Fixed-Time Synchronization of Chaotic Neural Networks with Mixed Delays via Dynamic Event-Triggered Control

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INFORMATION

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ABSTRACT

This paper proposes a novel dynamic event-triggered control scheme to address the fixed-time synchronization problem for chaotic neural networks (NNs) with mixed delays. Firstly, an adaptive threshold mechanism is embedded into the dynamic event-triggered control, which occupies less communication resource in comparison with the periodic-triggered control and enables the exclusion of Zeno phenomena. Secondly, by combining Lyapunov stability theory with fixed-time convergence criteria, a sufficient condition for the fixed-time synchronization of such chaotic NNs is established. Particularly, an explicit upper-bound estimation of the settling time is derived, which solely depends on controller parameters and is independent of the initial condition. Theoretical analysis indicates that the error system can converge to a predefined neighborhood of the origin within a fixed time. Finally, numerical simulations further substantiate the feasibility and superiority of the proposed methods.

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Nomenclature

The mathematical notations used throughout the manuscript is summarized as below. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

Notations	Implications
$\mathbb{R}$	The set of real number
$\mathbb{R}^n$	The n-dimensional vector space
$\mathbb{R}^{n\times n}$	The n-dimensional real matrix space
$\mathbb{Z}$	The integer set
the superscript T	The transpose operator
$\|\cdot\|$	The spectral norm of a matrix
$\lvert\cdot\rvert$	The Euclidean norm of a vector
$diag(\cdots)$	A diagonal matrix

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$\mathrm{sgn}\,(\cdot)$	The signum function
$D^{+}e\,(t)$	The upper right-hand Dini derivative of $e\,(t)$

1 Introduction

Neural network (NN), as a complex network, has been extensively investigated owing to its wide applications in signal processing, image processing, pattern classification, associative memories, and other areas [1,2]. With the ongoing in-depth study on the NNs, synchronization problems of the NNs have gradually gained considerable attention over the past decades, see [3–5] and references therein. As is well known, the presence of time delays is unavoidable in real-world neural network implementations [6,7]. Sometimes delays are so minor that they have negligible effects on transmission results and can be ignored. However, in most cases, time delays cannot be neglected, particularly when transmission is congested and the communication distance is remote. As is known to all, time delays may result in some complex dynamic behaviors such as oscillation, divergence, chaos, instability, or other poor performance of the NNs. Therefore, it is essential to examine the delayed NNs. On the other hand, time delays in the NNs mainly result from the finite switching speed of amplifiers and/or signal propagation. These causes often lead to mixed delays, including discrete delays, leakage delays, and distributed delays, existing simultaneously in the NNs. It should be pointed out that the NNs with mixed delays are more general than those with only one type of delay.

Synchronization refers to that the states of drive–response systems tend to consistency. During the past several decades, research on synchronization of the chaotic NNs has reached a relatively advanced stage and has been widely used in diverse fields, including biological systems, secure communication, optics, and information processing [8,9]. Rakkiyappan et al. [10] examined the exponential synchronization of Markovian jumping NNs with time-varying delays and sampled control. By employing a suitable Lyapunov–Krasovskii functional and the convex techniques, two exponential synchronization criteria were derived. However, in practical applications, it is always hoped to achieve synchronization of the NNs as soon as possible, and we aim to realize synchronization within a finite convergence time. In Yang and Cao [11], a finite-time synchronization method was employed to achieve synchronization among complex networks, which examines the impact of random perturbations on the finite-time synchronization. Liu et al. [12] focused on the finite-time synchronization problem concerning the time-lagged complex-valued NNs. The settling time for the finite-time synchronization is significantly influenced by the initial conditions of the system, as noted in references [11–13]. To solve this problem, Polyakov introduced the concept of a fixed-time synchronization in [14]. Zheng et al. [15] delved into the fixed-time synchronization for discontinuous competitive NNs with time-varying delays, and a less conservative upper bound of the settling time was achieved.

A variety of control schemes have been proposed to tackle the synchronization problems of the chaotic NNs, including impulse control [16], sliding mode control [17,18], event-triggered control (ETC) [19,20], and intermittent control [21]. Among them, the ETC has its unique advantages in reducing communication resources and saving computational energy consumption by updating control inputs only when system states or errors satisfy predefined triggered conditions.

Unlike traditional periodic or continuous control schemes, an event-triggered mechanism significantly minimizes redundant signal transmissions, which makes it particularly suitable for resource-constrained network environments. Nevertheless, in dynamic scenarios, external disturbances may lead to misjudgment of triggered conditions, causing a decrease in synchronization accuracy. Moreover, improperly designing static triggered thresholds has a risk of inducing the Zeno behavior and results in controller saturation or system instability. But overly conservative thresholds may prolong updating intervals and amplify synchronous error accumulation.

To solve such problems, researchers introduced a dynamic event-triggered control (DETC), for details, see [22–24]. Different from the conventional ETC frameworks, the DETC incorporates dynamic variables to adjust autonomously triggered conditions based on real-time system states or external disturbances. The block diagram of master–slave NNs with the DETC is depicted in Fig. 1. Such mechanisms not only mitigate the Zeno behavior but also balance between resource efficiency and control performance. In [22], the DETC is used to achieve the finite-time synchronization for semi-Markov neural networks with time-varying delay. However, the authors seldom consider global optimization strategies for dynamic threshold parameters.

