

EVALUATION OF DIFFERENT COMPUTATIONAL MODELLING STRATEGIES OF A MASONRY VAULT WITH BUTTRESSES AND BACKFILL

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Abstract. In this paper, a critical comparison of two of the most common methods for the analysis of masonry vaults up to collapse is carried on. Innovative 2D Discrete Element (DE) and Finite Element (FE) models have been adopted aiming at capturing the main features of masonry single curvature structures mechanical behavior. Two numerical strategies are adopted: a discrete element model and a continuous homogenized model. The first approach provides an estimate of the collapse load and failure pattern of masonry based on ad-hoc implemented algorithm. The second approach is formulated in the framework of multi-surface plasticity and implemented in a FE code for the path-following non-linear analysis of masonry wall described as continuous anisotropic plate. The two numerical approaches are adopted to reproduce the Experimental Campaign of a full-scale Catalan vault with buttresses and backfill, statically loaded at 1/3 span. The failure modes and the crack patterns obtained by the proposed models are compared between them and with that observed in the experimental test as well as the load-displacement response curves. Both the proposed models efficiently capture the behavior of the vault and, in particular, the backfill deformation and load-spreading effect, hinges position and formation order.

1 INTRODUCTION

The masonry architectural heritage is constituted by unique characteristics that make assessing its preservation particularly challenging. Due to their load-bearing capacity, masonry vaults have been widely used since Roman times in a large amount of historical construction. Despite its key role in modern restoration practice, assessing stability conditions of masonry

curved structures is not a trivial task [1]. While the sole arch structural behaviour is only driven by the hinges creation, when the system involves backfill and abutments, mechanical behaviour is more sophisticated. Indeed, the load spreading effect of backfill and the increase of stiffness due to the presence of spandrel walls have a beneficial effect on the load bearing capacity of the structure [2].

Numerous numerical methods have been proposed for curved structures modelling. An accurate model of the structure should allow for the description of existing damage and alterations in the structure, including cracks, disconnections, crushing, deformation and damage evolution.

The selection of the most appropriate method to use depends on, among other factors, the desired level of accuracy and simplicity, knowledge of the input properties in the model and the computational load. Preferably, the approach selected to model the masonry should provide the desired information reliably within an acceptable degree of accuracy and at minimum cost. However, the selection of a suitable analysis method is not a trivial task.

In this paper, the result of a DEM model [3] and of a FEM model [4,5] are validated through the comparison of a full-scale experiment [6]. Collapse mode, crack pattern, hinge formation and load bearing capacity are shown and compared.

2 THE EXPERIMENTAL SETUP

The experimental campaign carried out in [6] consisted of one unreinforced vault and three externally reinforced vaults with shoulders, spandrel wall and backfill. The vaults were built with clay bricks and lime mortar, while the backfill consisted of gravel grains laid manually on the vault.

The abutments were contracted with a reaction wall and a steel frame on both sides. The specimens were then statically loaded by means of a hydraulic actuator with loading and unloading cycles until failure.

In this paper, only the unreinforced specimen is considered and the loading and unloading cycles are neglected in both modelling approaches.

The experimental results provide a failure mode driven by four hinges located alternatively at the extrados and intrados of the vault. The load capacity was recorded at 5.8KN with the maximum displacement recorded at the two central hinges and consisting of 38.1mm (upwards) and 36.1 mm (downwards).

3 MODELLING APPROACHES

Two bi-dimensional modelling strategy are herein adopted: the former, is based on a micro-modelling approach, based on an Explicit Code [7], while the latter employs a continuous method [4,5].

3.1 Overview of modelling with DEM

The Discrete Element Model is carried out by means of the commercial software UDEC [8]. Both rigid and deformable blocks are employed. The former, are adopted for the bricks, a part from the vault voussoirs, for which elastic bodies are modelled in order to increase the contacts discretization and the backfill, represented as a continuum elasto-plastic block (Figure 1).

