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LOWER BOUND LIMIT ANALYSIS OF MASONRY PLATES IN TWO-WAY BENDING BY MEANS OF FULL 3D ELEMENTS

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Abstract. *The paper provides a novel Lower Bound (LB) Limit Analysis (LA) Finite Element (FE) model for the study at failure of masonry walls in two-way bending by means of full 3D elements. The method of hexahedral discretization is used, while assuming infinite resistance and a quadrilateral interface where all plastic dissipation occurs. It can more accurately analyze the collapse mechanism of masonry panels in the process of two-way bending. It chose two cases to study. They are three series of panels with and without perforations tested at collapse at the University of Adelaide Australia and four series of solid and perforated panels tested at the University of Plymouth UK. The feasibility of the research method was verified. The obtained research results show that the use of the method proposed allows to provide a safe prediction of the existing LA code based on kinematics theorem, with a small computational burden, and the obtained results are more in line with the actual situation and have better practical effects.T*
The original abstract is titled: "Out of Plane Lower Bound Limit Analysis"

1 INTRODUCTION

According to post-earthquake surveys of masonry buildings around the world by current scholars, the collapse of current masonry structures is most likely to occur in out-of-plane failures with rigid wall sections with variable shapes. The rotation and translation of the wall will form a predetermined kinematic chain. Factors such as the texture of the masonry structure, the membrane load and the interlocking between perpendicular walls all have an impact on the shape of its deformation [1]-[11].

At present, the research of domestic and foreign scholars mainly focuses on the in-plane research of the wall, while the experimental research of the out-of-plane loaded masonry is relatively small. The UK code of practice for the use of masonry BS 5628 [12][13] was probably the first to propose the use of the yield-line theory [14] to estimate the load carrying capacity of masonry panels in two-way bending under different boundary conditions. However, this method cannot be applied to the masonry buildings under the current earthquake. Likewise, the Italian national code on constructions [15] prescribes to evaluate the seismic vulnerability of existing buildings by means of the so-called linear kinematic analysis. This method may overestimate the out-of-plane bearing capacity, and the estimated vulnerability is often unsafe.

At present, the research of scholars at home and abroad mainly focuses on the in-plane research of the wall, while the experimental research on the out-of-plane loading of masonry is relatively lacking. In 1997, British scholars first proposed the use of the yield-line theory to estimate the bearing capacity of masonry panels under bidirectional bending under different boundary conditions. They have written this method into the UK code of practice for the use of masonry BS 5628 [12][13]. But with the development of technology and research, this method is no longer suitable for masonry buildings under the current earthquake. Likewise, there are separate codes for assessing the seismic vulnerability of existing masonry structures in Europe, such as the Italian national code on constructions [15]. The limitations of these assessment methods are that they tend to overestimate the out-of-plane bearing capacity of masonry structures and are too idealistic and often unsafe for the analysis of building vulnerability.

To overcome these shortcomings, scholars have proposed more advanced mathematical model calculation methods. These methods fall into two main categories: incremental non-linear techniques and limit analyses. Incremental non-linear techniques require users to be proficient in mathematical theory and professional skills, combined with expensive calculation soft armor, and the calculation time is long. In contrast, limit analysis techniques are more time- and labor-saving in evaluating the failure behavior of out-of-plane loaded masonry panels. Even so, there is still a lack of a modeling research method based on the lower bound (LB) theorem of limit analysis.

The research of this paper is to try to fill the blank of this part. It provides a novel Lower Bound (LB) Limit Analysis (LA) Finite Element (FE) model for the study at failure of masonry walls in two-way bending by means of full 3D elements. The method of hexahedral discretization is used, while assuming infinite resistance and a quadrilateral interface where all plastic dissipation occurs. At this interface, the three internal actions of bending moment, out-of-plane Kirchhoff shear and torque act on the plastic field to dissipate power. Imposes equilibrium on rigid elements and enforces admissibility at the interfaces between adjoining elements. The results obtained by this method are straightforward.

