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EFFECT OF STEP SURFACE IMPERFECTIONS ON BOUNDARY LAYER TRANSITION

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Abstract. Surface imperfections due to manufacturing tolerance, such as steps, gaps, and waviness, can cause early boundary layer transition, thus reduce the practicability of the natural laminar flow (NLF) technology for aircraft drag reduction. In this study, a new numerical methodology is proposed and implemented to predict the effect of backward-facing step (BFS) and forward-facing step (BFS) on boundary layer transition over NLF aircraft body. The method is based on a hypothesis that the effect of surface imperfections on boundary layer flow can be mimicked as that of surface roughness due to the relevant small size of the imperfection compared to the local boundary layer thickness. The predicted transition location on BFS/FFS surface agrees very well with the experimental validation data. It implies the applicability and capability of this approach for transition location prediction over more general surface imperfection arrangements. The study thus provides an efficient, practical prediction tool to determine the manufacturing allowance on surface imperfection dimension on NLF body for aerodynamic engineers.

1 INTRODUCTION

Laminar flow control (LFC) or hybrid laminar flow control (HLFC) techniques have been widely used to reduce the drag on aircraft and thus enhance fuel efficiency by sustaining laminar flow over a bigger part of the aircraft body. However, even small surface imperfections, such as steps and gaps at junctions, waviness, and bulges can cause early laminar to turbulent transition and thus reduce the region of laminar flow. As manufacturing is only possible with certain tolerances and imperfections are unavoidable, it is necessary to specify the manufacturing tolerance for the shape and dimension of these imperfections to ensure achievement of laminar flow. This can be done only by quantification of the effects of surface imperfections on the stability of laminar boundary layer. Both experimental and numerical investigations have been performed for such purpose.

Experimental studies were conducted at low-speed incompressible flow conditions on the effect of different types of surface imperfections on boundary layer transition over natural laminar flow body. Fage[1] examined the effects of built-up ridges, essentially an integral backward-facing step (BFS) and forward-facing step (FFS) in a low speed wind tunnel using both flat plate and an aerofoil model. Smith and Clutter[2] performed a low-speed study of 2D and 3D surface excrescences and concluded that the critical step Reynold number for a 2-D wire trip was in the range of $Re_{h,crit} = [43, 260]$. Braslow[3] summarized a number of experimental data sets and concluded that 2D roughness, of the type incorporating an integrated forward and backward-facing step has a critical step Reynolds number $Re_{h,crit} \approx 200$. Drake et al[4] produced low-speed experimental database over a flat plate model with backward-facing step, forward-facing step and wave excrescence. Gaster and Wang[5] conducted low-speed wind tunnel test on the effect of BFS and FFS on boundary layer transition. In compressible flow range, systematic experiments were conducted by Costantini et al[6, 7] at a subsonic Mach number of $Ma = 0.77$ in DNW-KRG wind tunnel. The test conditions are relevant for low-sweep NLF surfaces. The surface imperfections were shown to reduce the extent of the laminar region and transition was observed to move gradually towards the step location with increasing step Reynolds number and relative step height. Flight experiments in subsonic conditions with Mach number $Ma = [0.5, 0.8]$ were conducted by Drake et al[8]. Roger[9] conducted experimental investigations on a pressure-plotting flat plate with forward-facing step in a NPL low-density tunnel at stream Mach number $M_0 \approx 2$.

In terms of numerical study, Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES) are necessary to provide fundamental flow physics and transition mechanism around the surface excrescences over an aircraft body. However, the high computing cost for the complex aircraft configurations at high Reynolds number make these methods unaffordable in real engineering applications. Alternative numerical tools have been developed to predict boundary layer transition over surface imperfection only, such as the e^N method[10, 11]. Crouch[12] and Gaster[5] have used the e^N method to investigate the effects of step excrescence on boundary layer transition. It was found that in the presence of a step, the Tollmien-Schlichting (TS) instability amplification can be locally enhanced, resulting in larger TS-wave amplitudes and a forward movement of the transition location. This forward movement in the transition location was captured by a reduction in the critical N-factor for transition onset ΔN, as modelled by $N_{tr} = N_0 - \Delta N(\frac{h}{\delta^*})$. N_{tr} and N_0 are the critical transition N-factors with step and without step, respectively. $\frac{h}{\delta^*}$ is the ratio between the step height h and the boundary layer displacement thickness δ^*at the step location[12]. Crouch[13] applied the e^N approach to capture the superposition of a forward-facing and a backward-facing step in the form of a rectangular protrusion or a wide gap to assess the applicability of this approach for more general step arrangements. The influence of the FFS on laminar to turbulent transition was studied with both DNS and the e^N method by Edelmann and Rist[14] for a compressible subsonic flow with $Ma = 0.15 - 0.8$ at varying step locations and step heights. Perraud et al[15-17] performed numerical and experimental investigations in the ONERA F2 low speed wind-tunnel in 2D and 3D mean flows to assess the tolerable heights of surface imperfections (steps, gaps and humps) in various types of laminar flows. It is noted that the major limitation of using the e^N method is that it is not compatible with modern CFD tools. Each time when one of the components in the e^N method is changed, such as new boundary layer calculation method, improved stability diagrams, new experiments tests, the whole method will have to be recalibrated[18].

For low Reynolds number flow, the Reynolds Averaged Navier Stokes (RANS) turbulence models have been proved to perform significantly well in the flow simulation over BFS. The reattachment length downstream the step is well predicted comparable to the experimental data by using the $k - \omega$ model and a refined mesh[19]. However, such model can't capture the boundary layer transition due to the fully turbulent flow assumption. The boundary layer transition over BFS/FFS surface normally occurs at very low Reynolds numbers and would be a free shear flow transition. The available RANS-based turbulence transition models for the boundary layer flow, such as the correlation-based transition models of Menter et al[20, 21], can't account for the mechanism of free shear flow transition, thus can't be used directly to capture the boundary layer transition caused by the BFS/FFS imperfection.

