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A METHOD OF PARAMETERS IDENTIFICATION FOR THE COUPLED DRY FRICTION MODEL FOR PNEUMATIC TIRES

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Abstract. The theory of multi-component dry friction accounting for the coupled kinematics of relative motion in finite contact spots is applied to the modelling of pneumatic tires. The analytical models of the combined dry friction accounting for the anisotropy of the dry friction factors as well as the distribution of the contact pressure close to the real one are introduced. The exact integral relationships between the dry friction force and torque and the generalized velocities, i. e. the speed of sliding and angular velocity of spinning, appears as a result of integration over the contact spot and are too complex to be implemented analytically into the engineering practice. The proposed approximate models are based on the fractional approximations and could be interpreted as rheological models with low number of constitutive constants. These constants could be identified after the solution of some specific inverse problems with input data given by the diagrams of the dry friction forces and torque obtained as a result of simple physical tests. Here these inverse problem are formulated and solved using the perturbed benchmark solutions of the corresponding direct problems with the factors obtained after the numerical solution of the contact problem for the heterogeneous pressurized tire. The possibility of stable solution for the inverse problems on the basis of the proposed approach is shown. The presented models as well as the method if constitutive constants identification could be applied for more detailed investigation of unsteady rolling regimes of pneumatics which are characterized by the non-vanishing sliding and spin.

1 INTRODUCTION

The “shimmy”, i.e. the high-amplitude coupled lateral and torsional vibrations of controllable nose landing gears of aircrafts is a well-studied phenomenon [1, 2]. At the same time the shimmy-like oscillations of rigidly fixed main landing gears is not so widely known and moreover not investigated in details in spite of the fact that they were often observed during operation of some modern aircrafts, moreover such oscillations resulted destructions of torque links of gears and even caused hard failures of aircrafts’ frames in some registered cases. The main specific features of such instable rolling regimes are the following ones:

- the oscillations appear at initial stages of landings shortly after touchdown (i.e. during non-steady wheels rolling with longitudinal sliding);
- the significant sliding of wheels at unstable regimes is proved by tires' tracks at runaways surfaces.

Thus, the classical shimmy model based on the non-holonomic constraint following from the assumption of vanishing slip and spin of a wheel [1, 2] fails evidently, therefore a qualitatively hew theory is required. Such a model should take into account as well dry friction effects as deforming of pneumatic tires. A first attempt of accounting for the finite dimension of the area of contact of a wheel and a road and, consequently, for the coupling dynamic effects of the combined kinematics of relative motion of interacting wheel of an landing gear and a runaway surface under the condition of intensive sliding was proposed in [3] where the dynamics of the asymmetric one-wheel main landing gear was studied in. It was shown that the shimmy phenomenon can be provoked only by dry friction forces under the conditions of combined kinematics. The coupled dry friction theory was further developed in several works [4-14]. In spite of efficiency of the approach [3] it cannot be implemented in the engineering practice since the Hertzian distribution of the contact pressure has almost nothing to do with the real one observed for severely deformed pressurized pneumatic tires (except very small loads). The first improved model accounting for the random distribution of the normal contact stresses inside of contact patches was introduced in [15]. Thus, the second step required to adopt the theory of coupled dry friction to the problems of rolling stability for pneumatic tires consists in the use of accurate approximations of the contact pressure; indeed, this distribution impacts significantly on the friction parameters and, consequently, on the rolling stability domain obtained by the appropriate computations [5, 6, 7, 13]. Such a distribution could be obtained on the background of the 3D finite element modelling in statics [16] or using geometrically nonlinear versions of quasi-3D refined shell theories [17, 18].

