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Tracing non-linearities in the load bearing behavior of structural membranes: an investigation of typical shapes and load cases

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Summary. One of the prevalent challenges in the design, numerical analysis and verification of structural membranes lies in the non-linearity of their load-response curves. Structural analysis has to be performed with a geometrically non-linear approach, due to the interaction of form and forces and thus a linear extrapolation or combination of analyis results is not possible. The appropriate modeling of the environmental impacts (such as wind and snow) also has a significant influence on the analysis results. Furthermore, non-linear material behavior can be of interest.
The resulting load-response curves (e.g. stresses, deformations) are typically non-linear and their interpretation towards the underlying safety requirements is not straight forward. In addition, the prestress also has a major influence on membranes' structural behavior. However, the current European regulations for the proof of the limit states (ULS and SLS) of any building requires a simplified categorization of the structural behavior.
This research investigates the load-bearing behavior of typical re-occurring membrane shapes in the context of current verification requirements. Typical load cases are applied and the structural behavior is shown under consideration of the mentioned non-linearities.

Keywords: CAD-integration, Isogeometric B-Rep Analysis, Non-linearities, Parametric Design, Structural Analysis, Verification of Structural Membranes.

1 INTRODUCTION

The design and analysis of structural membranes is a challenging task for architects and engineers, consisting of three major analysis steps: formfinding, structural analysis and cutting pattern generation. Being highly interactive, these steps lead to the iterative design cycle for structural membranes.
This paper focuses on the characteristic non-linear load-response curves of prestressed tensile membrane structures and the consequences for verification along European code guidelines, as partly presented in [2]. After a brief introduction of the non-linearities that need to be considered in the design and analysis of membranes, the prevalent semi-probabilistic safety-

concept is summarized. The case study with the typical membrane shapes is described, followed by the obtained results, a conclusion and outlook.

1.1 Types of non-linearity

Structural membranes typically show a geometrically non-linear behavior, i.e. the deformation of the shape cannot be neglected in analysis as it influences the structure's stiffness properties. However, the prevalent large deformations are expected to lead to small strains, therefore the highly non-linear material behavior is mostly not modeled in engineering practice, but simplified to linear elasticity.
The common assumptions made in the design and analysis of structural membranes are described in detail in e.g. [4,5].

1.2 Prevalent European verification guidelines

The semi-probabilistic safety concept of the Eurocodes was mainly developed for linear structural behavior and applies safety factors both to the applied loads and the material resistance for evaluating the ulitmate limit state, see [3]. For non-linear cases, a categorization into load-response curves, that either increase over- or underproportionally (simplified expression: over- and underlinear) needs to take place. Depending on this type of non-linearity, the design effect of action is either determined by introducing a safety factor to the characteristic action or effect of action, as shown in Figure 1. Amongst others, Philipp [11] and more recently, Bakeer [6] provided an explanation for the inapplicability of this categorization for non-linear systems. Furthermore, it was shown by Fußeder et al. [1] that the application of the described regulations to tensile structures is problematic and can lead to unsafe results.