In particular, each masonry unit is modelled considering the exact geometry and comprising

half of the mortar joint on each side. Contacts represent the potential fracture lines, where cracking and sliding take place. At each edge of the interface, the constitutive behaviour is defined by non-linear springs located at each corner. In the normal direction, springs behaviour is purely elastic in compression and brittle in tension as far as the tensile strength, is overcome. In the tangential direction, the behaviour is governed by the Coulomb slip law:

$$\tau \leq c + \sigma \cdot \tan(\varphi) \quad (1)$$

Where τ and σ are the shear and normal stresses, respectively, c is the cohesion and φ is the friction angle. When the maximum stress is attained, the tensile strength and cohesion are reduced to zero and the behaviour in tangential direction is purely frictional. The input mechanical parameters for the contacts are: Normal stiffness k_n , Shear stiffness k_s , Friction angle φ , Tensile strength f_t and Cohesion c .

The elastic normal stiffness at the contact between blocks K_n is estimated from the Young moduli of both bricks (E_b) and mortar (E_m), by lumping at the contact the overall deformation as follows:

$$K_n = \left(\frac{h_b}{E_b} + \frac{h_m}{E_m} \right)^{-1} \quad (2)$$

where h_b and h_m are, respectively, the height of the brick in the direction perpendicular to the contact surface and the thickness of the mortar joint. The same holds for the shear stiffness K_s as a function of the shear moduli G_b and G_m . For joints located on the buttresses, considering the presence of backfill for half of the thickness of the specimen, the mechanical properties of the contacts are set to provide an average overall stiffness between masonry and backfill.

The equations of motion are integrated through an explicit algorithm, considering finite displacement and updating the contacts between the elements that can be lost or created during the analysis.

3.1.1 Arch, Backfill and Load Model

According to the experimental setup, the vault is made of 25 rigid blocks which form the arch itself lying on the two abutments and constrained on both sides with buttresses. The live load is modelled through an ad hoc algorithm, in which the load cell is represented as a rigid block to which a constant vertical velocity, small enough to mitigate the effects of vibration, is applied. The resulting load is determined as the vertical reaction monitored at the contact between the block and the backfill.

The soil backfill is modelled as a Mohr Coulomb frictional no-cohesive and no-tension continuum deformable material, discretised with a triangular mesh grid having 30mm average distance between the grid points. According to experimental tests, the mechanical parameters provided for the backfill are the density equal to 12KN/m³ and the friction angle equal to 39°, whereas the Young's modulus and the Poisson's ratio for fine-grained soil are assumed respectively as 0.3GPa and 0.3. An initial condition of a passive soil stress vertical gradient due backfill self-weight is applied, whereas dilatancy is neglected. In order to improve plasticity calculations and overcome the 'volumetric locking', the non-linear backfill behaviour is

improved through the 'nodal mixed discretization' on stress, based on the averaging of volumetric strains and stresses in the elements around each node [9].

As regards, the strength properties of the joints, a constant friction angle of 31° is adopted, while $f_i = 0.2$ MPa and $c = 0.3$ MPa are set at vault joints.

Blocks and joints belonging to the buttresses are modelled considering homogenised mechanical properties deriving from the bricks and the backfill. Furthermore, joints belonging to the interface between the vault and the backfill are modelled as frictional joints with $\varphi=39^\circ$, corresponding to the backfill friction angle. For further details of the numerical model, readers might refer to the cited work [3].