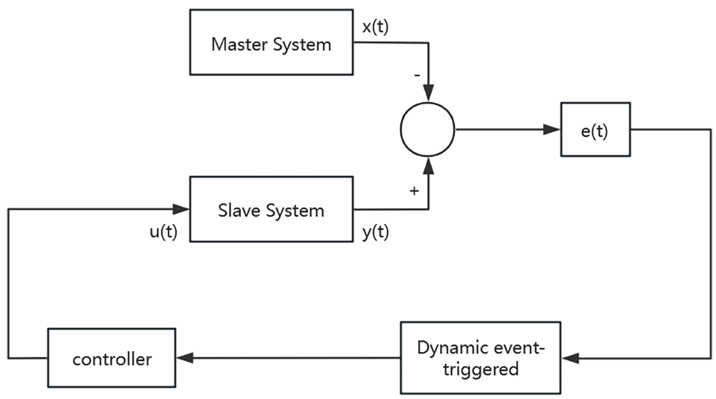

Figure 1: Master–slave synchronization under the DETC

Motivated by the above observations, we inject fresh impetus into the fixed-time synchronization for the chaotic NNs with mixed delays in this paper. The primary contributions of this paper are summarized as two aspects.

(1) A novel dynamic event-triggered mechanism with adaptive thresholds is proposed. It integrates synchronization errors as an exponentially decaying term, enabling automatic adjustment of triggered frequency based on real-time error trends. Such a mechanism occupies fewer communication resources than existing works in the literature and avoids the Zeno behavior.

(2) A new fixed-time synchronization controller is designed for the chaotic NNs. Combining Lyapunov stability theory with fixed-time convergence criteria, this controller can effectively realize synchronization within a settling time whose upper-bound is less than the prior results and is irrelevant to the initial states.

The rest of this paper is organized as follows. In Section 2, the model of the NNs with mixed delays is presented, along with some definitions and presumptions. The fixed-time synchronization for the NNs systems with mixed delays and its rigorous derivation are developed in Section 3. In Section 4, a numerical example is conducted. The conclusion of this paper is drawn in Section 5.

2 Preliminaries

Consider NNs with mixed delays described by

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + Cf(x(t-d(t))) + D\int_{t-\tau(t)}^{t} f(x(s))\,ds \tag{1}$$

where $x(t) = \begin{bmatrix} x_1(t) & x_2(t) & \cdots & x_n(t) \end{bmatrix}^T \in \mathbb{R}^n$ is the neuron state vector, $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T$ represents the neuron activation function satisfying the Lipschitz condition, namely, there exist a real positive constant L such that

$$|f(u) - f(v)| \leq L|u - v|, \forall u, v \in \mathbb{R}^n, u \neq v \tag{2}$$

$A = diag\begin{pmatrix} a_1 & a_2 & \cdots & a_n \end{pmatrix}$ with $a_i > 0\,(i = 1, 2, \ldots, n)$; B, C, $D \in \mathbb{R}^{n\times n}$ are connection weight matrices. $d(t)$ is the discrete delay and satisfies $0 < d(t) < d$, $\dot{d}(t) < d_{\max} < 1$, where d and d_{max} are known real constants. $\tau(t)$ is the distributed delay and satisfies $0 < \tau(t) < \tau$, $\dot{\tau}(t) < \tau_{\max}$, where τ and τ_{max} are known real constants.

To facilitate discussion, we take model (1) as the master system, and the slave system is represented by

$$\dot{y}(t) = -Ay(t) + Bf(y(t)) + Cf(y(t-d(t))) + D\int_{t-\tau(t)}^{t} f(y(s))\,ds + u(t) \tag{3}$$

where $y(t) = \begin{bmatrix} y_1(t) & y_2(t) & \cdots & y_n(t) \end{bmatrix}^T \in \mathbb{R}^n$ is the neuron state vector in the slave system (3), $f(y(t)) = [f_1(y_1(t)), f_2(y_2(t)), \ldots, f_n(y_n(t))]^T$ is the neuron activation function, $u(t)$ is the control input of the slave system.

Definition 1 [25]: *The master system (1) and the slave system (3) are said to be the fixed-time synchronization if there exists a stable time $T > 0$ such that the following equality (4) holds for any initial conditions*

$$\lim_{t\to T} |x(t) - y(t)| = 0 \tag{4}$$

Lemma 1 [26]: *Assume that a function $V(t)$ is non-negative and satisfies the following conditions:*

$$\begin{cases} \dot{V}(t) \leq -aV^q(t) - bV^p(t), & t \in [mT, mT + \theta T), \\ \dot{V}(t) \leq 0, & t \in [mT + \theta T, mT + T) \end{cases} \tag{5}$$

where $a > 0$, $b > 0$, $T > 0$, $0 < p < 1$, $q > 1$, $m = 0, 1, 2, \cdots$. Then $V(t) \equiv 0$, if

$$t \geq T_{\max} = \frac{1}{a\theta(q-1)} + \frac{1}{b\theta(1-p)} \tag{6}$$

In this case, $T_{\max}$ is said to be a settling time.

Lemma 2 [27]: *For any* $e_i > 0, i \in \mathbb{Z}_{[1,n]}$ *and* $0 < p \leq 1, q > 1$ *where* $\mathbb{Z}_{[1,n]} = 1, 2, \cdots, n$, *the following two inequalities hold*

$$\sum_{i=1}^{n} e_i^q \geq n^{1-q} \left(\sum_{i=1}^{n} e_i \right)^q, \quad \sum_{i=1}^{n} e_i^p \geq \left(\sum_{i=1}^{n} e_i \right)^p \tag{7}$$

Lemma 3 [28]: *Let* $X, Y \in \mathbb{R}^n$, *then there exists a positive constant* γ *such that the following inequality holds*

$$2X^T Y \leq \gamma X^T X + \gamma^{-1} Y^T Y \tag{8}$$

3 Fixed-Time Synchronizations of the NNs with Time Delays

In this section, we will study the fixed-time synchronization of NNs with mixed delay via DETC.

Define $e(t) = y(t) - x(t)$, then an error system can easily be derived as follows

$$\begin{aligned} \dot{e}(t) &= -Ae(t) + Bg(e(t)) + Cg(e(t - d(t))) \\ &\quad + D \int_{t-\tau(t)}^{t} g(e(s))\, ds + u(t) \end{aligned} \tag{9}$$

where $g(e(t)) = f(y(t)) - f(x(t))$.