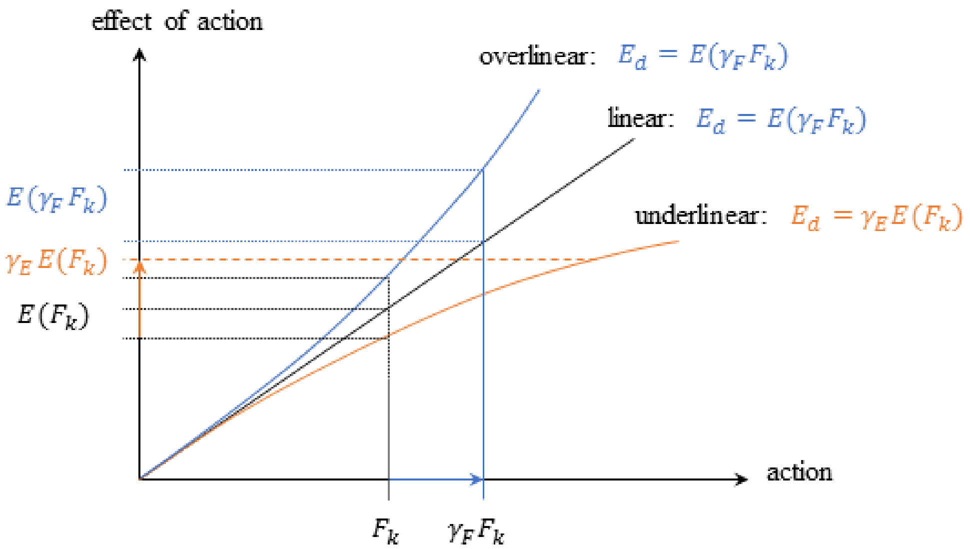

Figure 1: Characterization of non-linearity according to EC 0 and resulting action design regulations.

2 CASE STUDY

Typical membrane shapes have been defined in a number of publications, e.g. Knippers [4] and Wagner [5], the most common ones are shown in Figure 2: hypar, tent, saddle and cone. Even in large membrane structures, a variation or accumulation of these shapes can be found quite often. Therefore, their structural behavior can be expected to be representative for the investigated ranges of prestress and loading.
The shapes that are shown in this contribution resulted from a formfinding analysis with an isotropic prestress of 4 kN/m in the membrane and 30 kN in the edge cables. For reasons of comparability, the base layout was chosen to be 6 by 6 m, while the height of 2m was altered during the study in order to achieve a variation of curvature. The formfound structures were subjected to a uniformly distributed snow load and wind suction load, that were incrementally increased by the load factor λ to a characteristic intensity (at λ=1) of 0.6 kN/m² and 1.0 kN/m² respectively. The material model was a linear-elastic one, with a Young's modulus of 1000 1000 MN/m² for the membrane in both fibre directions and 205 MN/m² for the steel cables. All simulations were performed with Isogeometric B-Rep Analysis, i.e. the analyses were performed directly on the refined parametric CAD models. Parametrization could thus be used for geometric changes, as well as for alterations of mechanical parameters, as described in Goldbach [2].
The case study aimed at the categorization of the resulting stress-curves with respect to their non-linearity, as intended by EC 0 and described in Section 1.2. Previous investigations related to reliability in membrane design (see [2,7,9,10,13]) had shown the load intensity and prestress to be the main influential parameters. The main focus was thus put on a variation of those. Some results are summarized in the next section.