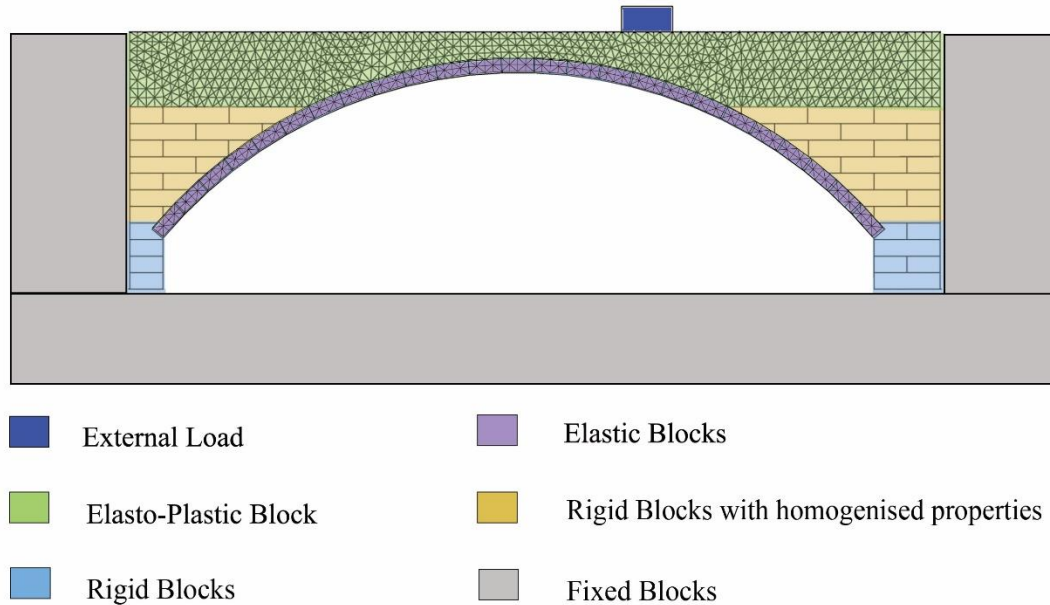


Figure 1. DEM Geometrical model.

3.1.2 Discrete Element Results

The DEM failure configuration of the masonry vault are shown in Figure 2. The failure mode obtained from the numerical model matches perfectly to the experimental one for both the bricks displacement and the backfill deformation. In Figure 3 the load-displacement curve and is shown for the two proposed modelling strategies and compared with the experimental data. The shown downward displacement is recorded between block 16 and 17. The first hinge opens at the intrados between arch bricks 16 and 17 in the ascendant branch at a load of 3.5kN with a downward displacement corresponding to 2.15 mm, which is close to that noticed experimentally (2.2mm, [6]). The second crack appears at the extrados between bricks 11 and 12, while the third and the fourth hinges open respectively, at the extrados and at the intrados between voussoirs 6-7 and 20-21, around the peak phase, exactly as noticed in the experimentally. As in the experimental test, the peak load is equal to 5.8kN. The numerical simulation provides a sudden decrease in the load after the peak value, which is not reflected in the experiment, and a corresponding underestimate of the load-displacement softening branch.

It is noteworthy that backfill constrains displacements during the pre-peak phase and increases the load-carrying capacity due to its load spreading effect.