2 METHODOLOGIES

According to the LB approach, a fully 3D masonry structural wall is considered that can accept out-of-plane presence and constrained in a way that promotes bidirectional bending. Suppose the structure is discretized by infinite resistance and rigid hexahedrons (Figure 1). These discretized hexahedral structures undergo all dissipation under the quadrilateral rigid-plastic interface connection. Defining out-of-plane shear, torsion, and bending moments as internal variables for each interface, the Kirchhoff-Love assumption is proposed to enforce the acceptability of internal effects on the interface between the interior and the boundary [17].

In Figure 1, there are four interface I, J, K, L between element i and adjacent elements. Here the research uses interface I for specific analysis (Figure 1). P1-P4 respectively represent the four vertices in interface I, and set the local reference system on different planes (Figure1, Figure 2), see Eq. (1):

$$\begin{cases} \mathbf{q}^I = \dfrac{P_2 - P_1}{\sqrt{(P_2 - P_1)(P_2 - P_1)^T}} \\ \mathbf{t}^I = \mathbf{k} \\ \mathbf{n}^I = \dfrac{(P_4 - P_1) \times (P_2 - P_1)}{\sqrt{[(P_4 - P_1) \times (P_2 - P_1)][(P_4 - P_1) \times (P_2 - P_1)]^T}} \end{cases} \tag{1}$$

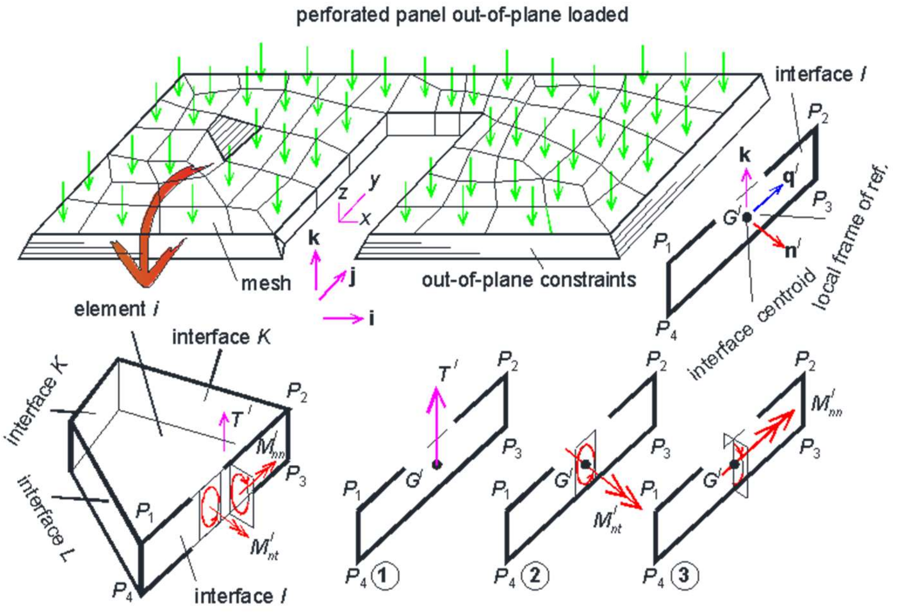


Figure 1: A full 3d panel which is discretized by infinite resistance and rigid hexahedrons.

Considering interface I, J, K, L, six equilibrium equations which contain three translations of the centroid and three rotations around the centroid for the element can be written see Eq. (2). It is worth noting that this equation applies to panel with large slenderness, where the panel is primarily affected by out-of-plane effects. If readers are interested, please refer to the paper [17].

$$\begin{cases} \sum_{M=I}^{L} T^M + \mathrm{l}\alpha_{\mathrm{l}h} + \alpha_{0h} = 0 \\ \sum_{M=I}^{L} (T^M \mathbf{r}^M \times \mathbf{k}\bullet\mathbf{e}_h + M_{nt}^M \mathbf{n}^M \bullet\mathbf{e}_h + M_{nn}^M \mathbf{q}^M \bullet\mathbf{e}_h) = 0 \end{cases} \tag{2}$$