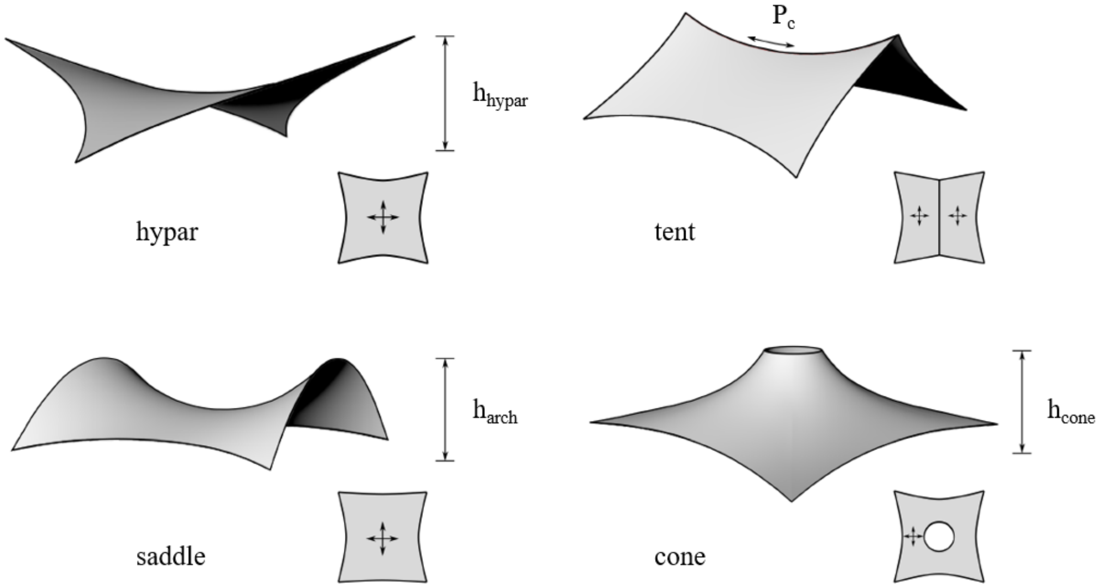

Figure 2: Typical membrane shapes, adapted from [2].

3 RESULTS

As can be seen in Figure 3, the principal stresses in all investigated membrane structures develop in a fairly linear manner (linear extrapolation indicated with dotted lines), until tension is lost in one direction and the load transfer thus shifts[1]. The stresses represented here are the overall minimum and maximum values, starting from the level of prestress. A shift in the prestress level did not affect this pattern, as can be seen for the hypar example shown in the bottom part of the figure. Linear extrapolations from the origin to the effect of the characteristic action at λ=1 are included in the bottom graphs. It can be seen that a classification following these linear curves is not intuitive.