To achieve the fixed-time synchronization, we design the event-triggered controller as

$$u(t) = -K\mathrm{sgn}(e(t_k)) \left(|e(t_k)|^{\alpha} + |e(t_k)|^{\beta} \right) \tag{10}$$

where $K \in \mathbb{R}, K > 0$ is the control gain parameter to be determined, $t_k \leq t < t_{k+1}$, $e(t_k)$ is discrete measurement sampling instant t_k and t_k satisfies the following $0 = t_0 < t_1 < \ldots < t_k < \ldots < \lim_{k\to\infty} t_k = +\infty$, α, β are constants satisfying $0 < \alpha < 1, \beta > 1$.

Substituting (10) into (9) yields

$$\begin{aligned} \dot{e}(t) &= -Ae(t) + Bg(e(t)) + Cg(e(t - d(t))) + D \int_{t-\tau(t)}^{t} g(e(s))\, ds \\ &\quad - K\mathrm{sgn}(e(t_k)) |e(t_k)|^{\alpha} - K\mathrm{sgn}(e(t_k)) |e(t_k)|^{\beta} \end{aligned} \tag{11}$$

To proceed with the control scheme, we construct a new event-triggered mechanism. The next triggered instants t_{k+1} is determined by

$$t_{k+1} = \inf \left\{ t > t_k | \, |e(t) - e(t_k)|^2 - \gamma_1 |e(t)|^2 \geq \gamma_2 e^{-at} \right\} \tag{12}$$

where γ_1, γ_2 and a are positive constants to be determined.

Theorem 1: *The slave system (3) is fixed-time synchronized onto the master system (1) within a fixed time* $T_{\max}$ *if the following inequality holds*

$$-2\|A\| + \lambda_1 \left\| BB^T \right\| + \lambda_1^{-1} L + \lambda_2 \left\| CC^T \right\| + \lambda_3 \left\| DD^T \right\| + 1 + \tau_{\max} \leq 0 \tag{13}$$

where $\lambda_1, \lambda_2, \lambda_3 > 0$ *are positive constants.*

Therefore, the error system (9) can achieve the fixed-time synchronization under the controller (10). The settling time can be estimated by

$$T_{\max} = \frac{1}{(1-\alpha)K} + \frac{1}{n^{\frac{1-\beta}{2}}(\beta-1)K} \tag{14}$$

Proof: The fixed-time synchronization of (1) and (3) is equivalent to the fixed-time stability of error system (9).

A candidate Lyapunov functional is chosen as

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

where $V_1(t) = e^T(t)e(t)$, $V_2(t) = \int_{t-d(t)}^{t} e^T(s)e(s)\,ds$, $V_3(t) = \int_{-\tau\max}^{0}\int_{t+\theta}^{t} e^T(s)e(s)\,dsd\theta$.

Then

$$\begin{aligned}
\dot{V}(t) &= 2e(t)^T\dot{e}(t) + e(t)^T e(t) - \left(1-\dot{d}(t)\right)e(t-d(t))^T e(t-d(t)) \\
&\quad + \tau_{\max} e(t)^T e(t) - \int_{t-\tau\max}^{t} e(s)^T e(s)\,ds \\
&= -2e(t)^T Ae(t) + 2e(t)^T Bg(e(t)) + 2e(t)^T Cg(e(t-d(t))) \\
&\quad + 2e(t)^T D\int_{t-\tau(t)}^{t} g(e(s))\,ds + e(t)^T e(t) \\
&\quad - \left(1-\dot{d}(t)\right)e(t-d(t))^T e(t-d(t)) + \tau_{\max} e(t)^T e(t) \\
&\quad - \int_{t-\tau\max}^{t} e(s)^T e(s)\,ds - 2e(t)^T K\mathrm{sgn}(e(t_k))|e(t_k)|^{\alpha} \\
&\quad - 2e(t)^T K\mathrm{sgn}(e(t_k))|e(t_k)|^{\beta}
\end{aligned}$$

By using (2) and Lemma 3, the following inequalities can be obtained

$$\begin{aligned}
2e(t)^T Bg(e(t)) &\le \lambda_1 e(t)^T BB^T e(t) + \lambda_1^{-1} g(e(t))^T g(e(t)) \\
&\le \lambda_1 e(t)^T BB^T e(t) + \lambda_1^{-1} L^2 e(t)^T e(t),
\end{aligned}$$

$$\begin{aligned}
2e(t)^T Cg(e(t-d(t))) &\le \lambda_2 e(t)^T CC^T e(t) + \lambda_2^{-1} g(e(t-d(t)))^T g(e(t-d(t))) \\
&\le \lambda_2 e(t)^T CC^T e(t) + \lambda_2^{-1} L^2 e(t-d(t))^T e(t-d(t))
\end{aligned}$$

$$2e(t)D\int_{t-\tau(t)}^{t} g(e(t))\,ds \le \lambda_3 e(t)^T DD^T e(t) + \lambda_3^{-1}\left(\int_{t-\tau(t)}^{t} g(e(s))\,ds\right)^T \left(\int_{t-\tau(t)}^{t} g(e(s))\,ds\right)$$

Utilizing Jensen integral inequality for the last term in the above inequality, one arrives at

$$2e(t)D\int_{t-\tau(t)}^{t} g(e(t))\,ds \le \lambda_3 e(t)^T DD^T e(t) + \lambda_3^{-1} L^2 \tau(t)\int_{t-\tau(t)}^{t} e(s)^T e(s)\,ds$$