In this study, a new numerical methodology is proposed. It is based on a hypothesis that the effect of surface imperfections on boundary layer transition could be mimicked as that of surface roughness, and thus the existing RANS-based turbulence transition models could be used to model the surface roughness effect on laminar flow transition. More details on the methodology are described in Section 2, following the results validation and analysis in Section 3. Conclusions and future work recommendations are provided in Section 4.

2 METHODOLOGY

2.1 Numerical method

It is found that the dimension (height) of step surface imperfection on NLF body is normally very small compared to the local boundary layer thickness. Therefore, the step excrescence can be regarded as surface roughness. A hypothesis is thus proposed, that is the effect of step excrescence on boundary layer transition can be mimicked as the effect of surface roughness on the boundary layer flow. A new numerical method is developed based on the hypothesis, in which a modified $\gamma - Re_\theta$ transition model for rough wall[20] would be employed and the BFS/FFS configurations will be applied as surface roughness on the smooth surface without step geometry immersed in the numerical model. It should be noted that the proposed method cannot reveal the boundary layer transition mechanism over the step imperfections. Instead, it will provide a computationally efficient approach to predict the transition location over the steps, thus help determine the manufacturing tolerance on the allowable step dimension on natural laminar flow body.

The details on the $\gamma - Re_\theta$ transition model is in Ref[20]. Rough wall is accounted for by a modification of the built-in correlation for $\widetilde{Re}_{\theta t}$ and is defined as:

$$Re_{\theta t,rough} = \widetilde{Re}_{\theta t} \cdot f(H) \tag{1}$$

$\widetilde{Re}_{\theta t}$ represents the transition momentum thickness Reynolds number for smooth surface. H is related to the geometric roughness height as input by the user. The new defined $Re_{\theta t,rough}$ is then used in the transition modelling[20].

From experimental study, BFS and FFS have shown different effect on boundary layer transition[4]. In the numerical simulation, an effective roughness profile, as described in H, is applied on the smooth surface to distinguish the BFS and the FFS. H is correlated closely with step configurations including step height and step location. A general form of H used in present model is shown as below:

$$BFS: \quad H = \begin{cases} 0 & (x \le x_h - C_{BFS} * h) \\ F_{BFS}(h) & (x_h - C_{BFS} * h < x < x_{tr,0}) \\ 0 & (x \ge x_{tr,0}) \end{cases} \tag{2}$$

$$FFS: \quad H = \begin{cases} 0 & (x \le x_h - C_{FFS} * h) \\ F_{FFS}(h) & (x_h - C_{FFS} * h < x < x_{tr,0}) \\ 0 & (x \ge x_{tr,0}) \end{cases} \tag{3}$$

Here, h is the step height, x_h is the step location from the model leading edge, $x_{tr,0}$ is the natural transition location on the smooth surface, x is the streamwise location on the surface from the leading edge. C_{BFS} and C_{FFS} are the calibration parameters for the BFS and FFS, presenting the interaction length before the step between the boundary layer and the surface steps. F_{BFS} and F_{FFS} are the calibration functions for the BFS and FFS, presenting the extent of step effect on boundary layer flow. These empirical parameters/functions are obtained from performing numerical tests and comparing the results with the available experiments taken from ref[4].

2.2 Geometric and numerical model

In Drake's experimental study[4], a symmetrical flat plate model with a super ellipse shaped leading edge and provisions for surface imperfection elements in various locations were designed. The chord length of the plate model is 1.2m (4feet), thickness 0.019m (0.75inch) and span 0.6m (23.5inch). A uniform step height across the span is applied on top surface of the model by using excrescence inserts with different heights at different locations. The step height and step locations tested in the experiments are shown in Table 1 and 2. In the numerical study, a 2D half plate model with the same chord length of 1.2m and a thickness of 0.009525m is used. The same BFS/FFS configurations are tested by various step height and step location combination. In total, there are 24 BFS cases and 24 FFS cases.

Table 1: Surface step heights

Step height (mm)	H1	H2	H3	H4	H5	H6
	2.540	2.286	2.032	1.778	1.397	0.889

Table 2: Surface step type and step locations

Step type\ Location (m)	A	B	C	D
BFS	0.1778	0.2540	0.3302	0.4064
FFS	0.1270	0.2032	0.2794	0.3556