Let us note that the 3D finite element simulation of the nonlinear dynamics of landing gears remains too resource consumptive even today to be implemented into the engineering practice at preliminary design stages; it is no doubt required at final ones when most of possible effects (e.g. tire friction heating, mass loss, etc.) should be accounted, but the simple models with few degrees of freedom like [1] are required to predict which effects have to be studied before the detailed quantitative dynamic analysis will be performed [19]. Thus, the development of a simplified shimmy model accounting as well for the dry friction with combined kinematics as for the deformed state is a topical problem. Indeed, some attempts of use of of the coupled dry friction theory with quasi-rigid wheel model and Hertz's solution for the contact pressure allowed not only to calculate the dry friction forces and torque but to estimate also the effects of the dry friction on the dynamics as it was made in [4,5,7-10]. One of first attempts of accounting for the contact pressure distribution close to real ones was made in [7]; the finite element solution was approximated by the Legendre polynomials, then the model coefficients were computed analytically. The coefficients definition remained the drawback limiting the practical use of the model [7]. The technique of experimental definition of these coefficients was presented in [19]. The validation procedure proposed below consists in two main stages: at the first one the coefficients are calculated using analytical formulae [4,5,7,9] with aid of numerical simulation of the contact pressure distribution, then these coefficients are defined from the numerical dependence for dry friction torque and force components on the slip and spin velocities.

2 GENERAL MODEL OF THE COUPLED DRY FRICTION

For a general case, let us consider the orthotropic dry friction given by the tensor $\mathbf{f}$ defined within its principal components f and κf, and the Cartesian frame Oxy attached to the centroid of the area S of contact of a wheel and a road; we consider below only axially symmetric contact areas having the average radius R; the appropriate base vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are collinear to the principal axes of $\mathbf{f}$. Finally, let us assume the static contact pressure distribution, $\sigma_0(x,y)=\sigma_0(\pm x,\pm y)$, be symmetric with respect to the center O of the area S.

Let the motion of a wheel be defined by the longitudinal velocity $\mathbf{v}_0=v\mathbf{e}_1$ (i.e. along the axis $O\xi$ of the global rest frame; for more details, see [5] and [7]), the angular velocity of rolling $\boldsymbol{\omega}_\tau=-\Omega_r\mathbf{e}_2$, and the velocity of spinning ω. The presence of a thread of any real tire is modeled by the different dry friction factors f along the tread and κf across it. The rolling friction is defined by the vector $\mathbf{d}$ representing the shift of the gravity center of the contact area S with respect to its geometric center O, therefore the deformed contact pressure distribution could be introduced as $\sigma(x,y)=\sigma_0(x,y)(1+k_x x+k_y y)$ [14] where k_x and k_y are projections of the vector $\mathbf{d}$ on axes x and y. We could define hence the friction-induced tangent strain at arbitrary point $M\in S$ as follows:

$$\boldsymbol{\tau}=\mathbf{f}\cdot\sigma(x,y)\frac{\mathbf{v}_\Sigma}{|\mathbf{v}_\Sigma|},\quad \mathbf{v}_\Sigma=\mathbf{v}_0+\boldsymbol{\omega}_\tau\times\mathbf{r},\quad \mathbf{r}=x\mathbf{e}_1+\mathbf{y}\mathbf{e}_2. \tag{1}$$

The resultant force vector can be determined by integration of the relationship (1) over the contact area S and represented as a sum $\mathbf{T}=T_{\parallel}\mathbf{e}_1+T_{\perp}\mathbf{e}_2$ of the longitudinal friction force $T_{\parallel}$ and the lateral one denoted as $T_{\perp}$. It was shown [9] that the lateral friction appears under the conditions of simultaneous slip and spin due to the coupling effects. The orthogonal transformation of rotation on the angle φ gives the new frame attached to the contact area, $\xi=x\cos\varphi-y\sin\varphi,\ \eta=x\sin\varphi+y\cos\varphi$, and enables to obtain the resultant integration over the fixed domain [9], therefore the appropriate expressions can be simplified keeping in mind that integrals will be zero for symmetric integration domains and odd integrand function by one of the arguments. As a result, for $\kappa=1$ we obtain for the friction force and torque

$$F_{\parallel}=f\iint_G\frac{\sigma_0(1+k_x x+k_y y)(v-\omega(x\sin\varphi+y\cos\varphi))dxdy}{\sqrt{v^2-2v\omega(x\sin\varphi+y\cos\varphi)+\omega^2(x^2+y^2)}} \tag{2}$$

$$M_v=f\iint_G\frac{\sigma_0\omega(x^2+y^2)-v(x\sin\varphi+y\cos\varphi)dxdy}{\sqrt{v^2-2v\omega(x\sin\varphi+y\cos\varphi)+\omega^2(x^2+y^2)}} \tag{3}$$

$$F_{\perp}=f\iint_G\frac{\sigma_0\omega(x\cos\varphi-y\sin\varphi)dxdy}{\sqrt{v^2-2v\omega(x\sin\varphi+y\cos\varphi)+\omega^2(x^2+y^2)}} \tag{4}$$

The model defined by integral formulae (2-4) is interpreted as “exact” one.