Setting

$$\lambda_2 = \frac{L^2}{1-d_{\max}}, \lambda_3 = L^2\tau_{\max}$$

one has

$$\begin{aligned}\dot{V}(t) \le & -2e(t)^T Ae(t)^T + \lambda_1 e(t) BB^T e(t) + \lambda_1^{-1} L^2 e(t)^T e(t) \\ & + \lambda_2 e(t)^T CC^T e(t) + \lambda_2^{-1} L^2 e(t-d(t))^T e(t-d(t)) \\ & + \lambda_3 e(t)^T DD^T e(t) + \lambda_3^{-1} L^2 \tau(t) \int_{t-\tau(t)}^{t} e(s)^T e(s)\, ds \\ & - 2e(t)^T \mathrm{K}\operatorname{sgn}(e(t_k)) |e(t_k)|^{\alpha} - 2e(t)^T \operatorname{sgn}(e(t_k)) |e_i(t_k)|^{\beta} \\ & + e(t)^T e(t) + \tau_{\max} e(t)^T e(t) - (1-d_{\max}) e(t-d(t))^T e(t-d(t)) \\ & - \int_{t-\tau_{\max}}^{t} e(s)^T e(s)\, ds \\ \le & -2e(t)^T Ae(t)^T + \lambda_1 e(t) BB^T e(t) + \lambda_1^{-1} L^2 e(t)^T e(t) \\ & + \lambda_2 e(t)^T CC^T e(t) + \lambda_3 e(t)^T DD^T e(t) \\ & - 2e(t)^T \operatorname{sgn}(e(t_k)) |e(t_k)|^{\alpha} - 2e(t)^T \operatorname{sgn}(e(t_k)) |e_i(t_k)|^{\beta} \\ & + e(t)^T e(t) + \tau_{\max} e(t)^T e(t)\end{aligned}$$

From (10) and (12), $e(t_k)$ can be expressed by $e(t)$ as

$$\begin{aligned}|e(t_k)| & \ge |e(t)| - |e(t) - e(t_k)| \\ & \ge |e(t) + | \sqrt{\gamma_1 |e(t)|^2 + \gamma_2 e^{-at}}\end{aligned}$$

Then

$$\begin{aligned} & 2e(t)^T \mathrm{K}\operatorname{sgn}(e(t_k)) |e(t_k)|^{\alpha} \\ & \ge 2e(t)^T \mathrm{K}\, 1_n |e(t_k)|^{\alpha} \\ & \ge 2e(t)^T \mathrm{K}\, 1_n |e(t)|^{\alpha} + 2e(t)^T \mathrm{K}\, 1_n \left(\gamma_1 |e(t)|^2 + \gamma_2 e^{-at}\right)^{\frac{\alpha}{2}} \\ & \ge 2e(t)^T \mathrm{K}\, 1_n |e(t)|^{\alpha}\end{aligned}$$

In an analogous manner, we have

$$2e(t)^T \mathrm{K}\operatorname{sgn}(e(t_k)) |e(t_k)|^{\beta} \ge 2e(t)^T \mathrm{K}_n |e(t)|^{\beta}$$

where $1_n = [1 \quad 1 \dots 1]^T$.

Based on Cauchy inequality, one reaches

$$2e(t)^T K 1_n |e(t)|^{\alpha} \le -2K\sqrt{n} |e(t)|^{\alpha+1}, -2e(t)^T K 1_n |e(t)|^{\beta} \le -2K\sqrt{n} |e(t)|^{\beta+1}$$

Thus

$$\dot{V}(t) \leq -2e(t)^T \|A\| e(t) + \lambda_1 e(t)^T \left\|BB^T\right\| e(t) + \lambda_1^{-1} L^2 e(t)^T e(t) + \lambda_2 e(t)^T \left\|CC^T\right\| e(t) + \lambda_3 e(t)^T \left\|DD^T\right\| e(t) + e(t)^T e(t) + \tau_{\max} e(t)^T e(t) - 2K\sqrt{n}\,|e(t)|^{\alpha+1} - 2K\sqrt{n}\,|e(t)|^{\beta+1}$$

$$\leq e(t)^T \left(-2\|A\| + \lambda_1 \left\|BB^T\right\| + \lambda_1^{-1} L^2 + \lambda_2 \left\|CC^T\right\| + \lambda_3 \left\|DD^T\right\| + 1 + \tau_{\max}\right) e(t) - 2K\sqrt{n}\,|e(t)|^{\alpha+1} - 2K\sqrt{n}\,|e(t)|^{\beta+1}$$

By inequality (13), one gets

$$\dot{V}(t) \leq -2K\sqrt{n}\,|e(t)|^{\alpha+1} - 2K\sqrt{n}\,|e(t)|^{\beta+1}$$

Using Lemma 2 yields

$$\left(\sum_{i=1}^n |e_i(t)|^2\right)^{\frac{\alpha+1}{2}} \leq \left(\sum_{i=1}^n |e_i(t)|^{\alpha+1}\right)$$
$$n^{\frac{1-\beta}{2}} \left(\sum_{i=1}^n |e_i(t)|^2\right)^{\frac{\beta+1}{2}} \leq \left(\sum_{i=1}^n |e_i(t)|^{\beta+1}\right)$$

Therefore

$$|e(t)|^{\alpha+1} = \left(|e(t)|^2\right)^{\frac{\alpha+1}{2}} \geq V^{\frac{\alpha+1}{2}}(t)$$
$$|e(t)|^{\beta+1} = \left(|e(t)|^2\right)^{\frac{\beta+1}{2}} \geq V^{\frac{\beta+1}{2}}(t)$$

$$\dot{V}(t) \leq -2K\left(\sum_{i=1}^n |e_i(t)|^2\right)^{\frac{\alpha+1}{2}} - 2Kn^{\frac{1-\beta}{2}}\left(\sum_{i=1}^n |e_i(t)|^2\right)^{\frac{\beta+1}{2}} \leq -2KV^{\frac{\alpha+1}{2}}(t) - 2Kn^{\frac{1-\beta}{2}} V^{\frac{\beta+1}{2}}(t) \quad (15)$$

Letting

$$a = 2K,\ b = 2Kn^{\frac{1-\alpha}{2}},\ p = \frac{1+\alpha}{2},\ q = \frac{1+\beta}{2},$$

then based on Lemma 1, we can derive (14).