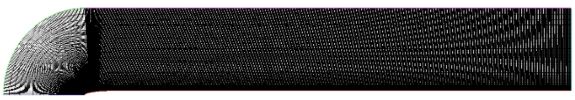

Figure 1: Computational domain and the mesh

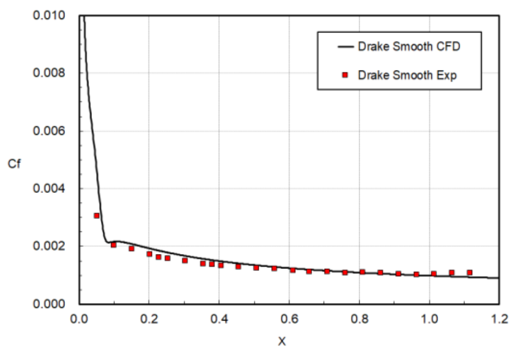

Figure 2: Skin friction profile on the smooth surface

In the numerical simulation, a 2D flow domain is created with the plate located on the bottom of the domain. Domain inlet and top boundaries are located 20 times of the plate thickness away from the plate. The outlet boundary is placed at the trailing edge of the plate. The (x, y) coordinate is along with the streamwise and wall normal direction, respectively. A 2D multi-block structured mesh is generated with refined mesh in the near-wall region ($y^+ \approx 1$) to capture the boundary layer flow accurately. The flow domain and the clustered mesh around the plate is shown in Figure 1. The flow condition is: Mach number $M = 0.0255846$, unit Reynolds number $Re = 5.68 \times 10^5/m$, freestream temperature $T_\infty = 295.9K$ and freestream pressure $P_\infty = 99898Pa$. The pressure gradient is $K = 0.04 \times 10^{-7}$, where K is defined as $K = \frac{\nu_e}{u_e^2}\frac{du_e}{dx}$[4] with u_e is the undisturbed boundary layer edge velocity and ν_e is the viscosity at the same location. A 2D incompressible flow solver combined with Menter's four equation $\gamma - Re_\theta$ transition model for rough wall as implemented in ANSYS Fluent 2019R3 solver is employed in the numerical simulation. The effective roughness profile H defined by user functions is applied on the smooth surface to mimic the effect of BFS/FFS on boundary layer transition.

2.3 Non-dimensional parameters

In the experimental[4] and numerical study, two independent non-dimensional parameters are used to relate all the other parameters, as defined as below:

(1) Step Reynolds number $Re_h = \frac{u_h h}{\nu_h}$

Where u_h is the velocity that would be observed at a step height h above the surface in an undisturbed laminar boundary layer at the step location, ν_h is the viscosity at the same location in the undisturbed boundary layer. It should be noted that in some literatures[6-7, 15-17] step Reynolds number is defined as $R_h = \frac{Uh}{\nu}$, where the freestream velocity U and the freestream viscosity ν is used in the calculation. Clearly, Re_h is dependent on both the step location and the step height and is more representative on step configurations and boundary layer flow conditions.

(2) Non-dimensional transition location $s = \frac{x_{tr} - x_h}{x_{tr,0} - x_h}$

It expresses the relative change in transition location x_{tr} as reduced from its natural transition location $x_{tr,0}$ on a smooth surface to the step location x_h[6]. The change of the parameter Δs represents the loss of laminarity due to the surface imperfection.

Other non-dimensional parameters which are commonly used in literatures[5-7, 12, 13, 15-17] include: transition Reynolds number Re_{tr}, which is based on the freestream velocity and the transition location x_{tr}; relative step height $\frac{h}{\delta^*}$, where δ^* is the boundary layer displacement thickness computed at the step location over the smooth surface.

3 RESULTS ANS DISCUSSION

3.1 Boundary layer flow on smooth surface

A 2D turbulence transitional simulation on the smooth surface is performed first for validation purpose. The skin friction coefficient C_f, which is commonly used as a crucial indicator to boundary layer transition[22], is predicted and validated, as shown in Figure 2. It is found that, from both the experimental measurement and the numerical simulation, the boundary layer on the smooth surface behaves as a laminar flow and no transition occurs over the plate. The predicted boundary layer velocity profiles at all step locations on smooth surface are analysed (not shown here) to determine the step Reynolds number Re_h.

3.2 Boundary layer transition on BFS/FFS surface

A series of turbulence transitional simulation on the rough surface is performed to mimic the effects of BFS and FFS. The results are shown in Figure 3-5 (BFS) and Figure 6-8 (FFS).

Figure 3 shows the skin friction profile for the BFS cases at step location B with different step heights. The vertical dashed lines indicate the step location. It is seen that the skin friction C_f profile is predicted very well by the numerical simulation. The predicted Laminar to turbulent transition region matches the experimental measurement. In the laminar region, C_f values match the experimental data and the profiles align with the ones on smooth surface perfectly. In the turbulent region, skin friction profiles align with the experimental data, but its values are overpredicted about 10% for most step heights. The predicted C_f profiles at the other three BFS locations (not shown here) present the same conclusions.