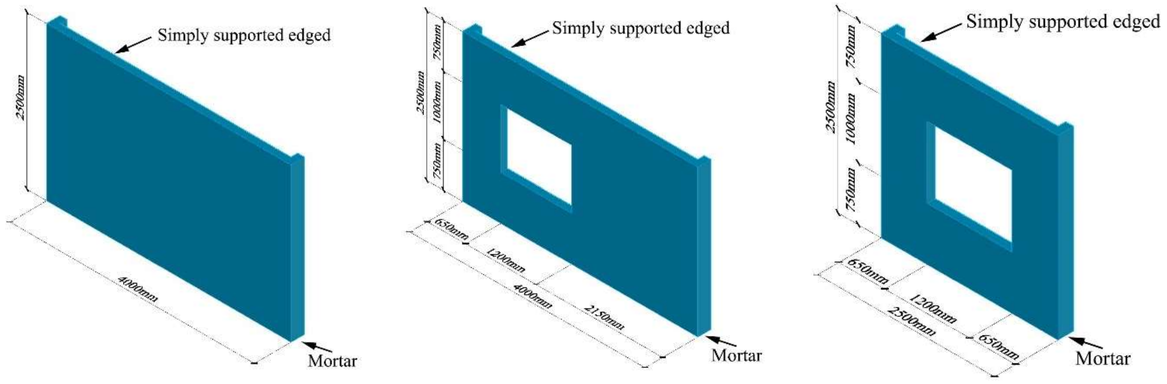
In this research, assume the material of the full 3D wall is an orthotropic rigid-plastic material, which is a Kirchhoff-love plate. Here, the Lower Bound approach in limit analysis theory can be used to solve the problem of homogenization of materials.

3 CASE STUDY

Two cases which have been deeply tested were chosen to do the LB numerical analysis. Compared the existing research results, the accuracy of the method could be approved.

3.1 University of Adelaide panels

The first case was from Ref.[18], an experimental campaign carried out in University of Adelaide. The paper chose 8 walls with different scales, positions of openings, and the restraint conditions. From Figure 2, the walls were named from W1 to W8, and the thickness of them are all 110mm. W1-W6 are rectangular panels measuring 4000 mm × 2500 mm (L × H), and W7-W8 are square panels measuring 2500 mm × 2500 mm (L × H). The interior of the W3-W8 panels contains an opening measuring 1200 mm × 1000 mm (L × H). The difference is that in W3-W6 panels, the openings are off-center, while in W7-W8, the openings are in the exact center of the panel. The pre-compression (σ_v) of the walls are also different. Among them, σ_v in W2, W5, W6, W8 is 0Mpa, σ_v in W1, W3, W7 is 0.10Mpa, σ_v in W4 is 0.05Mpa. In the Ref.[18], researchers used the mortar to connect the walls and the ground which can constraint the panels. It assumed that the top horizontal edges of all the walls (except W6) are simple supports.



-a -b -c

Figure 2: Geometry of W1-W8. -a: W1-W2, -b: W3-W6, -c: W7-W8.

Figure 3 to Figure 6 show the results obtained with the proposed LB model for the eight walls experimentally tested in [18]. Subfigures -a and -b show Lower Bound limit analysis Finite Element results, subfigures -c show the Comparison between experimental and numerical results.

Through the research results, it is possible to more accurately analyze how the behavior of such homogenized slabs is strongly bi-directional. Their cracks mainly follow sloping yield lines. The cracks start at the corners of the wall and gradually extend along the structure to the interior openings. Comparing the numerical simulations in this paper and references, the results of this study are very optimistic.