It follows that $V(t) \equiv 0$ from $t > T_{\max}$. Consequently, the error vector $e(t)$ will converge to zero within $T_{\max}$. Then, the slave system (3) is fixed-time synchronized onto the master system (1) within $T_{\max}$. This completes the proof of Theorem 1. □

Remark 1: *The upper bound on the settling time $T_{\max}$ is governed by the controller parameters α, β, and K. For fixed controller parameters α and β, one easily finds that the expression of $T_{\max}$ constitutes a strictly decreasing function with respect to K.*

Corollary 1: *When the information on $Bg(e(t))$ is unavailable in the error system (9), the error system (9) is converted as*

$$\dot{e}(t) = -Ae(t) + Cg(e(t - d(t))) + D\int_{t-\tau(t)}^{t} g(e(s))\,ds + u(t).$$

If the following inequality

$$-2\|A\| + \lambda_2 \left\|CC^T\right\| + \lambda_3 \left\|DD^T\right\| + 1 + \tau_{\max} \leq 0 \tag{16}$$

holds for $\lambda_2, \lambda_3 > 0$, *the error system can be stabilized within a fixed time under the controller (10). Moreover, the settling time can be estimated by*

$$T_{\max 1} = \frac{1}{(1-\alpha) K} + \frac{1}{n^{\frac{1-\beta}{2}} (\beta - 1) K}$$

When the time delays involving (9) are very trivial and can be ignored, we easily obtain Corollary 2 from Theorem 1.

Corollary 2: *In the absence of time delays, the error system (9) is changed as*

$$\dot{e}(t) = -Ae(t) + Bg(e(t)) + u(t)$$

If the following inequality

$$-2\|A\| + \lambda_1 \left\|BB^T\right\| + \lambda_1^{-1} L + 1 \leq 0 \tag{17}$$

holds for a positive constant λ_1, *under the controller (10) the error system can be stabilized within a fixed time. The settling time can be estimated by*

$$T_{\max 2} = \frac{1}{(1-\alpha) K} + \frac{1}{n^{\frac{1-\beta}{2}} (\beta - 1) K}$$

Theorem 2: *Under the condition of Theorem 1, the Zeno behavior of DETC (12) is avoidable.*

Proof: Based on Theorem 1, there exist $\omega, \lambda > 0$ such that $|e(t)| \leq \omega\, e^{-\lambda t}$.

By the definition of DETC, one obtains

$$\begin{aligned}
D^+ |e(t)| &\leq |\dot{e}(t)| \\
&\leq |Ae(t)| + |Bg(e(t))| + |Cg(e(t - d(t)))| + \left| D \int_{t-\tau(t)}^{t} g(e(s))\, ds \right| + |u(t)| \\
&\leq \|A\|\, |e(t)| + L\, \|B\|\, |e(t)| + L\, \|C\|\, |e(t - d(t))| + L\, \|D\| \left| \int_{t-\tau(t)}^{t} g(e(s))\, ds \right| \\
&+ K|e(t)|^{\alpha} + K|e(t)|^{\beta} \\
&\leq \left(\|A\| + \|B\|\, L + \|C\|\, L e^{\lambda d \max} + \|D\|\, L\tau_{\max} e^{\lambda \tau \max} + K\omega^{\alpha-1} e^{-\lambda(\alpha-1)t} + K\omega^{\beta-1} e^{-\lambda(\beta-1)t} \right) \omega e^{-\lambda t} \\
&\triangleq M
\end{aligned}$$

Then

$$\begin{aligned}
&|e(t_k) - e(t)| \leq \int_{t_k}^{t} |\dot{e}(s)|\, ds \leq M(t - t_k) \\
&|e(t_k)|^2 - |e(t)|^2 \leq \gamma_1 |e(t)|^2 + \gamma_2 e^{-at} \\
&|e(t_k)| - |e(t)| \leq \gamma_1 |e(t)| + \frac{\gamma_2}{\omega} e^{(\lambda - a)t}
\end{aligned} \tag{18}$$

In light of the event-triggered mechanism (12), we get

$$\begin{aligned} M\left(t-t_k\right) &\geq \left|e\left(t_k\right)\right|-\left|e\left(t\right)\right| \\ &\geq \gamma_1\left|e\left(t\right)\right|+\frac{\gamma_2}{\omega}e^{(\lambda-a)t} \end{aligned} \tag{19}$$

where $a>\lambda$. Combining (18) with (19), one infers that there exists $\delta>0$ such that

$$M^2\left(t_{k+1}-t_k\right)^2 \geq \left|e\left(t_{k+1}\right)-e\left(t_k\right)\right|^2 \geq \left(\gamma_1\omega^2+\gamma_2\right)e^{-at_{k+1}}$$

$$t_{k+1}-t_k \geq \frac{\gamma_1\left|e\left(t\right)\right|}{M} \geq \frac{\gamma_1\omega e^{-\lambda t}}{M} \geq \delta > 0$$

Therefore, there is no Zero behavior and this completes the proof of Theorem 2. □

4 Numerical Examples

In this section, a numerical simulation example is given to confirm the efficacy of the proposed methods.

Example: *Consider the delayed NNs in (9) with the following parameters:*

$$A=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, B=\begin{pmatrix}2.0 & -0.2\\ -5.0 & 3.0\end{pmatrix}$$

$$C=\begin{pmatrix}-1.5 & -0.2\\ -0.2 & -4.0\end{pmatrix}, D=\begin{pmatrix}-0.6 & -0.6\\ 4.5 & -3.0\end{pmatrix}$$

By setting the activation functions $f\left(x\left(t\right)\right)=\frac{1}{2}\left(\left|x\left(t\right)+1\right|-\left|x\left(t\right)-1\right|\right)$, the discrete delay $d\left(t\right)=\frac{e^t}{e^t+1}$, distributed delay $\tau\left(t\right)=0.5\sin^2\left(t\right)$, we can get $\mu=0.25$, $\tau=1$. The initial states are $x\left(0\right)=\left(0.4,-0.7\right)^T$, $y\left(0\right)=\left[-0.6,0.3\right]^T$, $\gamma_1,\gamma_2>0$, $a>0$ are parameters related to the dynamic event-triggered mechanism, then we choose $\gamma_1=0.015$, $\gamma_2=1$, $a=2$. According to the inequality (13) and $L=1$, $\lambda_1=0.1$, $\lambda_2=2$, $\lambda_3=0.5$, the controller gain can be designed to satisfy $K>0$. Then we choose $K=3$, $\alpha=0.5$, $\beta=1.5$, the chaotic behavior of the master system is shown in Fig. 2.