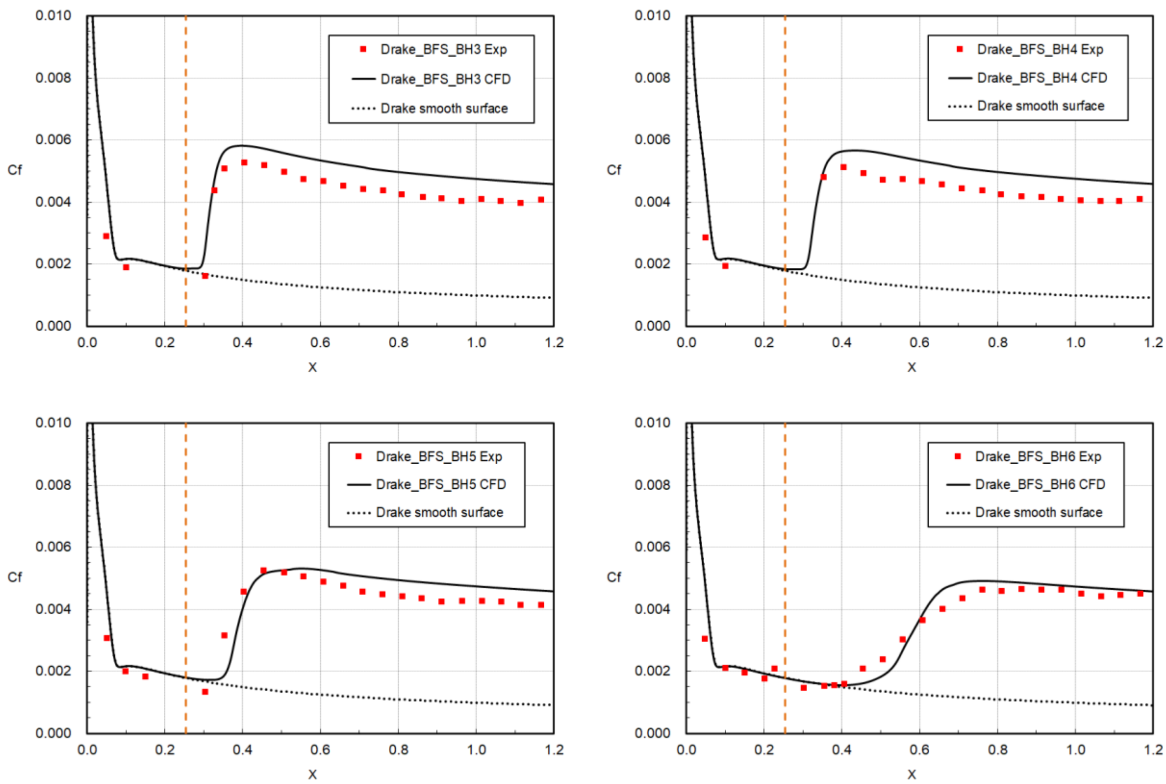

Figure 3: Skin friction profiles for the BFS at step location B

From both the experimental and the numerical study, it is found that the backward-facing step has significant effect on boundary layer flow resulting in a much earlier transition near the step location. The extent of the effect depends on the step configuration, i.e., the combination of the step location and the step height. It is necessary to determine the relations between the transition location and the step geometric characteristics. Here, the transition location criterion is defined as the location where the skin friction coefficient is greater than 1.5 times the measured smooth plate skin friction coefficient[4].

In Figure 4, the transition Reynolds number versus the step Reynolds number for the BFS cases are plotted at different step locations with different step heights. Reasonably good agreement between the experimental data and the CFD results is achieved at all step locations. Variation of the predicted transition location with the step configuration follows the same trend as observed in the experiments. It is found that, within the test range of step height of $h = [0.889, 2.540]mm$, transition location moves upstream towards the step location with the step height, but never reach the step location. As the step Reynolds number $Re_h < 200$, the effect of BFS reduces and the transition location moves downstream swiftly. As Re_h reaches a criterion value $Re_{h,crit}$, the BFS imperfection has no effect on boundary layer transition and the laminar flow is preserved on the rough surface. $Re_{h,crit}$ is determined as $Re_{h,crit} \approx 95$ in the experimental study and $Re_{h,crit} = 60$ in the numerical study.