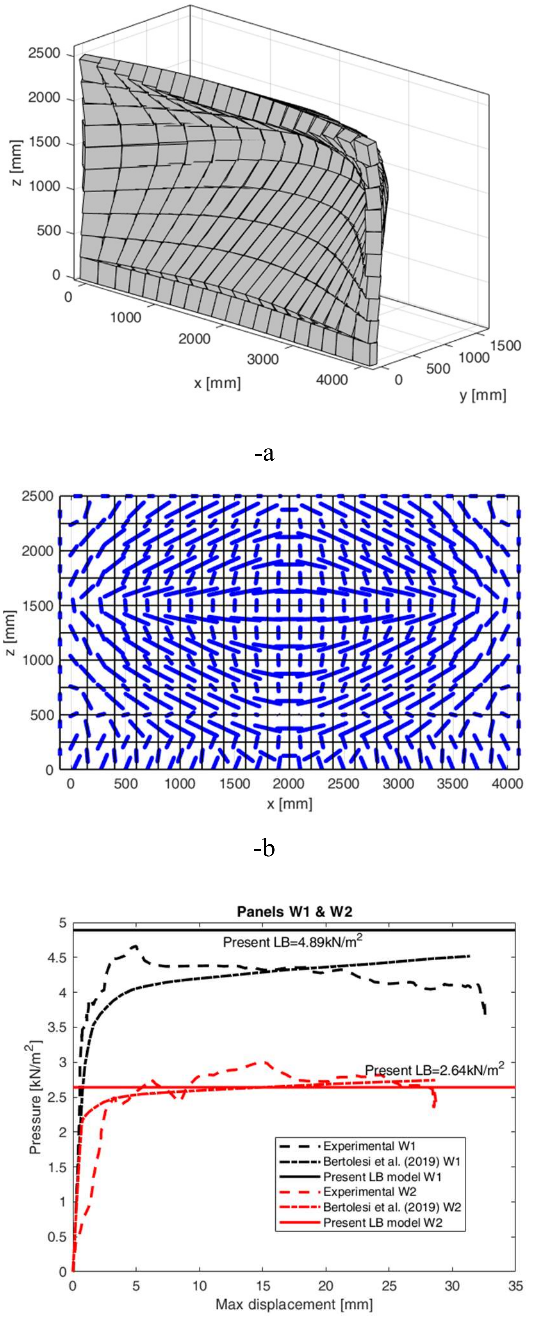


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Figure 3: University of Adelaide Panels W1-W2. -a, -b: Lower Bound limit analysis Finite Element results. -c: Comparison between experimental and numerical results

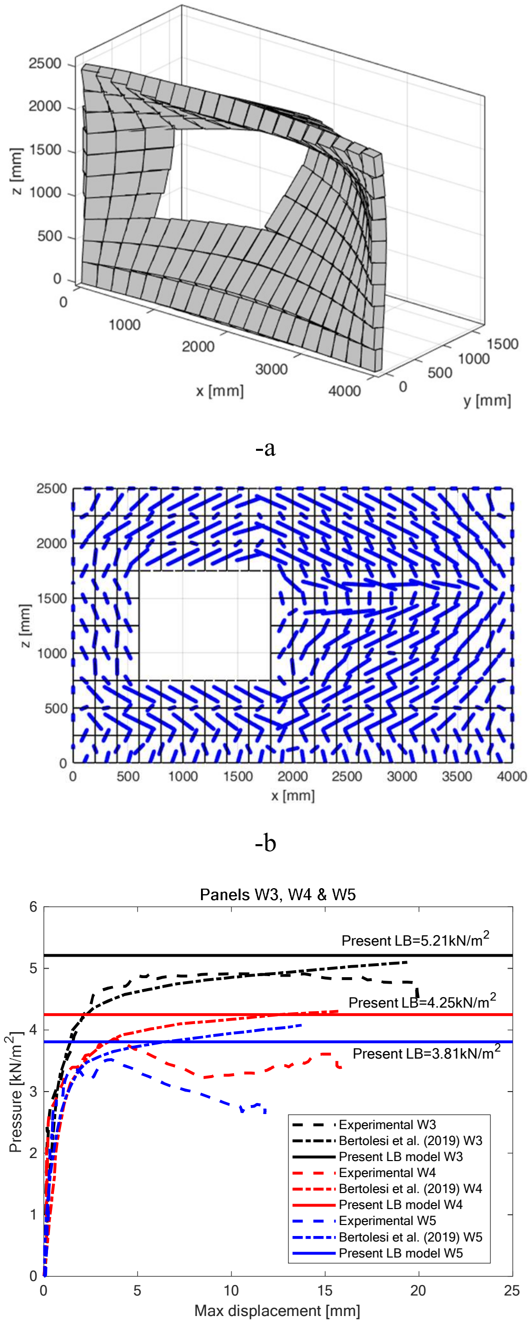

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Figure 4: University of Adelaide Panels W3-W5. -a, -b: Lower Bound limit analysis Finite Element results. -c: Comparison between experimental and numerical results