Fig. 3 demonstrates that the error system is stabilized under the DETC (12), with a settling time estimated via (14) as $T_{\max}=1.228$ (s). In [29], the synchronization error approaches to zero before 0.6 (s) while the proposed method in this paper does that before 0.5 (s). It is proved that the proposed method outperforms that in [29].

In order to implement Corollary 1, we obtain the gain parameters of the controller $K>0$ according to the inequality (17). Then we choose $K=2$, under the DETC (12), the error system is stabilized, which is depicted in Fig. 4. By the formula of $T_{\max 1}$, one estimates the settling time $T_{\max 1}=2.189$ (s). From Fig. 4, one finds that the synchronization error approaches to zero before 0.8 (s). This suggests that Corollary 1 is correct.

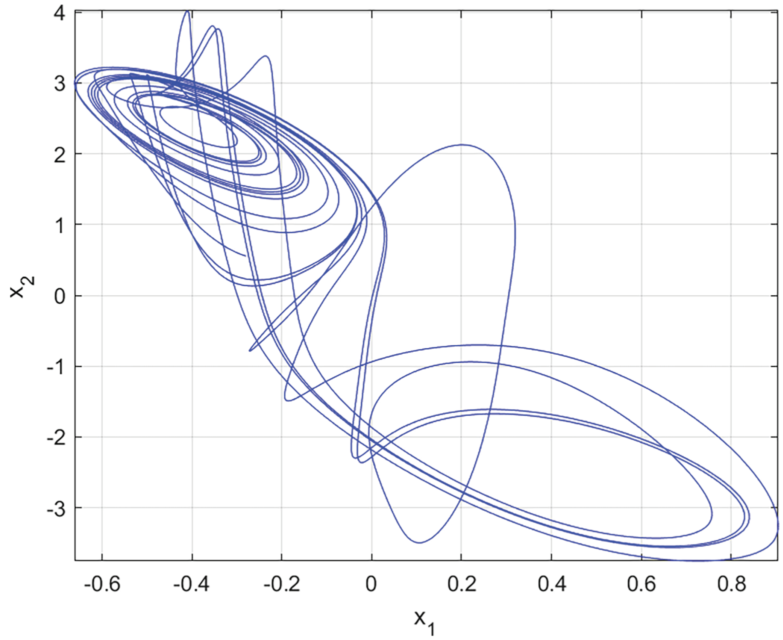

Figure 2: Phase portrait of the chaotic NNs with mixed delays

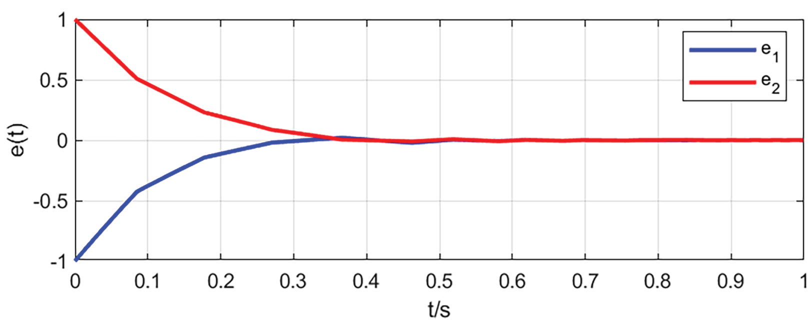

Figure 3: Synchronization errors $e(t)$ under the DETC

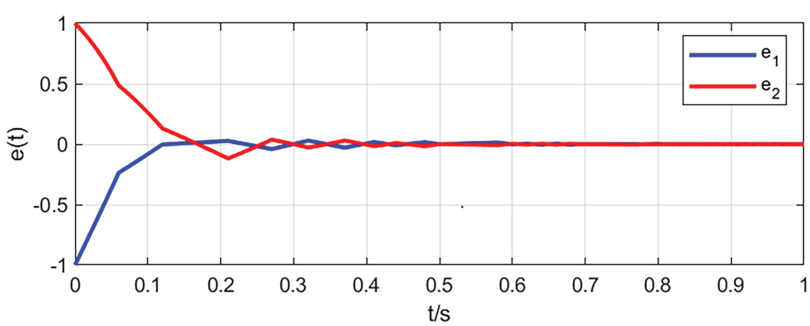

Figure 4: Synchronization errors $e(t)$ under the DETC for Corollary 1

To verify Corollary 2, we obtain the gain parameters of the controller $K > 0$ based on the inequality (18) and we choose $K = 4$. Then under the DETC (12), the error system is stabilized, which is shown in Fig. 5. In light of the formula of $T_{\max 2}$, one estimates the settling time $T_{\max 2} = 1.095$ (s). From Fig. 5, one gets the synchronization error approaches to zero before 0.48 (s). This indicates that Corollary 2 is also correct.