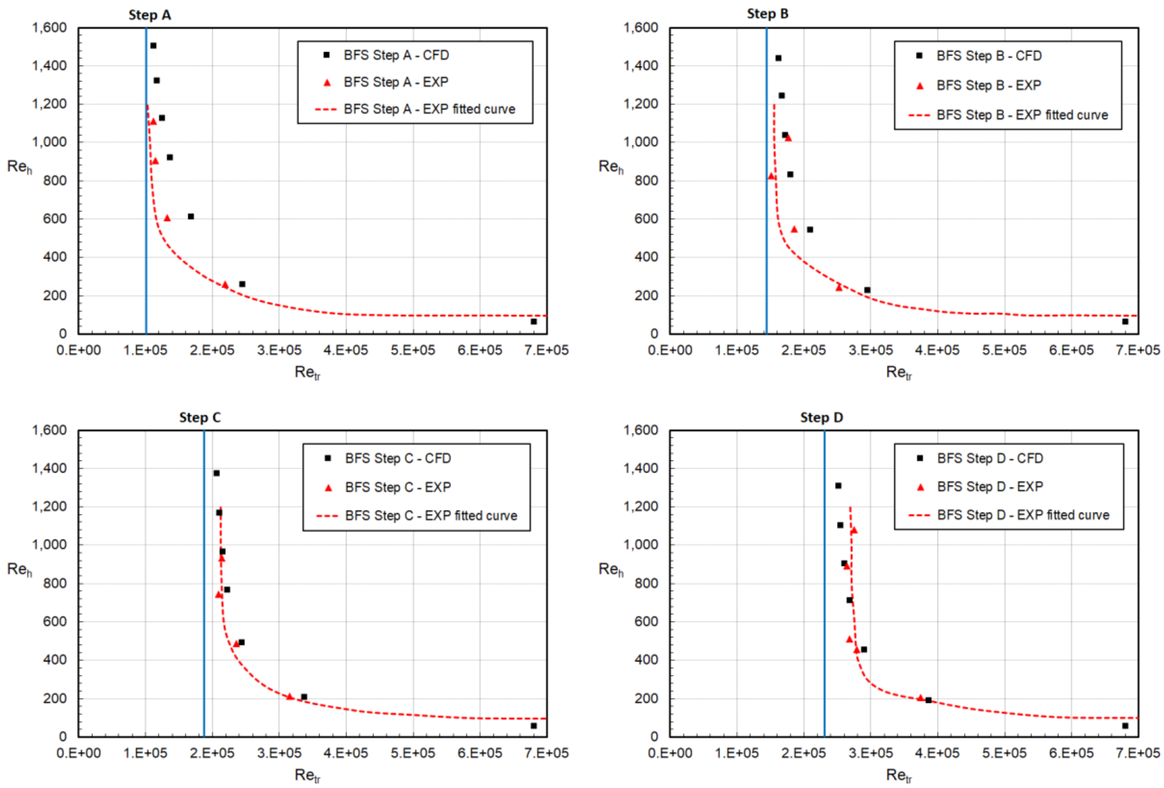

Figure 4: Transition locations for the BFS configurations

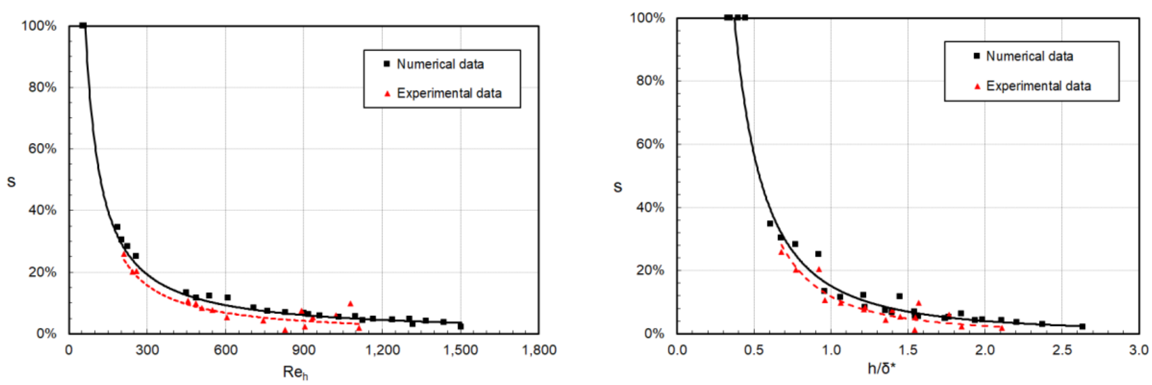

Figure 5: Non-dimensional transition location for the BFS as a function of the step Reynolds number (left) and the relative step height (right)

Non-dimensional transition location s as a function of step Reynolds number Re_h and relative step height $\frac{h}{\delta^*}$ is plotted in Figure 5. Within the test range of $Re_h = [0, 1500]$ and $\frac{h}{\delta^*} = [0, 3.0]$, the predicted non-dimensional transition location s agrees well with the experimental data. The trends of s match perfectly, while the value of s is slightly overpredicted. The non-dimensional transition location s decreases with step Reynolds number Re_h and relative step height $\frac{h}{\delta^*}$, which aligns with the observation in Figure 4. The allowable tolerance for the BFS on flat plate model can now be determined from the obtained relations. For example, using the power law approximation of the numerical data, a loss of the laminarity $\Delta s = 10\%$ (corresponding to $s = 90\%$) is found at $Re_h = 70$ and $\frac{h}{\delta^*} = 0.39$; $\Delta s = 20\%$ is found at $Re_h = 77$ and $\frac{h}{\delta^*} = 0.42$. Therefore, depending on the permitted loss of laminarity Δs for a certain surface, the corresponding value of Re_h and $\frac{h}{\delta^*}$ can be determined, thus the allowable step configuration including step location and step height in the manufacturing is determined. From Figure 5, the criteria for the step Reynolds number in the numerical simulation is determined as $Re_{h,crit} \approx 60$, and the criteria for the relative step height varies with step locations within a range of $\left(\frac{h}{\delta^*}\right)_{crit} = [0.33, 0.45]$.

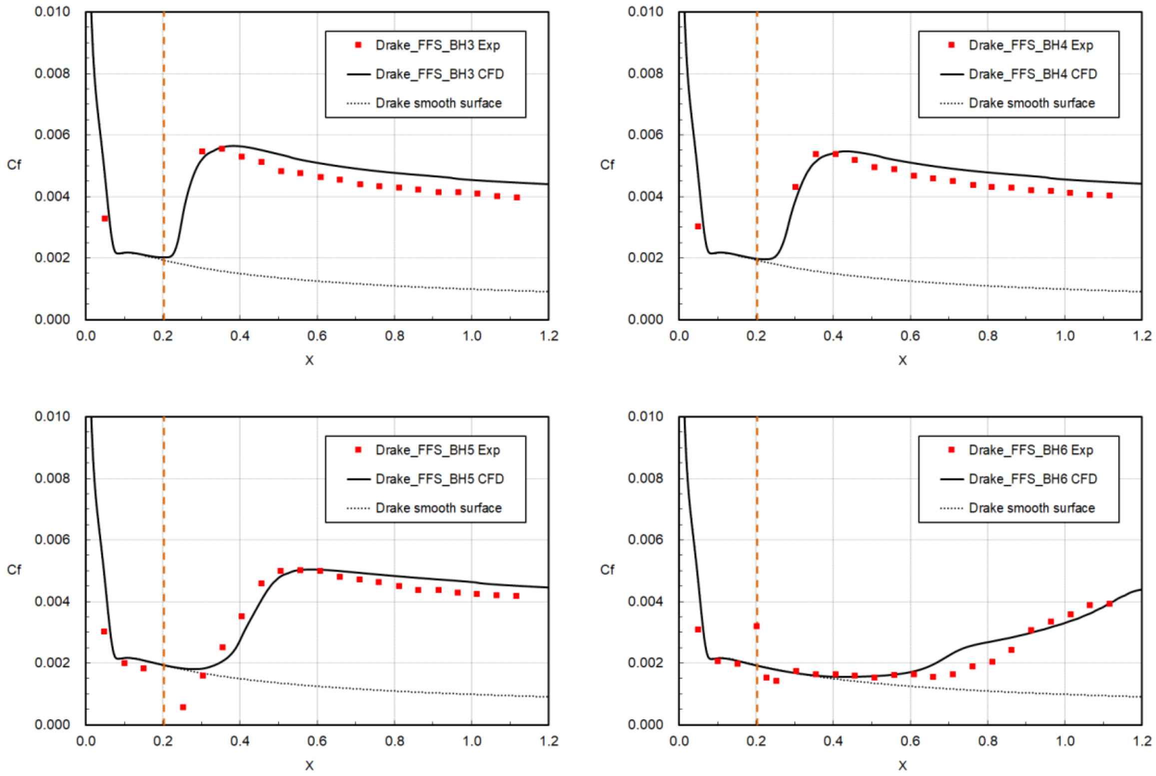

Figure 6: Skin friction profile for the FFS at step location B

Skin friction coefficient C_f profiles over FFS surface at step location B with different step height are shown in Figure 6. Similar to the BFS cases, the skin friction profile is predicted well and the predicted transition region and C_f values match the experimental data. FFS also shows significant effect on the boundary layer flow by prompting a much earlier transition near the step location. The extent of the effect depends on the combination of the step location and the step height.