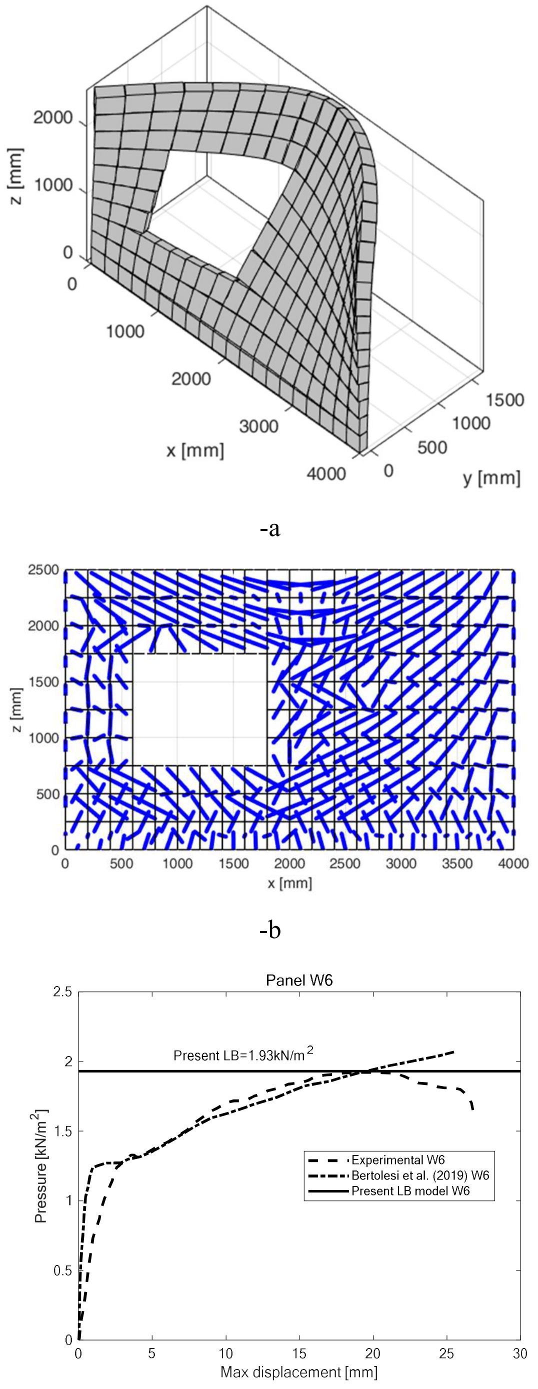

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Figure 5: University of Adelaide Panel W6. -a, -b: Lower Bound limit analysis Finite Element results. -c: Comparison between experimental and numerical results.

