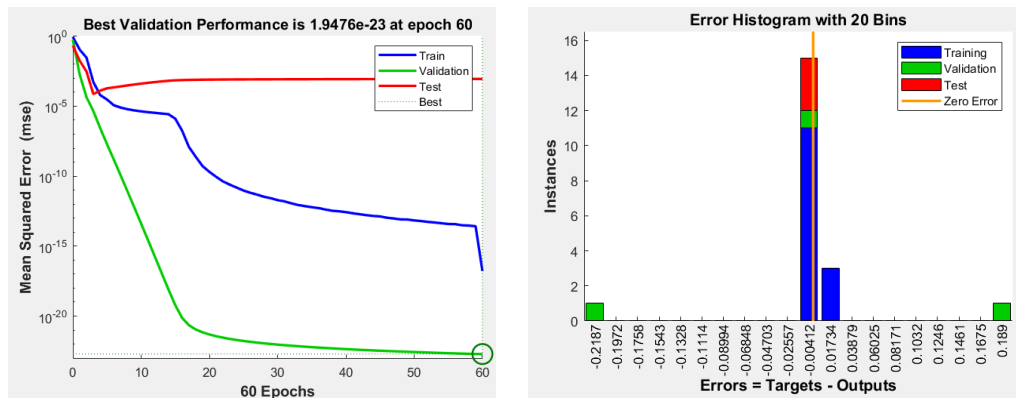


Figure 6: Validation of the ANN's best performance and bar graph of the neural network errors.

Figure 7 shows the graphs of the performance of the neural network for the four damage severity objectives (targets) that showed values between 99.7% and 99.9% precision, which indicates that good precision was obtained from the training of the neural network. Furthermore, it was observed that when the number of neurons in the hidden layers was decreased to 2, the accuracy was reduced by approximately 56% for one of the test targets. Which indicates that according to the amount of input data, it is necessary to evaluate the necessary number of neurons in the hidden layers to obtain better training of the neural network [25].

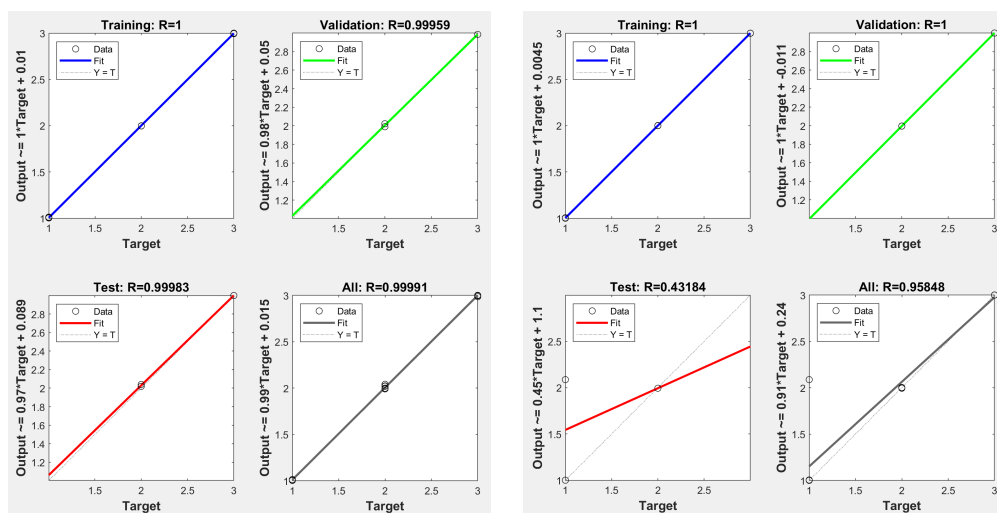


Figure 7: Neural network training results.

