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Considerations on the Updating Process in Density-based Topology Optimization Using the Modified Optimality Criteria Method

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Abstract. In this study, we performed the density-based topology optimization in static problem using our modified optimality criteria method. In topology optimization based on the homogenization or density methods, the optimality criteria (OC) method is often employed as update equation for design variables. But the OC method needs to set the weighting factor and move limit. In the modified OC method, they are not necessary. However, in the international paper we have already reported, the performance function did not always go down in 3-dimensional static problems, thus we discuss the updating process in detail in this study. As a result, we confirmed that even the OC method was employed, the performance function increased significantly during the update process. The increase in the performance function when using the modified OC method is more supressed than when using the OC method.

1. Introduction

In recent years, the structural optimization theory has been used to improve the performance of various products with the improvement of computatioanl performance and manufacturing technology[1]. The structural optimization theory is categorized into size optimization, shape optimization, topology optimization. Topology optimization is the best method in the structural optimization for weighting reduction and conseptual design[2]. However, the numerical analysis, including topology optimization, has a lot of difficulties. For example, difficulty level of theory, parameters for numerical analysis, robustness, and so on. We foucued on the number of parameters that the analyst and engineer have to set. Note that the parameters discussed here are required for numerical analysis. And not physical properties such as Young's modulus. The OC method[3], which is often used in topology optimization based on the homogenization or density method, requires the setting of weighting factor and move limit. We have developed a modified OC method that does not require them. However, the performance function when using the modified OC method, which is the proposed method, increased during the update process, as shown in Fig. 1. Based on the above points, the purpose of this study is to investigate the updating process in detail.

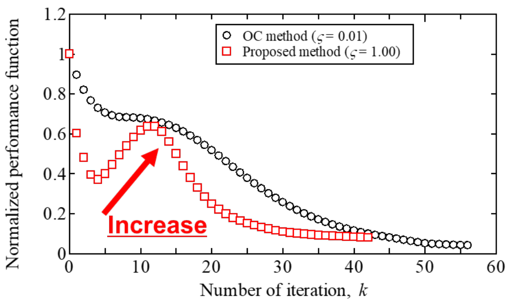


Fig. 1 History of normalized performance function in 3-dimensional static problem[4].

2. Formulation for density-based topology optimization

This section discusses about formulation for density-based topology optimization. Optimization problem, including the topology optimization, needs to set the goal. In this study, the goal is to minimize the strain energy that satisfies the target volume in design domain Ω. This is a fundamental problem in topology optimization and has been reported in many topology optimization studies [5]. The optimization problem is written as

$$\min_{e\in\Omega} \quad J = \sum_{e\in\Omega} J_e = \frac{1}{2}\{F\}^T\{U(\rho_e)\} \tag{1}$$

$$\text{subject to} \quad [K(\rho_e)]\{U(\rho_e)\} = \{F\} \tag{2}$$

$$V = \sum_{e\in\Omega} \frac{v_e \rho_e}{V_{\text{total}}} - \overline{\rho_e} \leq 0 \tag{3}$$

$$0 \leq \rho_e \leq 1 \tag{4}$$

where $[K]$, $\{U\}$, and $\{F\}$ are, respectively, the stiffness matrix, displacement vector at all nodes, and load vector at all nodes. Moreover, J_e, ρ_e, v_e, V_{total}, and $\overline{\rho_e}$ are, respectively, the performance function in element e, density in element e, an element volume, total volume in design domain Ω, average of initial density. The superscript T indicates transposition. Density-based topology optimization uses density as design variable and express the material distribution by representing the density value from 0 to 1, as shown in Eq. (4). Thus, the presence or absence of material is represented by the stiffness matrix $[K]$, as shown in

$$[K] = \rho_s [K_0] \tag{5}$$

In this study, the solid isotropic material with penalization (SIMP) method[6], which is one of the density method[7], is employed. In the SIMP method, the function ρ_s is written as

$$\rho_s = (1 - \rho_{\text{min}}){\rho_e}^p + \rho_{\text{min}} \tag{6}$$

where p and ρ_{min} are the penalization parameter for the SIMP method and parameter for numerical stability. The equation shown here is synonymous with the equation for the SIMP method described in other papers. The appropriate value of penalization parameter p is determined by Hashin-Shtrikman bounds[8] in 3-dimension, as shown in

