

VIBRATION CONTROL OF OFFSHORE WIND TURBINE TOWERS

ANDREAS E. KAMPITSIS¹

¹ National Technical University of Athens (NTUA)
Zografou Campus, GR-157 80 Athens, Greece
cvakamb@gmail.com

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Abstract. In this study, the dynamic response of a Offshore Wind Turbine (OWT) tower under wind loading is investigated adopting the vibration absorption concepts. Application of vibration control systems (VCS) to OWT towers has the potential to significantly improve the dynamics of the tower and the nacelle, increasing its reliability. The first vibration mitigation approach is the classical tuned mass damped (TMD) and it is used as benchmark. In the TMD approach an additional mass is installed inside the nacelle at the top of the OWT tower. Usually, a large additional mass is required to achieve significant effective damping, which may lead to undesirable second order effects and ultimately increase the tower displacement. In the second approach, a passive vibration absorption concept is implemented, namely the Extended KDamper (EKD). The KDamper device is a special mechanical configuration consisting of an additional mass, an artificial damper and both positive and negative stiffness elements. The OWT is subjected to extreme environmental loading such as the wind and sea waves. These OWT loadings, are usually calculated based on stochastic interpolation of meteorological data or by artificial time-histories. In this study, the Blade Element Momentum (BEM) theory is taken into consideration in order to produce the wind load time histories. The improvement on the dynamic response of the OWT tower using vibration control systems is illustrated. Moreover, this research shown that the EKD presents a significant increase of the damping ratio with a minimal value of added mass at the top of the tower.

1 INTRODUCTION

According to the Climate Change Conference, held in Paris 2015 (COP21), the European Union (EU) is committed to reduce CO_2 emissions by 2050 by 80% compared to 1990 levels. This requires drastic decarbonisation of the energy sector. To this end, Renewable Energy Sources (RES) play a key role. The EU is planning to achieve electric energy production from RES up to 97% of the total energy demand by 2050. According to the International Energy Agency (IEA) estimates, Wind and Solar Energy represent the most promising renewables and will soon be the cheapest sources of energy, achieving worldwide 40% penetration to the electric network by 2040 [1].

The installed Wind Turbines (WT) energy potential is currently estimated at 539GW, according to the World Wind Energy Agency (WWEA) [2]. An important part of the WT network is the offshore wind turbines (OWT). OWT are expected to increase significantly as European coasts and seas (including Greek islands) offer a large wind energy potential. The WWEA predicts that offshore wind farms of 150GW will operate in the EU by 2030, contributing 14% of the EU's total electricity consumption. The energy potential multiplies when the wind energy is combined with the energy of the sea Waves kinetic

energy. It is estimated that the possible power generated by the waves is 2TW, equal to the world's generated electricity [3]. Only the region of the Mediterranean [4] is expected to produce renewable energy to cover 50% of the current electricity consumption in the EU. Thus, EU has launched the Blue Growth program. That is the long-term strategy to support sustainable growth in the marine and maritime sectors.

However, the above predictions are very ambitious. It is noted, that in 2018 the annual rate of WT installation reached the lowest rate (10.8%) since the start of their industrial growth. Moreover, in recent years, several failures have been recorded in existing WT. It is estimated that the structural failure (tower, foundation) of the installed WT amounts up to 10% [5], significantly reducing their contribution to the energy network. The high demand for wind and wave energy, generates several practical challenges that prevent the achievement of the energy goals. These challenges stem mainly from the following factors:

- The large construction and foundation costs of the WT (25%-40% of the total cost). For offshore, the cost exponentially increases as the depth of the sea becomes greater.
- High cost of replacement/repair of the tower and foundation. It is noted that this cost is the highest one in WT industry and it is two times higher than the next in line which, is the electric generator.
- WT collapse due to earthquake excitations. Seismic events cause structural failures due to high dynamic stress loading exceeding structural strength.
- WT collapses due to wind or/and wave loading. This continuous and cyclic loading causes failure due to structural fatigue. .