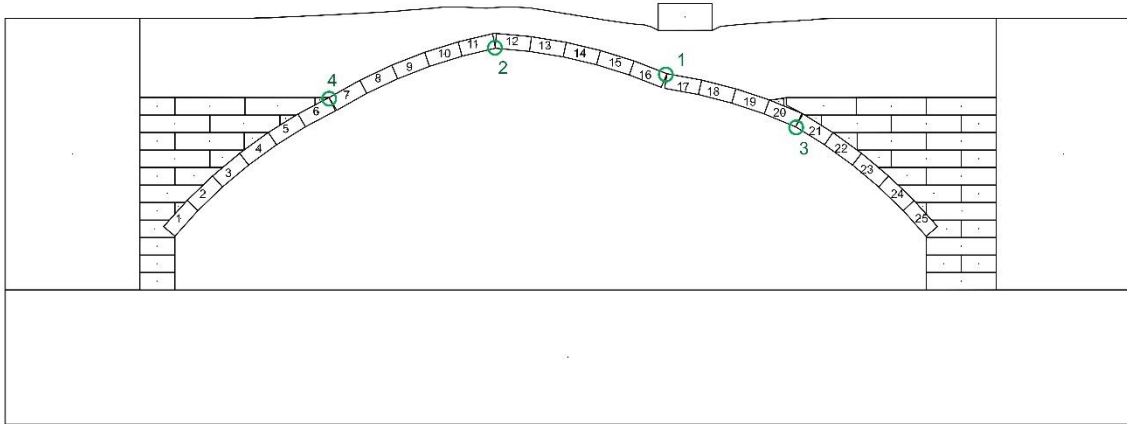


Figure 2. DEM failure mode and hinge formation order.

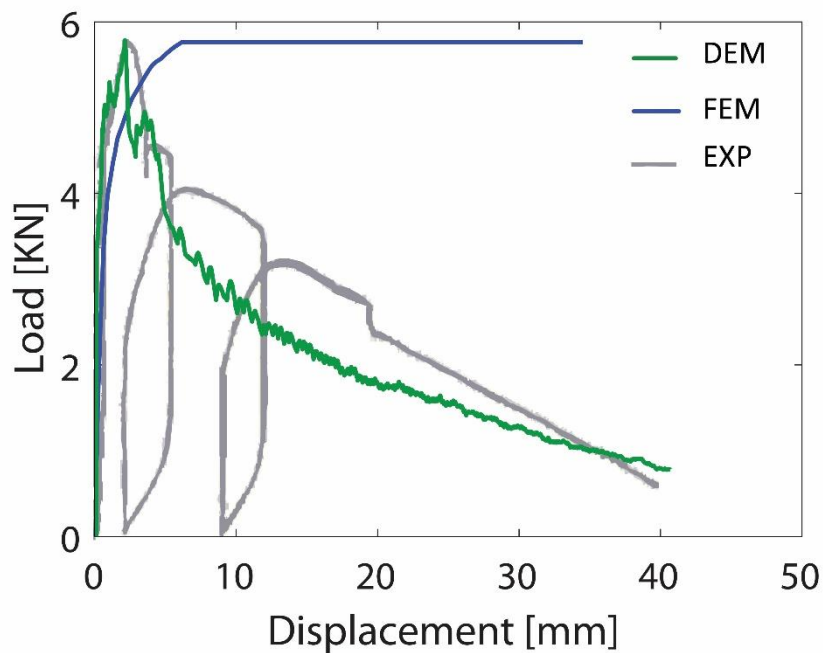


Figure 3. Load displacement curve of the Experimental test and the DEM-FEM modelling strategies.

3.1 Overview of modelling with FEM

The finite element model, adopted to describe the behavior of the vault, represents the masonry panel as an elasto-plastic homogenized Love-Kirchhoff elasto-plastic plate, with an associated flow-rule [4,5]. For further details, the readers can refer to the above aforementioned cited works [4,5]. While the mechanical behaviour of the backfill material is modelled by

considering it as a Mohr Coulomb medium. In Figure 4, the finite element discretization of the problem at hands in question is shown in Figure 5.