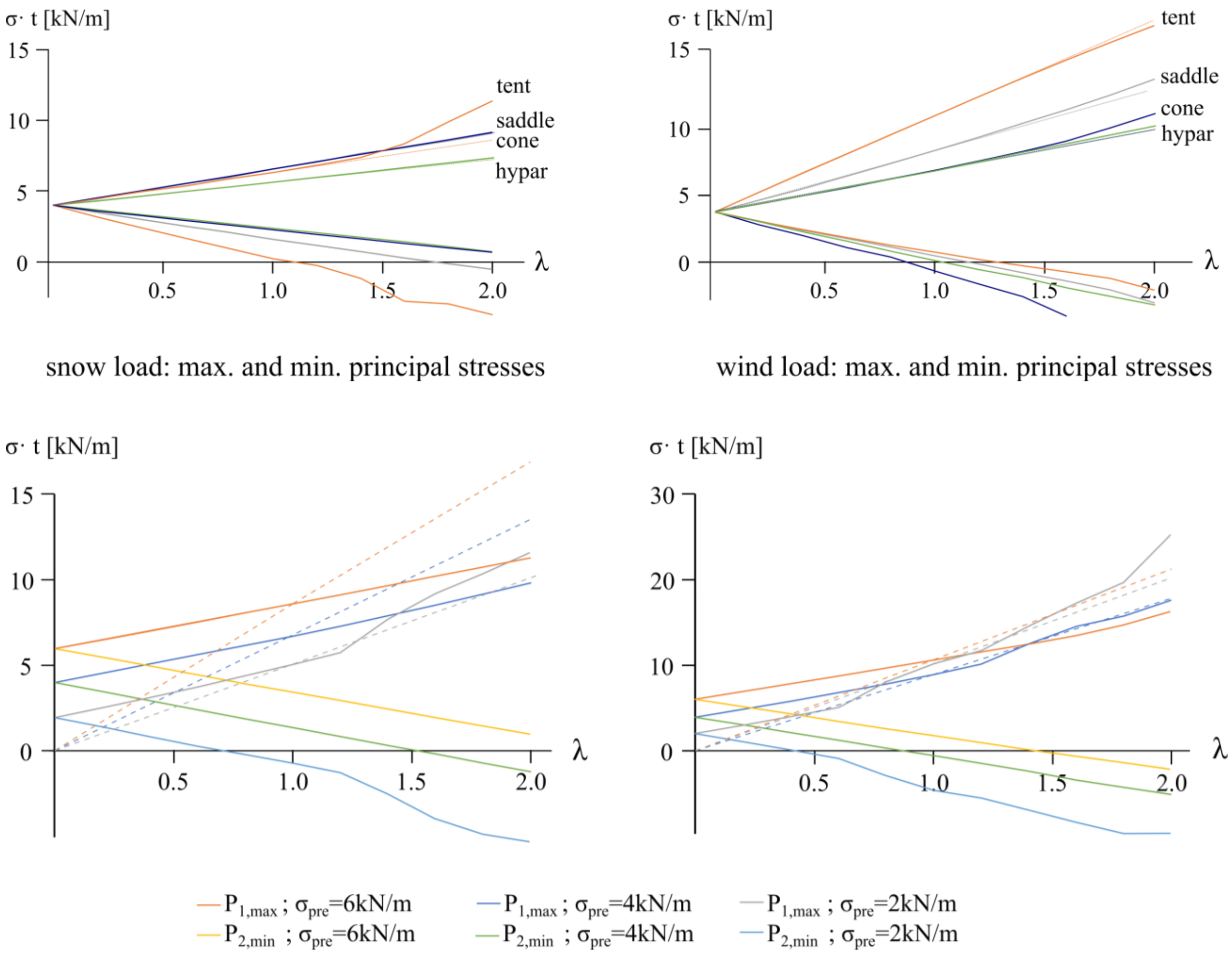

Figure 3: Top : Principal stresses developing under snow and wind suction load for four typical membrane shapes undergoing load controlled geometrically non-linear structural analysis.
Bottom : Principal stresses under snow and wind suction load for the hypar at different prestress levels undergoing load controlled geometrically non-linear structural analysis.
Both adapted from [2].

[1] It should be noted, that the analysis did not include a wrinkling model.

3.1 Conclusions related to non-linearity for the basic shapes

The case study showed, that the classification of the non-linear load response into over- or underlinear curves cannot be known a-priori, but has to be investigated for each shape and load case, as well as the type of response (stress or deformation). This can e.g. be seen in the top graphs of Figure 3: the maximum principal stress of the tent shaped membrane increases overproportionally under snow load and underproportionally under wind suction load (orange lines and dashed lines as linear comparison). Furthermore, the resulting stress curves were very close to linear responses as long as a taut state was kept – which is a prerequisite in membrane structures but not yet considered by the regulations of the European design codes.

Another aspect of the categorization into types of non-linearity that needs to be discussed is the origin of the linear curve used for the identification. The natural origin for many structures will be zero and this case is represented in EC 0 [3]. However, tensile membrane structures are subjected to external loads once prestress has been applied either mechanically or pneumatically. A linear curve that is used as a comparison should thus originate from the prestress level, as suggested in [2] and summarized in the next section.

3.2 Exemplary verification steps according to EC 0

In order to portray the consequences of the choice of non-linearity according to EC 0 with respect to the dimensioning points f_{Ed}, the maximum stress developing in a simple hypar subjected to uniformly distributed snow load is shown here as an academic example. As can be seen in Figure 4, the classification is ambiguous already. Taking a linear extrapolation from the origin to the characteristic effect of action $E(q_k)$ into consideration leads to an underlinear curve. A shift to the prestress level as the origin of a linear extrapolation curve yields an overlinear curve. Previous research on this topic proposed to e.g. take the curvature into account for the classification due to this ambiguity, see e.g. [11] for an overview.
Simply following the current instructions of EC 0 to reach a dimensioning point f_{Ed} results in the value $f_{Ed,underlinear}$ that is indicated in the figure below. A modification of the linear curve's origin to the level of prestress could be considered in two ways and leads to similar dimensioning points:

- $f_{Ed,overlinear}$ resulting from comparing the stress curve with a linear curve starting at the prestress level, i.e. a classification as overlinear

- $f_{Ed,modified}$ resulting from a classification as underlinear but subtracting the prestress level to reach a dimensioning point that considers the stress increase due to the external action.