3 APPROXIMATE MODEL OF THE COUPLED DRY FRICTION FOR A TIRE

The formulae approximating (2-4) for the longitudinal dry friction force, the lateral dry friction force, and finally for the dry friction torque could be written as follows [7, 19]:

$$F_{\parallel} = \frac{F_0 v}{\sqrt{v^2 + au^2}}, \quad F_{\perp} = \frac{k_x M_0 u}{2\sqrt{u^2 + \frac{1}{4} mv^2}}, \quad M_C = \frac{M_0 u}{\sqrt{u^2 + mv^2}} \tag{5}$$

Here the following factors are introduced:

$$N = 2\pi R^2 A^1, \quad F_0 = 2\pi f R^2 A^1, \quad M_0 = 2\pi f R^3 A^2 \tag{6}$$

$$\frac{1}{\sqrt{a}} = \frac{1}{F_0}\pi f R^2 A^0 = \frac{1}{2}\frac{A^0}{A^1}; \quad \frac{1}{\sqrt{m}} = \frac{1}{M_0}\pi f R^3 A^3 = \frac{1}{2}\frac{A^3}{A^2}. \tag{7}$$

$$A^m = \int_0^1 \sigma_0(\rho)\rho^m d\rho, \quad \rho = \frac{r}{R}.$$

4 CONTACT PRESSURE IN THE SPOT OF TIRE-ROAD CONTACT

Let us consider a typical pneumatic tire (e. g. see [14, 7]) with the boost pressure $p^{\star} = 200$ kPa and loaded by a vertical force. To obtain the contact pressure distribution as well as the contact spot diameter, the finite element simulation analogous to the one presented by the authors of [14] was performed. The solution presented below (100 nodes along the contact spot diameter) corresponds to the maximum vertical deformation of the tire at the center of the contact spot equal to $\delta = 0.040$ m, and the corresponding contact area diameter is equal to $R = 0.087$ m. The numerically obtained distribution of the dimensionless contact pressure $p = P / p^{\star}$ is strictly heterogeneous with local maxima near the contact spot boundary of and the minimum near its center (Figure 1).

To implement the approximate model of the dry coupled friction the analytical interpolation of this pressure distribution is required. Accounting for the symmetry of the contact pressure distribution, let us use the following formula:

$$p = \sum_{k=0}^{N} a_k \cos(\pi k \omega x), \quad x \in [-1,1], \quad \omega \in \mathbb{R} \qquad \mathbb{N} \tag{8}$$

The seventh order approximation (i. e. $N = 7$) is used below; the corresponding coefficients are shown in the Table 1.

Table 1: Coefficients of the trigonometric approximation of the numerical solution for the contact pressure ω

ω	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
0.848	0.685	0.398	-0.831	0.230	0.161	-0.027	-0.133	0.101

The approximated pressure is shown on the Figure 1 by the solid line. One could conclude that the approximation (8) with the coefficients given by the Table 1 offers a good approximation of the numerical solution; it seems to be better than the polynomial interpolation used in [7].