The transition Reynolds number versus the step Reynolds number for the FFS cases are plotted in Figure 7. In general, the predicted transition locations (Re_{tr}) matches the experimental data and their variation with the step configurations follows the same trend. With the increase of the forward-facing step height, the transition locations are moving upstream approaching the step location, but never reach it. As step height decreases, the transition locations move downstream until the step height criteria $Re_{h,crit}$ reached, below which there is no transition occurrence and laminar boundary layer maintains on the rough surface. The step height criteria correspond to $Re_{h,crit} \approx 180$ in the numerical and the experimental data.

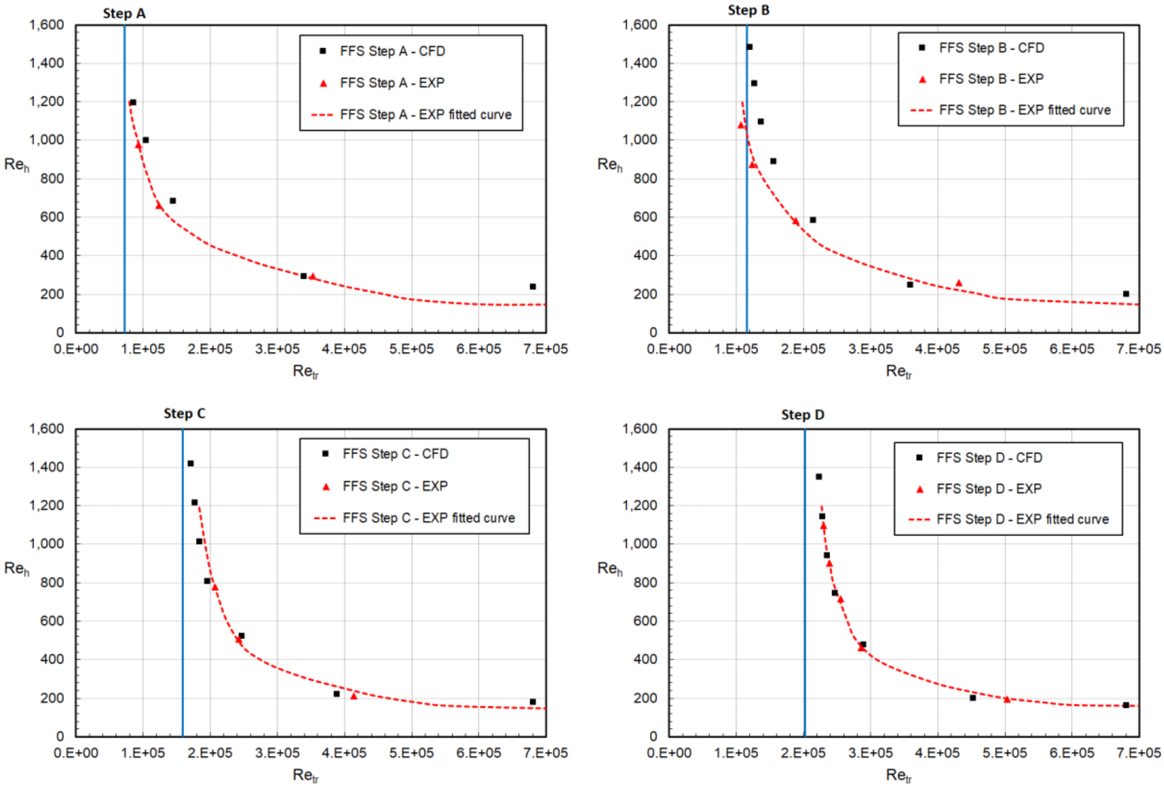

Figure 7: Transition locations for the FFS configurations

Analysis on the non-dimensional transition location s with the step Reynolds number Re_h and the relative step height $\frac{h}{\delta^*}$ over the FFS is shown in Figure 8. Apparently, the numerical data matches the experimental data very well within the test range of $Re_h = [0, 1500]$ and $\frac{h}{\delta^*} = [0, 2.5]$. As discussed in the BFS cases, the allowable tolerance for the FFS imperfection on the flat plate model can now be determined: a loss of the laminarity $\Delta s = 10\%$ (corresponding to $s = 90\%$) is found at $Re_h = 187$ and $\frac{h}{\delta^*} = 0.68$; $\Delta s = 20\%$ is found at $Re_h = 200$ and $\frac{h}{\delta^*} = 0.70$. From Figure 8, criteria value for the FFS step Reynolds number is determined as $Re_{h,crit} \approx 180$ for both the experimental and the numerical study. The criteria of the relative step height vary at different step locations within a range of $\left(\frac{h}{\delta^*}\right)_{crit} = [0.58, 0.98]$.