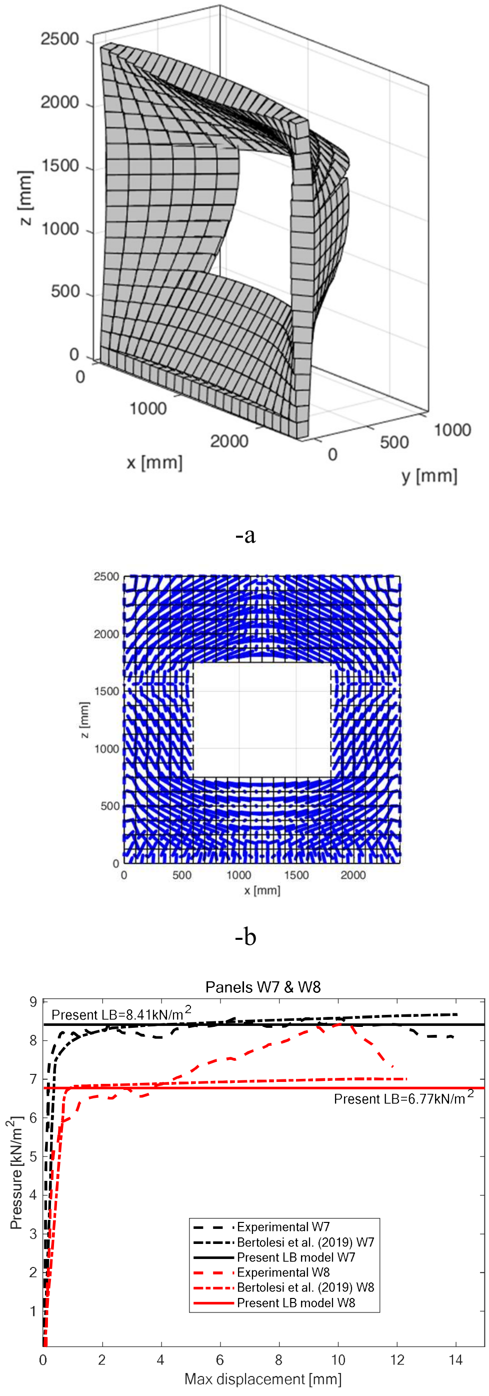

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Figure 6: University of Adelaide Panels W7-W8. -a, -b: Lower Bound limit analysis Finite Element results. -c: Comparison between experimental and numerical results.

3.2 University of Plymouth experimental tests

According to Ref. [19] and [20], the second validation carried out concerns four masonry walls experimentally tested at the University of Plymouth (UK), whose results are reported and discussed by Chong et al. and Southcombe et al. This test provides four walls SB01-SB04 whose dimensions are all 5600mm × 2475 mm (L × H), and their thickness are 102.5mm. The wall SB02 is a solid panel, while other three walls contain different sizes openings in difference position. Only the self-weight of masonry was considered in the simulations.

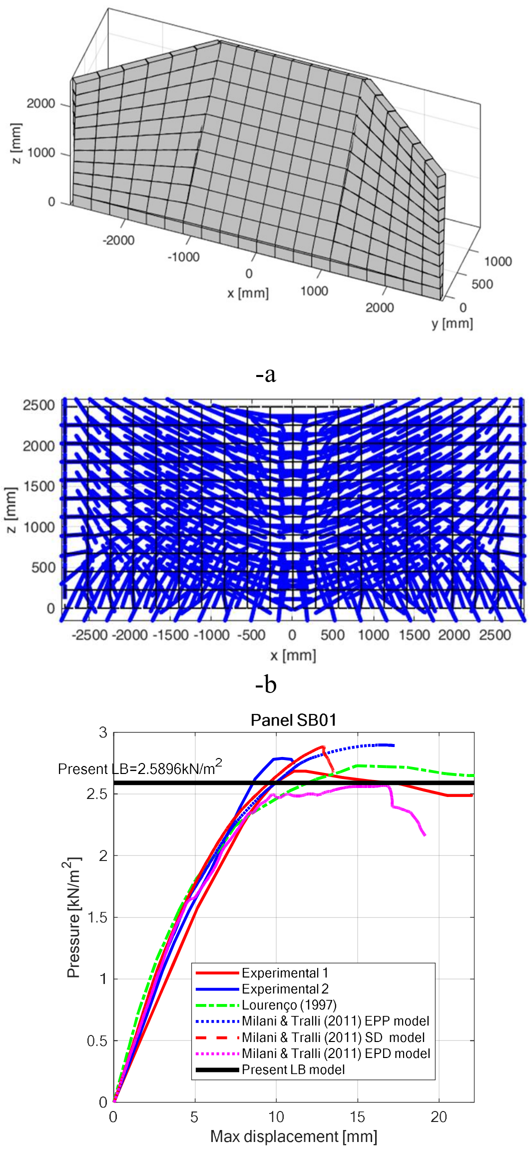

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Figure 7: University of Plymouth Panel SB01. -a, -b: Lower Bound limit analysis Finite Element results. -c: Comparison between experimental and numerical results.