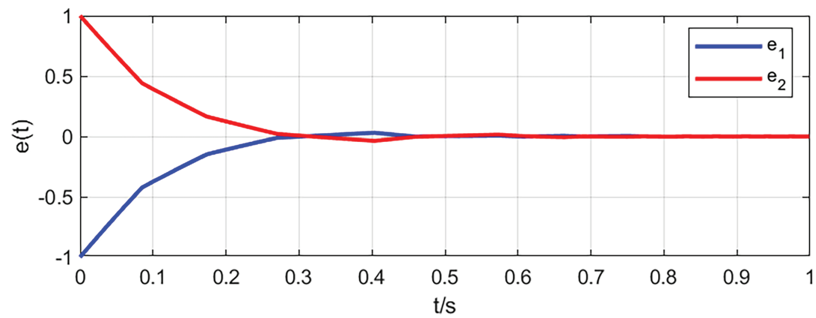

Figure 5: Synchronization errors $e(t)$ under the DETC for Corollary 2

Comparison between the updating instants generated by the proposed DETC and that of the static event-triggered control (SETC) in [30] is illustrated in Fig. 6, where the SETC changes correspondingly to

$$t_{k+1} = \inf\left\{t > t_k \left| |e(t) - e(t_k)|^2 - \gamma_1 |e(t)|^2 \geq 0 \right.\right\}$$

Note that triggered instants of the DETC are much less than that of the SETC in the beginning, but when time goes on, one finds that triggered instants generated by these two control schemes gradually become similar. The reason is that the DETC has an additional decreasing exponential threshold compared with the SETC. In conclusion, the DETC has a smaller number of triggered instants than the SETC without additional threshold.

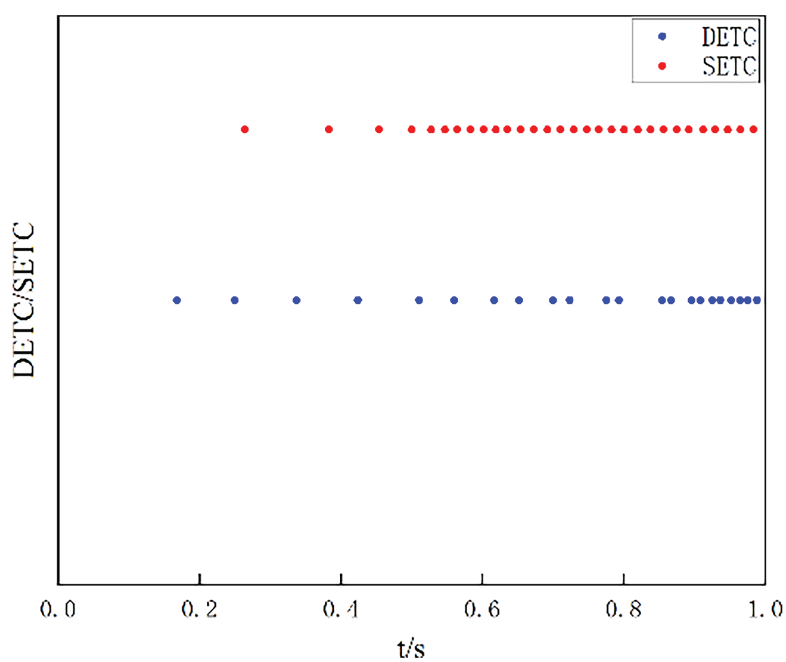

Figure 6: Triggered instant sequences of DETC and SETC

5 Conclusions

In this paper, a fixed-time synchronization controller with a convergence time independent of initial states is designed using the DETC. Under the controller, the chaotic NNs with mixed delays can effectively synchronize within a fixed-time. Unlike existing static event-triggered strategies, the proposed dynamic event-triggered mechanism with adaptive thresholds can save communication resources and eliminate the Zeno behavior. As mentioned in the Introduction, these advances align with the practical requirements in secure communication and biological systems. Numerical simulations show that fixed-time synchronization can be realized within a relatively short time frame and validate our theoretical framework.

Our future research is to address the fixed-time synchronization via a dynamic event-triggered mechanism for more complex NNs such as [31], particularly those with parameter uncertainties, and further to optimize dynamic threshold parameters.

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References

1. Hu J, Zhang G, Li Y, Wang T. Fixed-time stabilization and synchronization of fuzzy inertial neural networks via aperiodically semi-intermittent control. Neurocomputing. 2025;631(4):129720. doi:10.1016/j.neucom.2025.129720.
2. Zhang G, Rakkiyappan R. New exponential function based fixed/preassigned-time synchronization for a kind of neural networks with time-varying delays. J Nonlinear Dyn Appl. 2025;1(1):26. doi:10.62762/jnda.2025.937806.
3. Chen Y, Zhang N, Yang J. A survey of recent advances on stability analysis, state estimation and synchronization control for neural networks. Neurocomputing. 2023;515(4):26–36. doi:10.1016/j.neucom.2022.10.020.
4. Zhang Y, He Y, Long F, Zhang CK. Mixed-delay-based augmented functional for sampled-data synchronization of delayed neural networks with communication delay. IEEE Trans Neural Netw Learn Syst. 2024;35(2):1847–56. doi:10.1109/TNNLS.2022.3185617.
5. Liu P, Wang J, Zeng Z. Event-triggered synchronization of multiple fractional-order recurrent neural networks with time-varying delays. IEEE Trans Neural Netw Learn Syst. 2023;34(8):4620–30. doi:10.1109/TNNLS.2021.3116382.
6. Singh S, Kumar U, Das S, Alsaadi F, Cao J. Synchronization of quaternion valued neural networks with mixed time delays using Lyapunov function method. Neural Process Lett. 2022;54(2):785–801. doi:10.1007/s11063-021-10657-w.