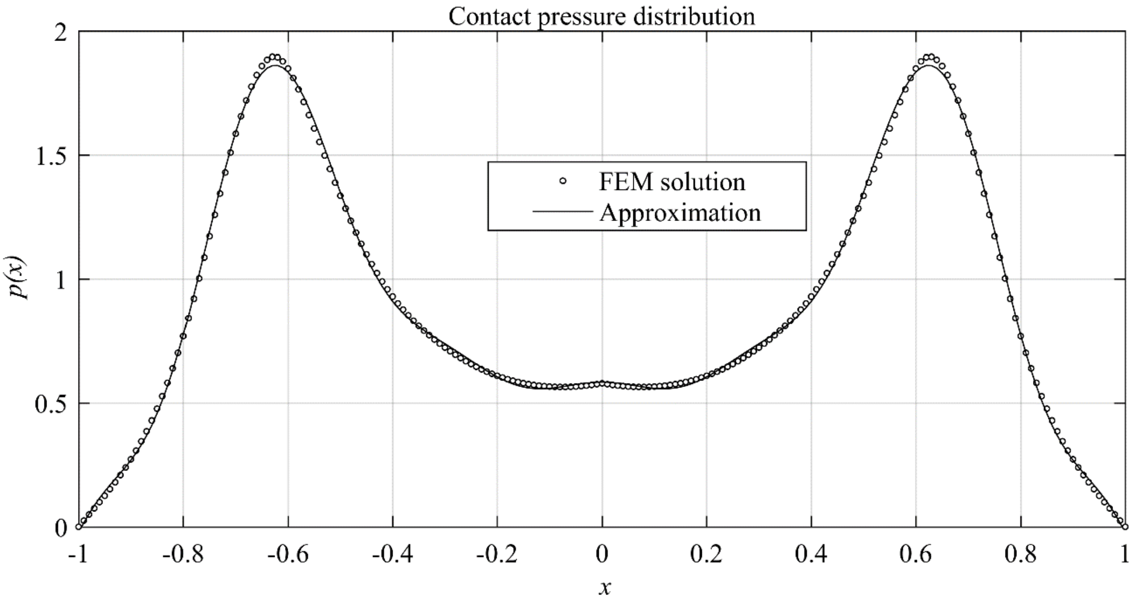

Figure 1: Contact pressure distribution along the contact spot: finite element simulation, trigonometric approximation

Let us assume hence the shape of the contact spot be close to a circle (accordingly to the numerical solution [14]), and let us consider the rolling defined by the angular velocity ω_y (see also [19]). Thus, the deformation of the contact pressure distribution could be taken into account by introducing the rolling correction factor k_x:

$$p_\Sigma = p_0(r,\theta)\left(1 + k_x x \left|\omega_y\right|^{-1} \omega_y\right) \qquad (9)$$

Here x is the dimensionless coordinate along the contact spot while k_x is the value of the shift of the gravity center of the contact pressure distribution due to the rolling.

Let us consider hence the contact pressure distribution corresponding to the steady-state rolling along the Ox axis. The numerically obtained solution is shown below on the Figure 2. The following formulae (see [19]) could be used for the rolling correction factor:

$$k_x = R\frac{s_x}{s}, \quad s = \pi R\int_0^1 \sigma_\nu(r,\theta) r^3 dr, \quad s_x = \frac{s_x^1}{s_x^2},$$
$$s_x^1 = R\int_0^{2\pi}\int_0^1 \sigma_\nu(r,\theta) r^2 \cos\theta\, dr d\theta, \quad s_x^2 = R\int_0^{2\pi}\int_0^1 \sigma_\nu(r,\theta) r dr d\theta. \qquad (10)$$