Therefore, according to the World Energy Agency [1], it has emerged as top priority to:

- Increase wind energy through the installation of WT and OWT (Political Recommendations 1).
- Enhance research to avoid WT failures (Economic Recommendations).
- Develop computational tools to simulate, analyse, and design WT (Technical Recommendations 1-7).
- Develop new innovative energy production structures (Political Recommendations 2).

Towards facing these challenges an innovative OWT vibration control system (VCS) that protects the OWT structure from all external loading is examined.

Extensive research has been conducted to investigate the vibration mitigation of WT towers by using various vibration control systems (VCS) [6]. VCS can be classified as passive [7–13], active [14], hybrid [15, 16], and semi-active control [17–21]. Passive VCS are the most commonly used in engineering practice [22–24] since they are simple to model, easy to implement and reliable to perform. From the numerous passive VCS, the Tuned Mass Damper (TMD) has received significant attention in the WT vibration mitigation literature [7, 9, 10]. The principle of the TMD is the degradation of the dynamic response of the structure through the transfer of energy to an additional mass. That mass along with a positive stiffness element and an artificial damper are placed at the top of the WT tower.

Variations of the TMD have been proposed in order to address the inherent nature of the OWTs. Lackner and Rotea [9] proposed the installation of two independent TMDs located in the nacelle, in order to capture both the fore-aft and the side-side modes. Stewart and Lackner [10] presented the beneficial dynamics of an OWT with TMD when significant wind-wave misalignment is presented, while Zuo

et al [25] proposed using multiple TMDs at certain locations along the tower in order to control the vibrations of OWT tower under the combined wind, sea wave and earthquake excitation. In almost all the aforementioned studies, the vibration control device is located in the nacelle at the top of the tower. However, in practice the installation of any CVS in the nacelle is challenging due to the limited available space.

An indirect approach to increase the inertia effect of the additional mass of the TMD, without increasing directly the mass itself, is the KDamper concept [26]. The KDamper is essentially an extension of the traditional TMD, by introducing an appropriate negative stiffness (NS) element to the oscillating mass of the TMD. The amplitude of the inertia force of the additional mass and the amplitude of the NS force are exactly in phase, due to the negative value of the implemented stiffness element [26, 27]. Although the KDamper oscillator incorporates a NS element, it is designed to be statically and dynamically stable. Once such a system is designed properly, it is shown to exhibit extraordinary damping properties, finding numerous applications in seismic isolation of bridges [28, 29], vibration absorption of WT towers [30–32], and implementation in the bases of structures as a stiff absorption base [27, 33, 34].

In this study, the dynamic response of a Offshore Wind Turbine (OWT) tower under wind loading is investigated adopting the vibration absorption concepts. The novel aspect of this study are summarized as following:

1. The dynamic response of an offshore wind turbine tower founded on a monopile is studied implementing geometrically non-linear (second order) FE model, while the monopile is embedded in a layered Winkler type soil profile represented by a Kelvin–Voigt element.
2. Two VCS are investigated intalled at the toip of the OWT, namely the TMD and the EKD.
3. The wind load time history is developed using the the blade element momentum theory incorporating Prandtl’s tip loss factor and Glauert’s correction.

2 OWT MODEL

The NREL baseline 5-MW nacelle and rotor [Figure 1](#). supported by a steel tower of 120 m height is examined [35, 36]. In order to consider the inertial forces applied by the mechanical parts (nacelle, rotor, and blades) an additional concentrated mass m_{op} is added at the top of the tower [36]. The WT is founded on a monopile embedded in a layered Winkler type soil profile represented by a Kelvin–Voigt element. The transient nonlinear dynamic response analysis of the tower–pile system is investigated through a *Beam of Variable Cross Section* over a *Beam on Winkler Foundation* formulation accounting for both kinematic and inertia interaction. The geometrically nonlinear equation of motion of the OWT is obtained as follows:

$$\begin{bmatrix} M_t & \mathbf{0} \\ \mathbf{0} & M_p \end{bmatrix} \begin{Bmatrix} \ddot{d}_t \\ \ddot{d}_p \end{Bmatrix} + \begin{bmatrix} C_t & \mathbf{0} \\ \mathbf{0} & C_p \end{bmatrix} \begin{Bmatrix} \dot{d}_t \\ \dot{d}_p \end{Bmatrix} + \begin{bmatrix} K_t(d) & K_{tp}(d) \\ K_{pt}(d) & K_p(d) \end{bmatrix} \begin{Bmatrix} d_t \\ d_p \end{Bmatrix} = \begin{Bmatrix} P_t(d) \\ P_p(d) \end{Bmatrix} \quad (1)$$

where $(\dot{})$ denotes differentiation with respect to the time t . Moreover, $[M_t]$, $[M_p]$, $[C_t]$ and $[C_p]$ are the mass and damping matrices of the OWT tower and monopile, respectively. $[K_t]$, $[K_p]$ and $[P_t]$, $[P_p]$ are the nonlinear stiffness matrices and force vectors containing the geometrically nonlinear terms, while $\{d_t, d_p\}^T$ denotes the vector of unknown nodal displacements. The key properties of the OWT steel tower are listed in [Table 1](#).

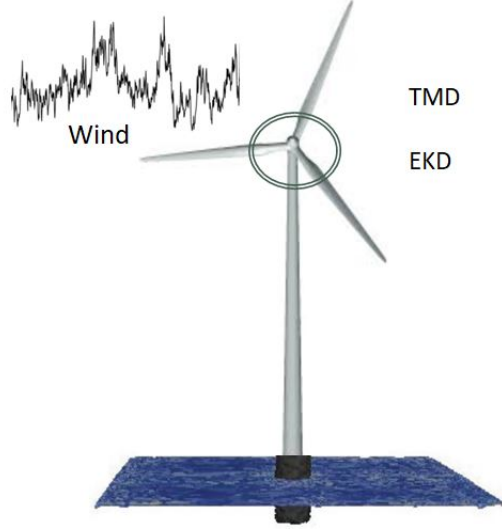


Figure 1: Offshore wind turbine tower on monopile foundation.

The first vibration mitigation approach is the classical TMD and it is used as benchmark since it is the most well studied in the literature. In the TMD approach an additional mass m_D is installed inside the nacelle at the top of the OWT tower. The mass is attached using a positive stiffness element k_D and a linear damper c_D . Alternatively, the EKD device [33], is used. The EKD consist of an additional oscillating mass connected to the nacelle with a positive stiffness element k_{PS} and to the tower with a negative stiffness (NS) element and an artificial damper k_{NS}, c_{NS} . The tower is also connected to the nacelle with a positive stiffness connection k_R . By introducing the vibration mitigation concept the tower related matrices of Equation 1 are modified respectively.

Table 1: Key properties of NREL baseline 5MW offshore wind turbine.

Property	Value		
Rating	5MW	Top thickness	25m
Rotor diameter	126m	Rated rotor	12.1rpm
Hub diameter	3m	Nacelle mass	240tn
Base diameter	8.43m	Rotor mass	110tn
Base thickness	48mm	Young's modulus	210GPa
Top diameter	3.87m	Steel density	8.5tn/m ³
Tower damping	2%	Blade damping	0.48%

3 DESIGN OF THE VCS

The design performance of most VCS strongly depends on the tuned frequency ratio and damping ratio. Over the years, numerous techniques have been proposed [25, 37–42] for the optimization of the key TMD or MTMD parameters. However, the complexity of the EKD configuration [33] renders the

conventional min-max (Hinf) approaches [43] ineffective, thus a constrained optimization approach is followed [26]. As for the negative stiffness is primarily achieved by special mechanical designs involving conventional positive stiffness prestressed elastic mechanical elements such as post-buckled beams, plates, shells, and precompressed springs [44, 45]. In the EKD configuration, the negative stiffness element is materialized via a precompressed springs mechanism. The implementation of such configuration is simple to design and construct while it also provides very good control of the negative stiffness influence on the VCS performance [26, 46].