$$p \geq \max\left\{15\frac{1-\nu}{7-5\nu}, \quad \frac{3}{2}\frac{1-\nu}{1-2\nu}\right\} \tag{7}$$

where ν denotes the Poisson's ratio. Next, the sensitivity is explained. The Lagrange function J^* is defined to solve this optimization problem, using the performance function shown in Eq. (1) and discretized governing equation shown in Eq. (2). By the gradient of the Lagrange function J^* with respect to the displacement u, the Lagrange multiplier vector $\{-\lambda\}$ is equal to $\{u\}$, which is the displacement vector at each node of element e. This relationship is called the self-adjoint. By substituting the self-adjoint, the gradient of the Lagrange function J^* with respect to the density ρ_e is rewritten as

$$\begin{aligned}\frac{\partial J^*}{\partial \rho_e} &= \frac{1}{2}\{\lambda\}^T \frac{\partial [K]}{\partial \rho_e}\{u\} \\ &= -\frac{1}{2}\{u\}^T \frac{\partial [K]}{\partial \rho_e}\{u\}\end{aligned} \tag{8}$$

Equation (8) is the sensitivity when design variables are given to elements. When the element has the density ρ_e that is the design variable, there is a checkerboard pattern in the density distribution. Various filter techniques have been developed as countermeasures[9]. In this study, the sensitivity filtering proposed by Borrvall[10] is employed. The OC method is used in the update equation, and the density is updated as shown in

$$\rho_e^{(k+1)} = \rho_e^{(k)} \left(\frac{\dfrac{\partial \overline{J^*}^{(k)}}{\partial \rho_e}}{-\Lambda^{(k)} \dfrac{\partial V^{(k)}}{\partial \rho_e}} \right)^{\eta} = \rho_e^{(k)} \left(A_e^{(k)}\right)^{\eta} \tag{9}$$

where the numerator in Eq. (9) is the filtered sensitivity, Λ is the extended Lagrange multiplier, and η is the weighting factor for update, respectively. The OC method is characteried by its ability to update the design variables more quickly than the steepest descent method. Although it is not used in this problem, the move limit, which is a method to suppress updates and stabilize them, is often employed when using the OC method. The move limit is written as

$$\rho_e^{(k+1)} = \begin{cases} \rho_e^L & \left(\rho_e^{(k)} \left(A_e^{(k)}\right)^\eta \leq \rho_e^L\right) \\ \rho_e^U & \left(\rho_e^{(k)} \left(A_e^{(k)}\right)^\eta \geq \rho_e^U\right) \\ \rho_e^{(k)} \left(A_e^{(k)}\right)^\eta & \text{(otherwise)} \end{cases} \tag{10}$$

$$\rho_e^L = \max\left\{\rho_e^{(k)} - \rho_{\text{move}}, 0\right\} \tag{11}$$

$$\rho_e^U = \min\left\{\rho_e^{(k)} + \rho_{\text{move}}, 1\right\} \tag{12}$$

Next, the modified OC method is explained. The modified OC method is derived by the concept of Newton's method. In the beginning, the OC method shown in Eq. (9) is expressed in natural logarithmic form to change to an additive formulation, as

$$\ln \rho_e^{(k+1)} = \ln \rho_e^{(k)} + \eta \ln A_e^{(k)} \tag{13}$$

From Eq. (13), $\ln \rho_e^{(k+1)}$ is equal to $\ln \rho_e^{(k)}$ if the number of updates increases. In other words, the second term on the right side in Eq. (13) should be zero. Thus, $\ln A_e^{(k)}$ is zero because η is not always zero. The Taylor expansion of $\ln A_e^{(k+1)}$ becomes as

$$\ln A_e^{(k+1)} = \ln A_e^{(k)} + \Delta\rho_e \frac{\partial}{\partial \rho_e}\left(\ln A_e^{(k)}\right) + o({\Delta\rho_e}^2) \tag{14}$$