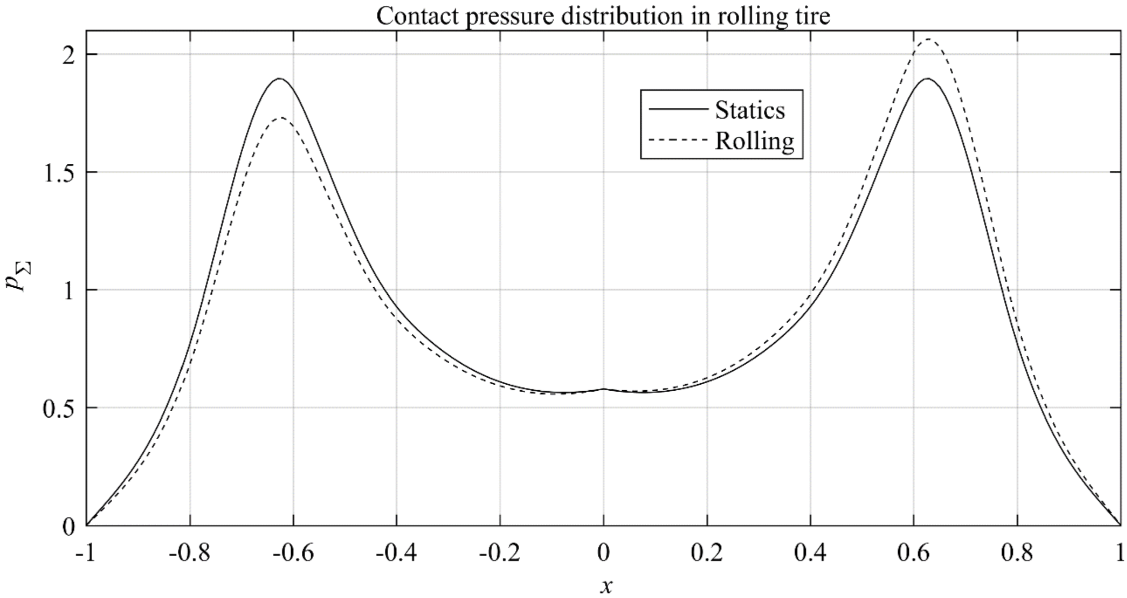

Figure 2: Contact pressure distribution along the contact spot: finite element simulation, trigonometric approximation

Given the pressure distribution (Figure 2) and using (10) we obtain $k_x = 0.14$.

5 PARAMETER IDENTIFICATION FOR THE ONE-DIMENSIONAL MODEL OF THE COUPLED DRY FRICTION

The factors given by (5-7) could be computed using the approximation (8) analytically:

$$A^m = \frac{1}{m+1}\sum_{k=0}^{N} a_k \left\{ -k\omega(k\omega)^{-m-5/2}\left(k\omega\pi^{-1/2-m}\cos(\pi k\omega) - \pi^{-m-3/2}\sin(\pi k\omega)\right) s_{m+3/2,1/2}(\pi k\omega) + \right. \\ \left. +\pi^{-1/2-m}(k\omega)^{-m-3/2} s_{m+1/2,3/2}(\pi k\omega)\sin(\pi k\omega)km\omega + \cos(\pi k\omega)\right\}, \tag{11}$$

here $s_{\mu,\nu}$ are Lommel's functions. Given boost pressure $p^{\star}$ [illegible]0 kPa and accounting for the formula (11), we obtain the following factors values:

Table 2: Example of the construction of one table

N, kN	F_0, kN	M_0, Nm	a, m$^{1/2}$	m, m
4.48	1.34	72.25	1.307	1.724

Let us introduce the following dimensionless variables for the dry friction model:

$$\upsilon = v/u, \quad \psi = \upsilon^{-1} = u/v. \tag{12}$$

Taking into account the formulae for the longitudinal dry friction force, lateral dry friction force, and the dry friction torque (5), and substituting into (5) the dimensionless variables (12), we obtain the one-dimensional dependencies for dry friction forces and torque [19]:

$$F_{\|} \quad \psi\left(\psi^2+a\right)^{-1/2}, \quad F_{\perp}=k_x M_0 \upsilon\left(4\upsilon^2+m\right)^{-1/2}, \quad M_C=M_0\upsilon\left(\upsilon^2+m\right)^{-1/2}. \tag{13}$$

Given F_0, M_0, k_x, a and m (see Table 2), we obtain the diagrams for the longitudinal dry friction force, lateral dry friction force, and the dry friction torque (13) (see Figures 3-5):