7. Yan H, Qiao Y, Ren Z, Duan L, Miao J. Master-slave synchronization of fractional-order memristive MAM neural networks with parameter disturbances and mixed delays. Commun Nonlinear Sci Numer Simul. 2023;120(12):107152. doi:10.1016/j.cnsns.2023.107152.
8. Wei F, Chen G, Zeng Z, Gunasekaran N. Finite/fixed-time synchronization of inertial memristive neural networks by interval matrix method for secure communication. Neural Netw. 2023;167:168–82. doi:10.1016/j.neunet.2023.08.015.
9. Goldbeter A, Yan J. Multi-synchronization and other patterns of multi-rhythmicity in oscillatory biological systems. Interface Focus. 2022;12(3):20210089. doi:10.1098/rsfs.2021.0089.
10. Rakkiyappan R, Chandrasekar A, Park JH, Kwon OM. Exponential synchronization criteria for Markovian jumping neural networks with time-varying delays and sampled-data control. Nonlinear Anal Hybrid Syst. 2014;14:16–37. doi:10.1016/j.nahs.2014.04.001.
11. Yang X, Cao J. Finite-time stochastic synchronization of complex networks. Appl Math Model. 2010;34(11):3631–41. doi:10.1016/j.apm.2010.03.012.
12. Liu M, Lu B, Wang J, Jiang H, Hu C. Finite-time and fixed-time synchronization of memristor-based Cohen-Grossberg neural networks via a unified control strategy. Mathematics. 2025;13(4):630. doi:10.3390/math13040630.
13. Ge C, Chang C, Liu Y, Hua C. Dynamic event-triggered exponential synchronization for neural networks with random controller gain perturbations. Int J Control Autom Syst. 2023;21(9):2927–37. doi:10.1007/s12555-022-0452-6.
14. Polyakov A. Nonlinear feedback design for fixed-time stabilization of linear control systems. IEEE Trans Automat Contr. 2012;57(8):2106–10. doi:10.1109/tac.2011.2179869.
15. Zheng C, Hu C, Yu J, Jiang H. Fixed-time synchronization of discontinuous competitive neural networks with time-varying delays. Neural Netw. 2022;153(5):192–203. doi:10.1016/j.neunet.2022.06.002.
16. Kowsalya P, Mohanrasu SS, Kashkynbayev A, Gokul P, Rakkiyappan R. Fixed-time synchronization of inertial Cohen-Grossberg neural networks with state dependent delayed impulse control and its application to multi-image encryption. Chaos Solitons Fractals. 2024;181(5):114693. doi:10.1016/j.chaos.2024.114693.
17. Cheng Y, Hu T, Xu W, Zhang X, Zhong S. Fixed-time synchronization of fractional-order complex-valued neural networks with time-varying delay via sliding mode control. Neurocomputing. 2022;505(1):339–52. doi:10.1016/j.neucom.2022.07.015.
18. Aouiti C, Hui Q, Jallouli H, Moulay E. Sliding mode control-based fixed-time stabilization and synchronization of inertial neural networks with time-varying delays. Neural Comput Appl. 2021;33(18):11555–72. doi:10.1007/s00521-021-05833-x.
19. Ping J, Zhu S, Liu X. Finite/fixed-time synchronization of memristive neural networks via event-triggered control. Knowl Based Syst. 2022;258(5):110013. doi:10.1016/j.knosys.2022.110013.
20. Chang Q, Park JH, Yang Y, Wang F. Finite-time multiparty synchronization of T-S fuzzy coupled memristive neural networks with optimal event-triggered control. IEEE Trans Fuzzy Syst. 2023;31(8):2545–55. doi:10.1109/tfuzz.2022.3228335.
21. Shanmugam S, Narayanan G, Rajagopal K, Ali MS. Finite-time synchronization of complex-valued neural networks with reaction-diffusion terms: an adaptive intermittent control approach. Neural Comput Appl. 2024;36(13):7389–404. doi:10.1007/s00521-024-09467-7.
22. Jin Y, Qi W, Zong G. Finite-time synchronization of delayed semi-Markov neural networks with dynamic event-triggered scheme. Int J Control Autom Syst. 2021;19(6):2297–308. doi:10.1007/s12555-020-0348-2.
23. Jia S, Hu C, Feng L, Shi T, Jiang H. Fixed/preassigned-time synchronization of quaternion-valued BAM neural networks: an event-based non-separation control method. Commun Nonlinear Sci Numer Simul. 2024;137(1):108145. doi:10.1016/j.cnsns.2024.108145.
24. Wang Y, Zong G. Dynamic event-triggered adaptive fixed-time practical tracking control for nonlinear systems through funnel function. IEEE Trans Automat Sci Eng. 2025;22:7008–17. doi:10.1109/tase.2024.3458176.

25. Jing T, Zhang D, Jing T. Finite-time synchronization of hybrid-coupled delayed dynamic networks via aperiodically intermittent control. Neural Process Lett. 2020;52(1):291–311. doi:10.1007/s11063-020-10245-4.
26. Gan Q, Xiao F, Sheng H. Fixed-time outer synchronization of hybrid-coupled delayed complex networks via periodically semi-intermittent control. J Frankl Inst. 2019;356(12):6656–77. doi:10.1016/j.jfranklin.2019.03.033.
27. Khalil HK, Grizzle JW. Nonlinear systems. Hoboken, NJ, USA: Prentice Hall; 2002.
28. Sui X, Yang Y, Wang F, Zhang L. Finite-time anti-synchronization of time-varying delayed neural networks via feedback control with intermittent adjustment. Adv Differ Eq. 2017;2017(1):229. doi:10.1186/s13662-017-1264-5.
29. Yang W, Huang J, Wen S, He X. Fixed-time synchronization of neural networks with time delay via quantized intermittent control. Asian J Control. 2023;25(3):1823–33. doi:10.1002/asjc.2805.
30. Tang W, Sheng Y, Xiao Q, Huang T, Zeng Z. Synchronization of complex dynamical networks on time scales via intermittent dynamic event-triggered control. IEEE Trans Syst Man Cybern Syst. 2024;54(5):2897–906. doi:10.1109/tsmc.2024.3352074.
31. Rakkiyappan R, Udhaya Kumari E, Chandrasekar A, Krishnasamy R. Synchronization and periodicity of coupled inertial memristive neural networks with supremums. Neurocomputing. 2016;214:739–49. doi:10.1016/j.neucom.2016.06.061.