4 WIND LOAD

The loading due to the wind is taken into account as follows, the OWT tower is considered to be subjected to the horizontal force \bar{F}_N due to the wind at its top, as presented in [36]. The horizontal force $\bar{F}_N(r, t)$ acting at a position r along the wind turbine blade can be obtained by the following relation [47].

$$\bar{F}_N(r, t) = \frac{1}{2} \rho_{air} C_N(r) c_{bl}(r) \bar{V}^2(t) \quad (2)$$

where $C_N(r)$ is the coefficient computed by the corresponding lift $C_L(r)$ and drag $C_D(r)$ coefficients and $\rho_{air} = 1.2 \times 10^{-3} \text{ tn}/\text{m}^3$. The values of the latter coefficients depend on the airfoil characteristics of the blades and their distribution with respect to the angle of attack of the wind velocity $\bar{V}(t)$ vector passing through the blade profile can be retrieved from. It is noted that the velocity vector is assumed to have a uniform spatial distribution over the actuator disc. $c_{bl}(r)$, $\beta(r)$ is the chord and the pitch angle of the blade profile varying along the blade length r , the distributions of which are obtained according to the blade type employed to equip the turbine. $C_N(r)$ is given as:

$$C_N(r) = C_L(r) \cos \phi(r) + C_D(r) \sin \phi(r) \quad (3)$$

where $\phi(r)$ is the wind flow angle [47]. In order to evaluate $\phi(r)$, the blade element momentum theory incorporating Prandtl's tip loss factor and Glauert's correction [47] is employed with an assumption of constant angular velocity of the blades Ω_{bl} . Subsequently, breaking down into a mean component V_m and a fluctuating component $V(t)$, the corresponding mean and fluctuating components of $\bar{F}_N(r, t)$ can be obtained as:

$$F_{Nm} = \frac{1}{2} \rho_{air} C_N(r) c_{bl}(r) \bar{V}_m^2 \quad (4a)$$

$$F_N(r, t) = \frac{1}{2} \rho_{air} C_N(r) c_{bl}(r) (2V_m V(t) + V^2(t)) \quad (4b)$$

The mean velocity is obtained by employing a basic velocity at the altitude of $10m$, V_b and applying the corresponding regulations of EC1, Part1,4 [48], Moreover, in order to take into account, the wind velocity fluctuation at the altitude of l_t an artificial velocity time history is generated applying the procedures presented in [49–52] assuming a value of standard deviation σ . After having established $\bar{F}_N(r, t)$, the total concentrated force exerted on the top of the tower can be computed as:

$$\bar{F}_N(t) = 3 \int_0^{r_{bl}} \bar{F}_N(r, t) dr \quad (5)$$

5 NUMERICAL RESULTS

On the basis of the numerical procedures presented, a computer program has been written and an extensive case study has been made to investigate the efficiency and the accuracy of the developed procedure as well as to highlight the structural aspects related to the various VCSs. The design criterion of the TMD and EKD is their mass ratio, which corresponds to the additional mass added to the system. In order to verify that this approach is indeed optimum for the implementation of the TMD to a OWT tower, the maximum top displacement over the TMD tuning frequency is illustrated in Figure 2, for various values of the TMD damping ratio, and the aerodynamic load presented previously. The mass ratio of TMD is selected for the considered analysis as 5%.