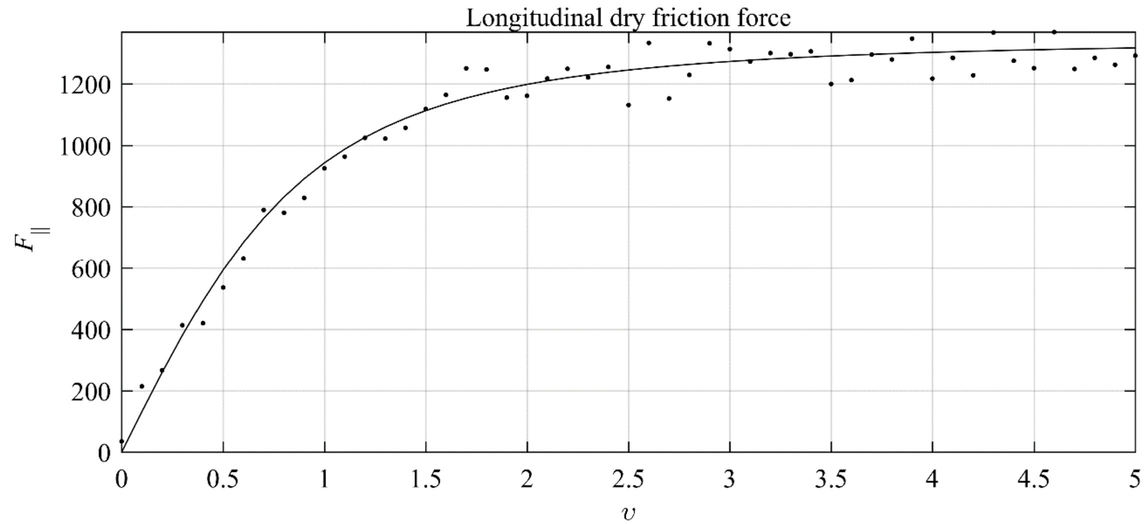

Figure 3: Dependence of the longitudinal friction force on the ratio between the dimensionless sliding velocity and spinning angular velocity, υ

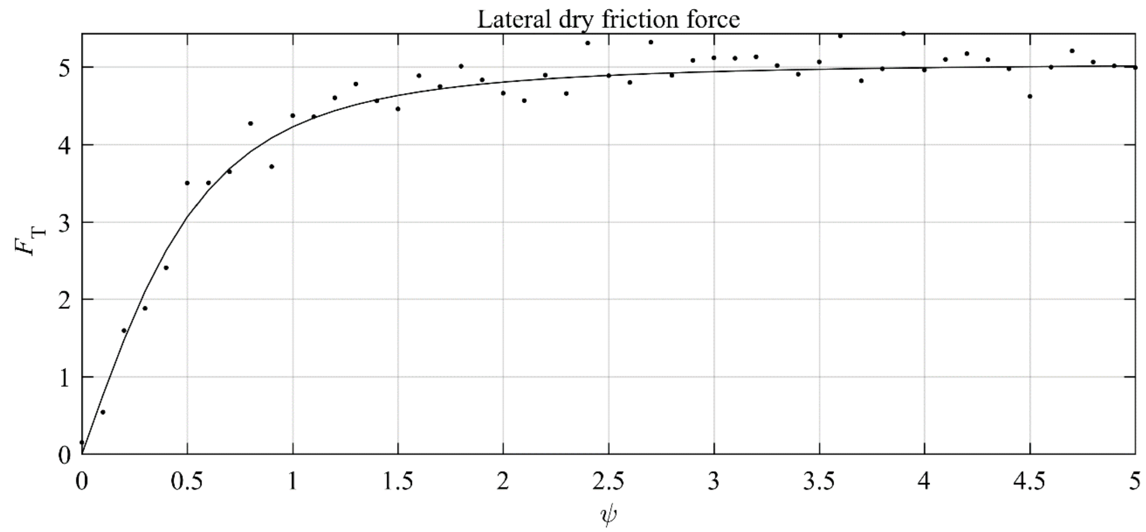

Figure 4: Dependence of the lateral friction force on the ratio between the spinning angular velocity and dimensionless sliding velocity, ψ.

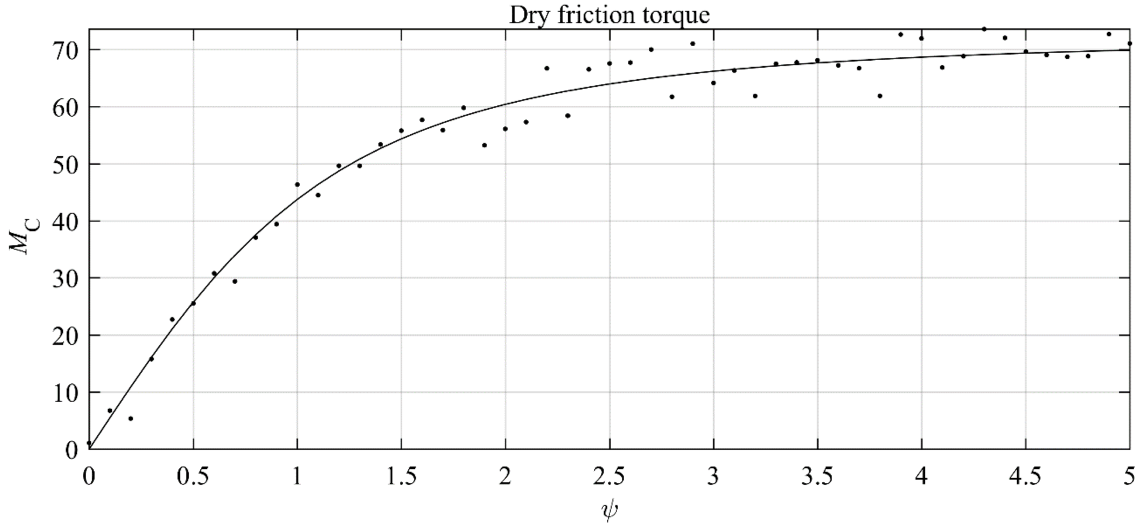

Figure 4: Dependence of the friction torque on the ratio between the spinning angular velocity and dimensionless sliding velocity, ψ.