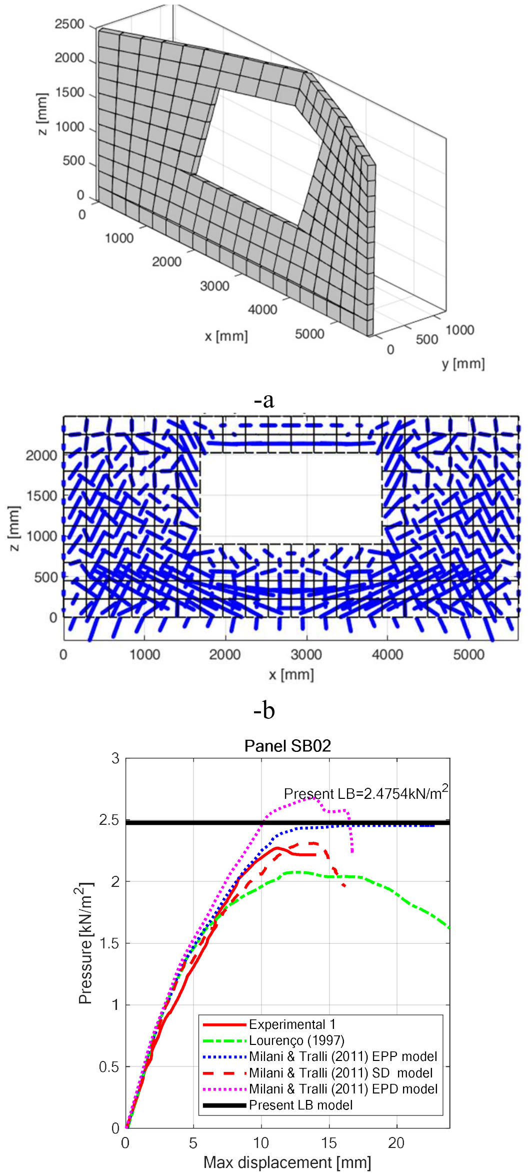

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Figure 8: University of Plymouth Panel SB02. -a, -b: Lower Bound limit analysis Finite Element results. -c: Comparison between experimental and numerical results.