Note that the function A_e is assumed to be univariate. Assuming that the third term on the right side in Eq. (14) is extremely small, $\Delta\rho$ can be expressed as

$$\Delta\rho_e = \left(-\frac{\partial}{\partial \rho_e}\left(\ln A_e^{(k)}\right)\right)^{-1} \ln A_e^{(k)} \tag{15}$$

Substituting Eq. (15), the update equation can be rewritten as

$$\begin{aligned} \ln \rho_e^{(k+1)} &= \ln \rho_e^{(k)} + \Delta\rho_e \\ &= \ln \rho_e^{(k)} + \left(-\frac{\partial}{\partial \rho_e}\left(\ln A_e^{(k)}\right)\right)^{-1} \ln A_e^{(k)} \end{aligned} \tag{16}$$

Returning Eq. (16) to the true number, the modified OC methdo can be obtained as

$$\rho_e^{(k+1)} = \rho_e^{(k)} \left(A_e^{(k)}\right)^{\left(-\frac{\partial}{\partial \rho_e}\left(\ln A_e^{(k)}\right)\right)^{-1}} \tag{17}$$

There are two points to be noted in using the modified OC method. First point is that the function A_e must be a positive value. This is also true for the OC mehtod, where the density ρ_e is imaginary if the function A_e is negative value. Second point is that the exponent in Eq. (17) is positive value. This is to find for the correct solution, just like Newton's method. The exponent depends on the optimization problem. In this study, the optimization problems is to minimize the strain energy in static problems and to satisfy a constant total volume at each iteration. Thus, the exponent is written as

$$\begin{aligned} -\frac{\partial}{\partial \rho_e}\left(\ln A_e^{(k)}\right) &= -\frac{\partial^2 \overline{J}_e^{*(k)}}{\partial {\rho_e}^2}\left(\frac{\partial \overline{J}_e^{*(k)}}{\partial \rho_e}\right)^{-1} \\ &\approx -\frac{\partial^2 J_e^{*(k)}}{\partial {\rho_e}^2}\left(\frac{\partial J_e^{*(k)}}{\partial \rho_e}\right)^{-1} \end{aligned} \tag{18}$$

Since the sensitivity filter is employed to avoid checkerboard pattern, unfiltered and filtered sensitivities are considered to be approximately equal. In this optimization problem, the exponent is rewritten as

$$\left(-\frac{\partial}{\partial \rho_e}\left(\ln A_e^{(k)}\right)\right)^{-1} \approx -\frac{-p\left(\rho_e^{(k)}\right)^{-1} J_e^{(k)}}{p(p+1)\left(\rho_e^{(k)}\right)^{-2} J_e^{(k)}} = \frac{\rho_e^{(k)}}{p+1} \tag{19}$$

Substituting Eq. (19) into Eq. (17), the modified OC method is expressed as

$$\rho_e^{(k+1)} = \rho_e^{(k)}\left(A_e^{(k)}\right)^{\frac{\rho_e^{(k)}}{p+1}} \tag{20}$$

3. Computational flow

This section describes the computational flow for the density-based topology optimization. The flow is as follows.

Step 1: Input of computaitonal condition and setting a model.

Step 2: 3-dimensional linear elastic analysis based on the finite element method is performed to obtain the displacement U (See Eq. (2)).

Step 3: The performance function is calculated with the obtained displacement (See Eq. (1)).

Step 4: If $k \geq k_{\max}$ is satisfied, the computational is finalized. Oterwise, go to the next step.

Step 5: Sensitivity analysis is performed using the gradient of Lgarange function J with respect to density ρ_e (See Eq. (8)).

Step 6: Sensitivity filter is applying for reducing checkerboard pattern in density distribution.

Step 7: The density is updated by the OC method or modified OC method (See Eq. (9) or Eq. (20)), and $k = k + 1$. Then return to Step 2.