Let us consider hence these diagrams $F_{\parallel}(\psi)$, $F_{\perp}(\psi)$, and $M_C(\psi)$ as measured ones; thus, the aforementioned model factors a, m, M_0, F_0 could be determined using various numerical optimizations methods (e.g. the nonlinear least squares fitting [18]) from these curves. The appropriate “noised” diagrams are simulated here by applying the random distribution with amplitude equal to 20%. The appropriate “simulated test diagrams” are shown above on Figures 2-4 by dotted lines.

Thus, denoting the values of ratios between the sliding velocity and spinning angular velocity, ψ and its inverse value, υ, by x_k while the quantities y_k are interpreted as “measured” values of the longitudinal dry friction force $F_{\parallel}$, lateral dry friction force $F_{\perp}$, and dry friction torque M_C (Figures 3-5), we have the following condition [19]:

$$\min \sum_{k=1}^{N} \left\| f\left(x_k\right) - y_k^2 \right\| \tag{14}$$

that leads to the estimates of the model factors a, m, M_0, F_0; the dry friction coefficient can be obtained as follows:

$$f = \frac{F_0}{N}. \tag{15}$$

The fitting (14) of the simulated test data results in the following values of the factors of the proposed coupled dry friction model obtained with 95% confidence bounds (see Table 3). The results shown in the Table 3 show a good correlation between the theoretical values and the ones obtained after the simulated experimental tests.

Table 3: Defined coefficients of the coupled dry friction model

Coefficient	Theoretical value	"Experimental" value	Confidence bounds
f	0.3	0.301	0.2958, 0.3061
F_0	1344	1348	1325, 1371
M_0	72.25	72.63	71,19, 74.07
a	1.030	1.024	0.8738, 1.1740
m	1.720	1.609	1.371, 1.847
k_x	0.140	0.139	0.1376, 0.1404

Let us note that the simplest model of the contact interaction of the tire with the road combined with the Amonton-Coulomb dry friction model cannot define neither the lateral force nor the dry friction torque, moreover it overestimates the dry friction force value:

$$F_0 \approx \pi R^2 p^{\star} f \; - \; [illegible].427\,\text{kN}. \qquad (16)$$

6 CONCLUSIONS

- The new model of the dry friction with coupled kinematics of the relative motion of interacting solids is considered accounting for the real contact pressure distribution;
- The real pneumatic tire is investigated, and the contact pressure is obtained from the finite element simulation of the quasi-static deformed state of the pressurized tire;
- The trigonometric approximation of the contact pressure in severely deformed tire is proposed, and the corresponding analytical representation of the coefficients of the dry friction model are constructed;
- The dependencies of the longitudinal dry friction force, lateral dry friction force, and dry friction torque on the ratio of the sliding velocity and spinning angular velocity are constructed using dimensionless variables;
- The method of identification of the coefficients of the developed dry friction model using the experimental dependencies of the longitudinal dry friction force, lateral dry friction force, and the dry friction torque on the ratio of dimensionless sliding velocity and spinning angular velocity is proposed;
- The experimental data are simulated by adding the random noise to the theoretically computed diagrams, and the model coefficients are obtained from these perturbed diagrams by nonlinear least squares procedure;
- The rolling friction coefficient as well as other factors of the coupled dry friction model based on fraction-rational approximations are computed using the numerical simulation of the steady rolling of the tire corresponding to the possible test data;
- The good correlation between the exact model coefficients and the ones obtained from the simulated test curves is shown, and the principal possibility of identification of the model parameters after typical tests for a rolling tire is shown.
- The further development of the proposed approach may consist in the accounting of both vertical and lateral deformations of the tire as it was proposed in [21].

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