3.3 BFS VS. FFS

Further analysis on the different effect of BFS and FFS on boundary layer transition is performed, as presented in Figure 9. Within the test range of $Re_h = [0, 1500]$ and $\frac{h}{\delta^*} = [0, 2.5]$, the effect of the BFS on boundary layer transition is more significant than that of the FFS, indicating by a much lower criteria value of $Re_{h,crit}$ and $\left(\frac{h}{\delta^*}\right)_{crit}$ for the BFS than that for the FFS. The BFS imperfection with a relatively small step height near the leading edge will cause early boundary layer transition and a significant loss of laminarity. It implies that the allowable manufacturing tolerance for the BFS on the NLF body is much lower than that for the FFS. In contrast, the manufacturing tolerance for the FFS imperfection is much higher. For any FFS excrescence with $Re_h < 180$, there is no effect on transition and the laminar flow extension will maintain. At the same non-dimensional step height Re_h, the effect of the BFS on the loss of laminarity is nearly double the effect of the FFS.

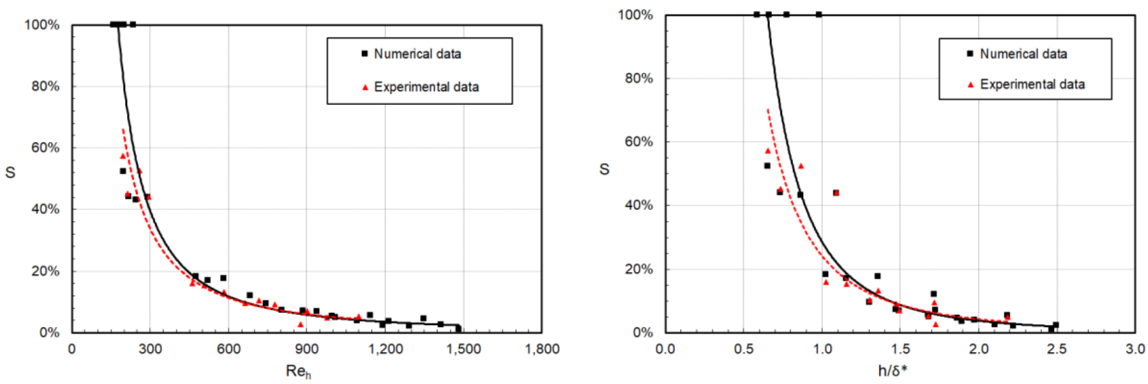

Figure 8: Non-dimensional transition location of the FFS as a function of the step Reynolds number (left) and the relative step height (right)

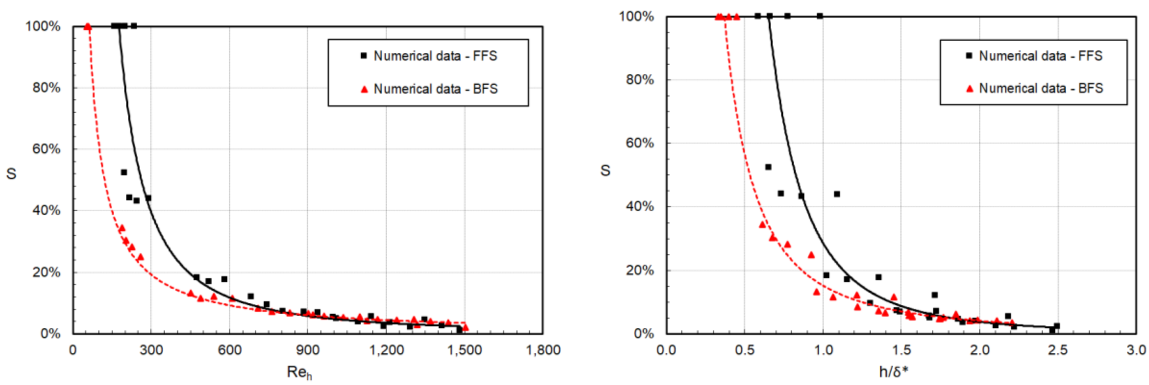

Figure 9: Comparison of the effect of BFS and FFS on boundary layer transition

4 CONCLUSIONS AND RECOMMENDATIONS

Surface step, including backward-facing step and forward-facing step, is one of the most encountered surface imperfections on aircraft body. It would result in significant effect on the boundary layer transition over natural/hybrid laminar flow aircraft body, thus reduce the practicability of the NLF technology for aircraft drag reduction. A guide is therefore needed for step imperfection dimension and shape allowance in the manufacturing that permits laminar flow to be maintained.

In this study, a new numerical method is proposed and implemented to predict the effect of backward-facing step and forward-facing step on boundary layer transition over NLF body. The method is based on the hypothesis that the effect of BFS/FFS on boundary layer transition could be mimicked as the effect of surface roughness due to the relevant small step height compared to the local boundary layer thickness. The modified $\gamma - Re_\theta$ transition model for rough wall is employed and the steps are applied as surface roughness by using user-defined correlations. A series of BFS and FFS test cases are simulated and validated; boundary layer transition locations are predicted and analysed. Four main conclusions can be drawn from the present analysis:

(1) The proposed method of mimicking the BFS/FFS as surface roughness and employing the modified $\gamma - Re_\theta$ transition model for rough wall can capture the boundary layer transition over the step surface imperfection very well. The relations between the loss of the laminarity and the step configurations are obtained, and the manufacturing allowance for steps over NLF body is determined.

(2) Within the test range of $Re_h = [0, 1500]$ and $\frac{h}{\delta^*} = [0, 3.0]$, the BFS and FFS presents significant effect on boundary layer transition, resulting in earlier transition approaching the step location with step Reynolds number and relative step height.

(3) The criterion value of the step Reynolds number is determined as $Re_{h,crit} \approx 60$ for the BFS and $Re_{h,crit} \approx 180$ for the FFS on flat plate model in an incompressible flow.

(4) The effect of BFS on boundary layer transition is more significant than that of FFS. Therefore, the allowable manufacturing tolerance for the BFS is much stringent than that for the FFS on NLF body.