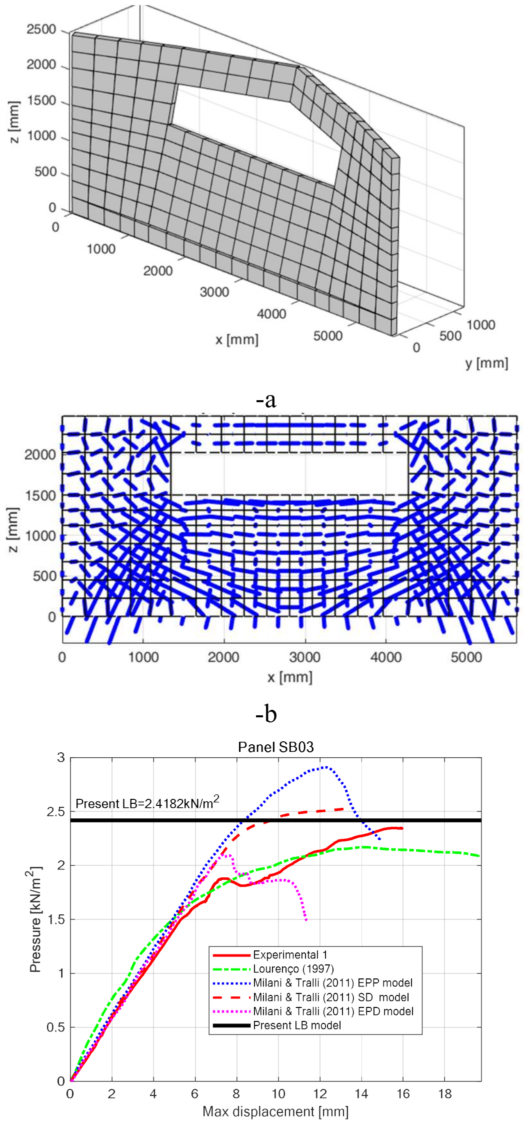

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Figure 9: University of Plymouth Panel SB03. -a, -b: Lower Bound limit analysis Finite Element results. -c: Comparison between experimental and numerical results.

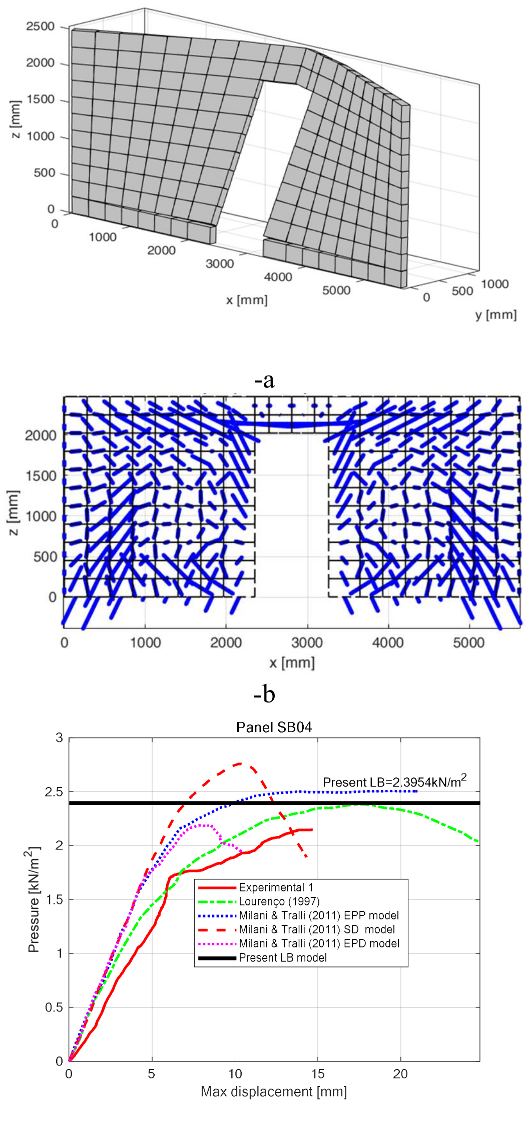

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Figure 10: University of Plymouth Panel SB04. -a, -b: Lower Bound limit analysis Finite Element results. -c: Comparison between experimental and numerical results.

From Figure 7 to Figure 10, the results of the walls SB01-SB04 were summarized. The meanings of the three subfigures -a, -b, -c are the same as in Case 1.

The analysis results show that the LB analysis method proposed is very simple and fast to predict the distribution of collapse loads, failure mechanisms and internal effects of full 3D masonry walls. Comparing existing simulation data, even with relatively unrefined meshes, is reliable and computationally inexpensive.

4 CONCLUSIONS

The paper has presented a novel Lower Bound (LB) Limit Analysis (LA) Finite Element (FE) model for the study at failure of masonry walls in two-way bending by means of full 3D elements. The method of hexahedral discretization is used, while assuming infinite resistance and a quadrilateral interface where all plastic dissipation occurs. It can more accurately analyze the collapse mechanism of masonry panels in the process of two-way bending.

The feasibility of the research method is verified by experiments applying the LB limit analysis method to two existing masonry walls cases. The use of the LB limit method allows to provide safe predictions about existing LA codes based on kinematic theorems, with the main advantages of low computational burden and reliable and accurate results. In future research, the authors will work on extending the method from a single wall bidirectional slab analysis to the structural analysis of monolithic masonry buildings. It is hoped that this method can benefit the development of engineering research technology more widely.

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