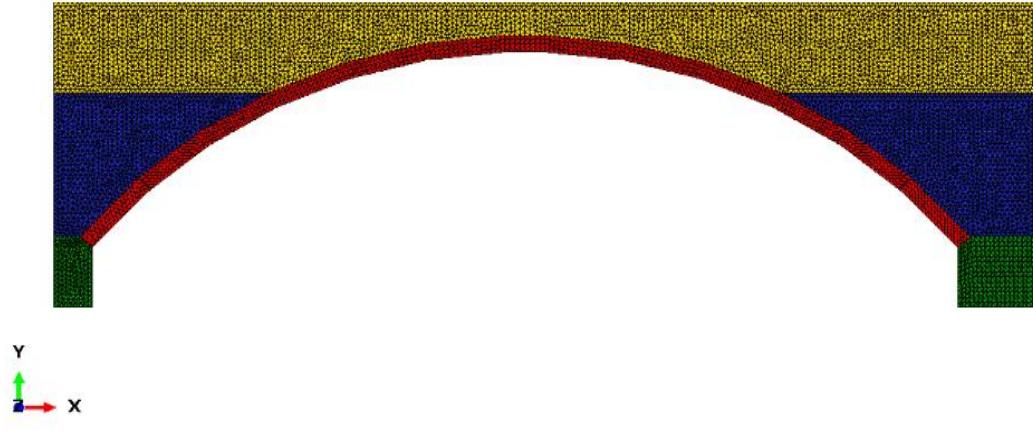


Figure 4: Finite element model.

The vault is discretized by means of triangular shell elements, but in this case the model is subject only to in-plane actions. In the same figure, the different colors correspond to the different masonry or backfill sections.

According to [4,5], the mechanical parameters needed to define the nonlinear behaviour of masonry are friction and cohesion of the joints and block sizes. These parameters, along with the mechanical parameters for backfill material, are derived from experimental work: a friction angle of 31° and 39° is adopted, for masonry and backfill, respectively, while a cohesion of zero is chosen for both masonry and backfill. Dimension of 120 cm and 55 cm are assumed for brick width and height, respectively.

Numerical simulation is performed by applying the self-weight and sequentially the vertical force.

3.1.1 Finite Element Results

The results of the numerical simulation are shown in Figure 5, where the plastic strain distribution corresponding to the opening of the head joints at the collapse, i.e. after the developing of all the four hinges, are shown. The load bearing capacity of the vault, provided by the finite element model is 5.6 KN (Figure 3). According to homogenization theory, on which the finite model is based, the value provided is a lower bound.

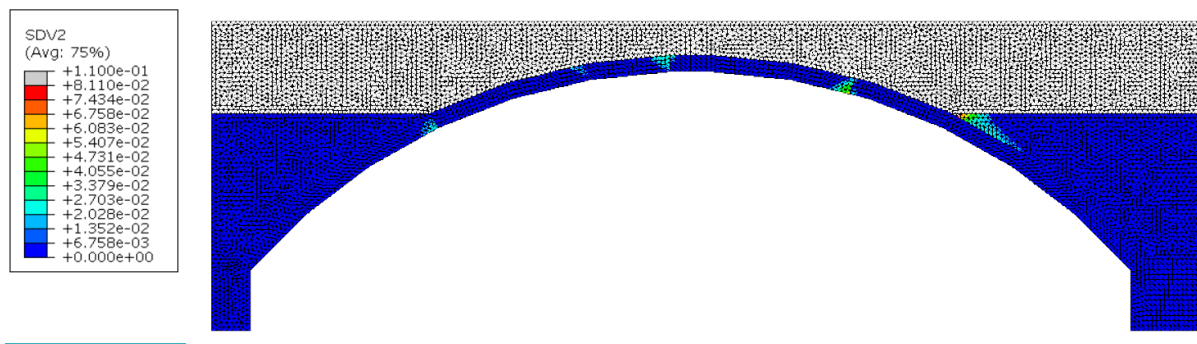


Figure 5: Plastic strain distribution after the hinge formation.

4.0 RESULTS DISCUSSION AND CONCLUDING REMARKS

In this paper, two 2D numerical approaches to model the mechanical behaviour of a vault under asymmetrical static loading are presented and compared.

According to experimental tests [6], both numerical methods capture the structural response. In fact, a good match is provided for both the bearing capacity and the collapse mode.

The continuous model requires little input information and a less time-consuming calibration. The main mechanical response is achieved, although the softening behaviour cannot be modelled. In contrast, the construction of the DEM geometric model and the calibration of the mechanical properties at the interface are computationally demanding, but the hinge position and formation order as well as the post-peak behaviour are well captured.

The simulations presented demonstrate the potential of the numerical strategies employed and provide a solid understanding of the mechanical response of masonry vaults with soil fill and spandrel walls.

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