The simple hypar example already shows that the design of membrane structures according to the rules of EC 0 can be very conservative. Fußeder et al. [1] also explained in detail, how the design can be unsafe due to the present non-linearities and the presence of prestress in structural membranes.

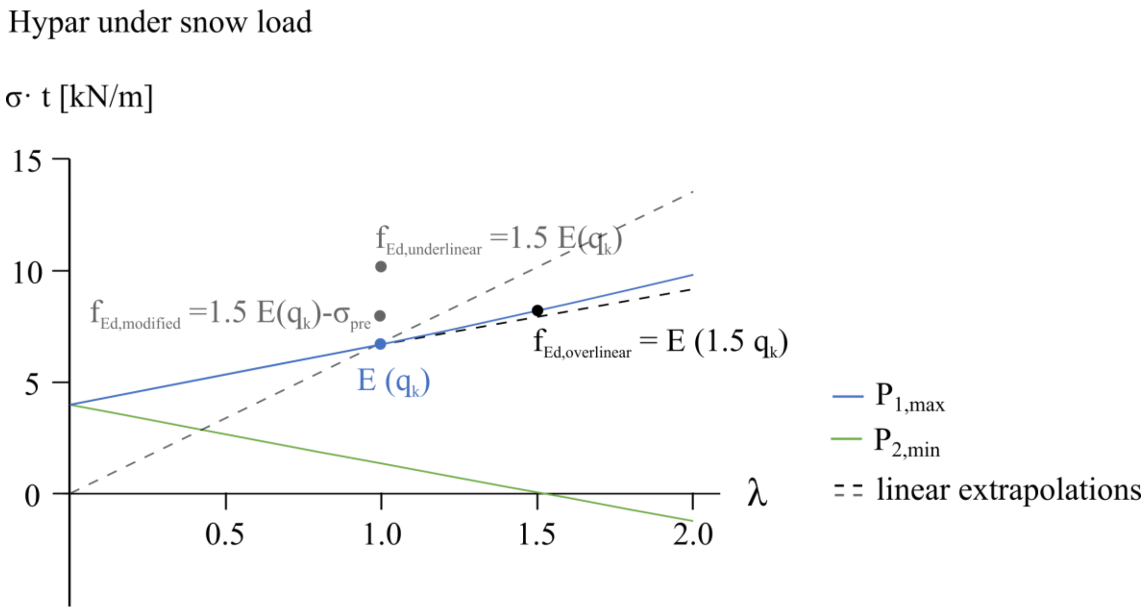

Figure 4: Dimensioning points f_{Ed} computed from principal stresses for a hypar under snow load. Linear extrapolations of an origin (grey : zero, black : level of prestress) to the effect of characteristic action $E(q_k)$ depicted as dashed lines, design effects of action indicated by balck and grey points.

12 CONCLUSIONS

This contribution shows how the non-linear behavior of structural membranes cannot be predicted to be under- or overlinear for design. Typical basic shapes are investigated in different load scenarios to conduct an overview of representative load-response curves. Furthermore, different methods to reach dimensioning points according to an interpretation of EC 0 are outlined. With the current developments towards implementing a design code on a European level ([8,12]), this topic is highly relevant and calls for more studies.

Additional aspects that need to be investigated for the development of reliable design regulations are the effects of non-linearity in loading directions and intensity (e.g. wind and ponding), as well as the usage of simplified material models, as indicated in [14]. Péntek et al. [15] have reached first conclusions on the topic of wind engineering for lightweight structures and more research is currently conducted on FSI simulations on membranes. A deepened understanding of how the flexibility of structural membranes in wind affect the pressure distribution and dynamic behavior plays an essential role for future design regulations.

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