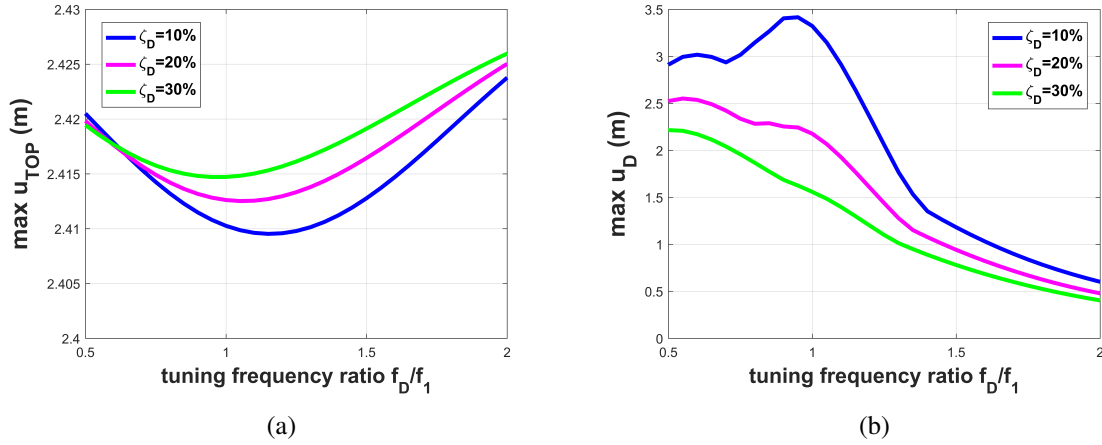


Figure 2: Optimum TMD tuning. (a) Maximum top tower displacement, and (b) TMD stroke, over the TMD nominal frequency ratio for various values of damping ratio.

The OWT tower nonlinear dynamic responses and the nacelle's response variables, considering the TMD and the EKD are illustrated in Figure 3. The EKD designs manage to greatly improve the dynamic behavior of the OWT tower. The effective damping ratio of the WT tower increases up to 10% with the EKD system with only 0.1% additional mass. The nacelle's dynamic response variables are also improved, with the exception of the nacelle's relative velocity, where a slight increase is observed. In addition, EKD greatly reduce the NS element stroke and the relative to the top of the tower displacement of the oscillating mass of the proposed devices, making the configuration design more realistic.

6 CONCLUSIONS

In this paper, two dynamic vibration absorber options are examined for improving the OWT tower dynamic behavior and increasing the effective damping, i.e. the conventional TMD and the EKD. The following conclusive comments can be made:

1. The geometrically nonlinear dynamic model of the OWT tower develop herein is suitable for real-life WT applications.
2. The proposed model obtains accurate results, providing a simple and efficient tool.

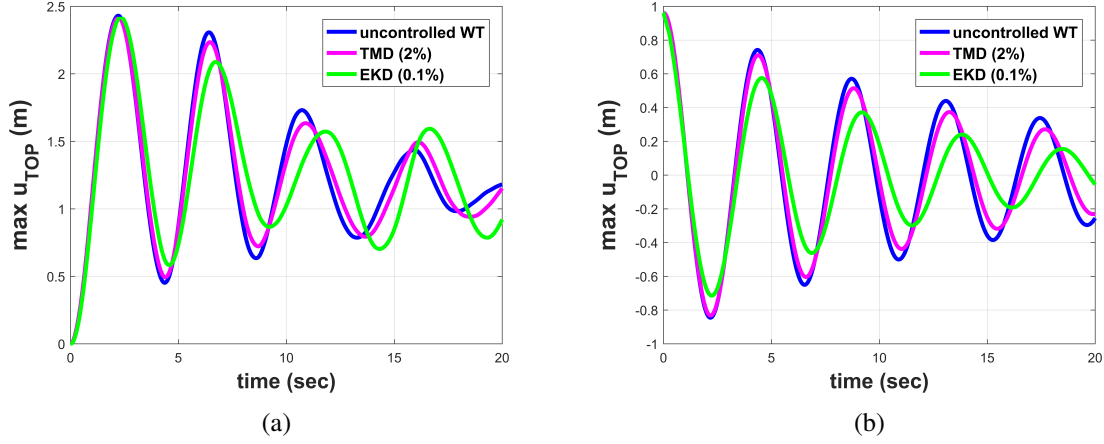


Figure 3: Dynamic responses of the controlled OWT: (a) top tower displacement due to the aerodynamic load, (b) top tower displacement considering a free vibration with initial conditions.

3. The EKD provided significant increase in the effective damping of the OWT tower as compared to the TMD.
4. The KDamper-based configurations are much more effective than the conventional TMD.
5. Monopile system leads to a more flexible structure (as compared to a fixed one) and affect the performance design of the VCSs.

7 ACKNOWLEDGMENTS

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