The damage scenarios, as well as the predictions of the severity of the damage and accuracy obtained from the neural network for the 6 test scenarios are presented in Table 3. It can be seen that 5 scenarios were predicted with damage and 1 without damage, likewise the accuracy of the predictions was greater than 85% in general with values of up to 98%, indicating that the ANN training was satisfactory to obtain significant results. The accuracy of the neural network was directly proportional to the amount of data used for training. The prediction for moderate (91%, 92%) and severe (95%, 97%) damage severities was higher because 20 damage scenarios were used and 13 of them presented moderate and severe severity, therefore the prediction for these severities were greater.

Table 3: Damage and severity scenarios obtained

Predicted Output Data			
Scenario	Damage Presence	Damage Severity	Precision
1	Yes	Moderate Damage	91%
2	Yes	Serious Damage	97%
3	Yes	Serious Damage	95%
4	Yes	Minor Damage	89%
5	Yes	Moderate Damage	92%
6	No	No Damage	85%

Table 4 shows the analysis and comparison of the results for the 6 test scenarios. The predefined damages were compared with the results predicted by the neural network and it was observed that the ANN managed to correctly define the six damage states, as well as their severity. The neural network obtained 100% accuracy in damage predictions and an average accuracy of 91.5% in damage severity predictions. Likewise, it was observed that the lowest precision obtained was 85% in the scenario without damage, this is because in the training data there was only 1 scenario with the label without damage.

Table 4: Defined test damage scenarios and severity results

Defined Data		Predicted Data	
Scenario	Damage Severity	Damage Severity	Precision
1	Moderate Damage	Moderate Damage	98%
2	Serious Damage	Serious Damage	89%
3	Serious Damage	Serious Damage	88%
4	Minor Damage	Minor Damage	97%
5	Moderate Damage	Moderate Damage	92%
6	No Damage	No Damage	85%

After analyzing the results obtained, it was possible to validate that the ANN model shown in this study can be functional and also has high precision in the identification of structural damage, thus serving as a basis for other research on new methodologies for monitoring the structural health (SHM). Feed-forward Backpropagation ANNs can learn from the information and refine their predictive ability over time, which could result in more accurate and reliable estimates. Additionally, ANNs are more versatile than conventional methodologies since they

can identify more complex interactions between input and output variables. This can be particularly valuable in situations where the fundamental relationships between variables are not adequately understood or are non-linear. Finally, ANNs can be trained with large amounts of data and even years of monitoring (big data), allowing the inclusion of a wide number of quantitative and qualitative input variables, which could generate more robust predictions. Consequently, neural networks can offer engineers a powerful instrument to detect and evaluate structural damage in beam-like elements throughout their useful life and could eventually lead to new findings and advances in the field of SHM.

6 CONCLUSIONS

- In this paper, an ANN was applied to identify and quantify the severity of structural damage by varying the frequencies of a cantilevered concrete beam with a rectangular cross section. To train the neural network, 20 scenarios with different levels of damage were simulated, which obtained the vibration frequencies for the first 8 modes that guaranteed a significant participation of the mass of the beam. Furthermore, 6 additional test scenarios were used to validate the performance and accuracy of the neural network.
- The ANN model was able to predict damage and classify it according to its severity as mild, moderate and severe. In addition, this demonstrates the success of using dynamic properties such as vibration frequencies to train an ANN with high precision and its subsequent identification and evaluation of the severity of the damage. The performance of the neural network was greater than 91.5%, which means that it has a high reliability value. By analyzing the MSE and the mean of the results, it has been shown that the ANN model provides a more effective prediction for damage identification than traditional high-order and linear regression models. Which is an important advance in the field of SMH since it allows monitoring by varying modal parameters (non-destructive) of the state of structural integrity in concrete beams. Regarding the training of the ANN, the number of targets and especially the number of neurons in the hidden layers directly influence the number of epochs that the network must simulate to validate a minimum MSE that guarantees considerable precision of the ANN. Therefore, it is advisable to analyze the amount of input data and the number of targets that the neural network will have to define the number of neurons necessary for better efficiency of the ANN.
- Finally, the results of this research will be used to evaluate real structures such as buildings, hospitals, schools and others. Furthermore, AI tools have been shown to optimally assist in detecting, locating and quantifying structural damage in resistant structural elements.

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