A couple of recommendations for the future work include (a) extending the methodology to other type of surface imperfection applications, such as waviness, gaps and integral BFS and FFS, to assess the applicability of this approach for more general surface imperfection arrangements; (b) calibrating further the empirical correlations for the effective roughness profile H in subsonic and transonic flight conditions, aiming to provide a convenient prediction tool determining the manufacturing allowance of surface imperfections for aerodynamic engineers.

REFERENCES

[1] Fage, A., The Smallest Size of a Spanwise Surface Corrugation Which Affects Boundary-Layer Transition on an Aerofoil, RAE R&M 2120, (1943).

[2] Smith, A. M. O., and Clutter, D. W., The Smallest Height of Roughness Capable of Affecting Boundary-Layer Transition in Low-Speed Flow, Douglas Aircraft Co. Engineering Report ES 26803, (1954).

[3] Braslow, A. L., Review of the Effect of Distributed Surface Roughness on Boundary Layer Transition, AGARD Report 254, (1960).

[4] Drake A., Bender, A., Solomon, W. and Vavra, A., Prediction of Manufacturing Tolerances for Laminar Flow, AFRL-VA-WP-TR-2005-3060, (2005).

[5] Wang, Y. X. and Gaster, M., Effect of Surface Steps on Boundary Layer Transition, *Experiments in Fluids*, (2005) **39**: 679-686.

[6] Costantini, M., Risius, S., Klein, C., and Kühn, W., *Effect of Forward-Facing Steps on Boundary Layer Transition at a Subsonic Mach Number, New Results in Numerical and Experimental Fluid Mechanics X*, edited by Dillmann, A., Springer, Cham, (2014), 203–213.

[7] Costantini, M., Risius, S., and Klein, C., *Experimental Investigation of the Effect of Forward-Facing Steps on Boundary Layer Transition*, Procedia IUTAM 14, edited by M. Medeiros, Elsevier, Amsterdam, (2015), 152–162.

[8] Drake, A., Westphal, R. V., Zuniga, F. A., Kennelly, R. A. and Koga, D. J., Wing Leading Edge Joint Laminar Flow Tests, NASA TM 4762, (1996).

[9] Roger, E., An Experimental Investigation of the Interaction between a Forward-Facing Step and a Laminar Boundary Layer in Supersonic, Low-Density Flow, ARC-RM-3506,

(1965).
[10] Van Ingen, J. L., A Suggested Semi-Empirical Method for the Calculation of the Boundary Layer Transition Region, Univ. of Tech., Rept. UTH1-74, Delft, The Netherlands, (1956).
[11] Smith, A. M. O. and Gamberoni, N., Transition, pressure gradient and stability theory, Technical Report ES-26388, El Segundo, Calif.: Douglas Aircraft Company, El Segundo Division, (1956).
[12] Crouch, J. D., Kosorygin, V. S., and Ng, L. L., *Modelling the Effects of Steps on Boundary-Layer Transition*, 6th IUTAM Symposium on Laminar-Turbulent Transition, edited by R. Govindarajan, Fluid Mechanics and Its Applications, Springer, Dordrecht, The Netherlands, (2006), **78**: 37–44.
[13] Crouch, J. and Kosorygin, V., Surface Step Effects on Boundary-Layer Transition Dominated by Tollmien–Schlichting Instability, *AIAA Journal*, (2020) **58**(7).
[14] Edelmann, C. and Rist, U. *Impact of Forward-Facing Steps on Laminar-Turbulent Transition in Subsonic Flows*, New Results in Numerical and Experimental Fluid Mechanics IX, Springer International Publishing, (2014) **124**:155-162.
[15] Perraud, J and Séraudie, A., Effects of steps and gaps on 2D and 3D transition, European Congress on Computational Methods in Applied Sciences and Engineering, ECCOMAS 2000, Barcelona, 11-14 September 2000.
[16] Perraud, J., Arnal, D., Séraudie, A. and Tran D., Laminar-Turbulent Transition on Aerodynamic Surfaces with Imperfections, RTO AVT Specialists' Meeting on "Enhancement of NATO Military Flight Vehicle Performance by Management of Interacting Boundary Layer Transition and Separation", Prague, 4-7 October 2004, RTO-MP-AVT-111.
[17] Perraud, J., Arnal, D and Kuehn, W., Laminar-turbulent transition prediction in the presence of surface imperfections, *Int. J. Engineering Systems Modelling and Simulation*, (2014) **6** (3/4):162–170.
[18] Van Ingen, J.L., The e^N method for transition prediction. Historical review of work at TU Delft, 38th Fluid Dynamics Conference and Exhibit, Seattle, Washington, 23-26 June AIAA2008-2830, (2008) 1-49.
[19] Menter, F., Two-equation Eddy-Viscosity Turbulence Models for Engineering Applications, *AIAA Journal*, (1994) **32**(8):1598-1605.
[20] Langtry, R., Menter, F., Likki, S., Suzen, Y. and Huang, P., A Correlation-Based Transition Model Using Local Variables—Part II: Test Cases and Industrial Applications, *Journal of turbomachinery*, (2006) **128**: 423-434.
[21] Menter, F. R., Smirnov, P. E., Liu, T. and Avancha, R., A One-Equation Local Correlation-Based Transition Model, *Flow, Turbulence and Combustion*, (2015) **95**: 583–619.
[22] Lin, Y., Robinson, T., Early, J., Riordan, D., Tweedie, J. and Magee, L., Implementation of Menter's Transition Model on an Isolated Natural Laminar Flow Nacelle, *AIAA Journal*, (2011) **49**(4): 824-835.