4. Computational conditions

This section discusses about computational conditions for topology optimization. In this study, the cantilever beam model shown in Fig. 2 is used as the calculation model in order to discuss the differences in updating between methods. The size of cantilever beam is $60[\mathrm{mm}] \times 10[\mathrm{mm}] \times 40[\mathrm{mm}]$, and one element is a hexahedron with a side of 1 [mm]. Table 1 shows the computational conditions for the topology optimization. Note that when the value of the move limit ρ_{move} is greater than or equal to 1.00, it is equivalent to not using the move limit ρ_{move} shown in Eqs. (10) to (12). In Cases 1 and 2 shown in Table 2, the initial density average is set to 0.5. Moreover, in cases 3 and 4, the initial density average is set to 0.3. Using the Cases 1 to 4, the results on updating for each method are discussed in the next section.

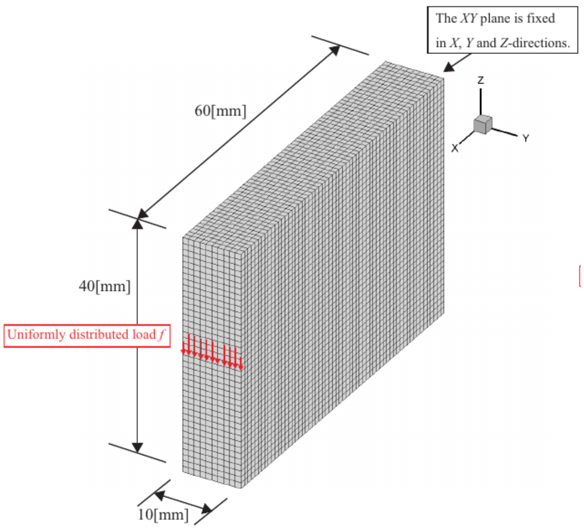


Fig. 2 Calculation model.

Table 1 Computational conditions.

Number of elements	24000
Number of nodes	27511
Weighting factor, η	0.75
Penalization parameter, p	6.0
Move limit, ρ_{move}	1.00
Filter radius	1.5
Maximum number of iteration, $k_{\max}$	150
Young's modulus [MPa]	1.0
Poisson's ratio, ν	0.3

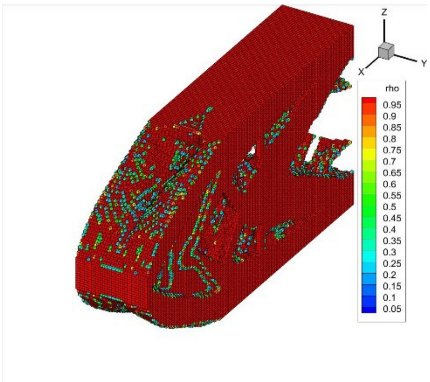
Table 2 Conditions for numerical experiments.

	Case 1	Case 2	Case 3	Case 4
Initial density average, $\overline{\rho_0}$	0.5	0.5	0.3	0.3
Update method	OC	Modified OC	OC	Modified OC

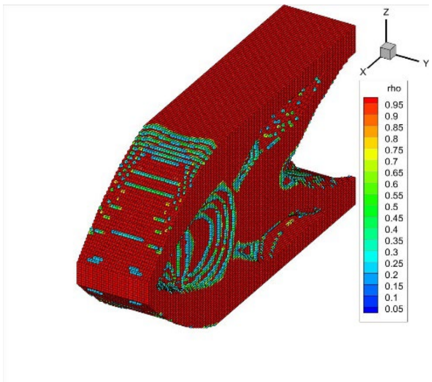
5. Results of numerical analysis

This section discusses about results of topology optimization. First, the results for Cases 1 and 2, where the initial density average $\overline{\rho_0}$ is set to 0.5, is discussed. Figure 3(a) shows the density distributions at final iteration in Cases 1 and 2, and Fig. 3(b) shows the history of performance function in Cases 1 and 2. It can be seen that the density distributions shown in Figs. 3(a) and 3(b) are similar results. From the history of performance function in Case 1 shown in Fig. 4, when the OC method is employed, the performance function increases significantly in the beginning iteration and then converges. On the other hand, when the modified OC method is employed, the increasing trend of the performance function is smaller than when using the OC method. Table 3 shows the maximum and minimum normalized performance functions in Cases1 to 4. From the results of Cases1 and 2 in Table 3, the maximum and minimum normalized performance functions are also similar value. Thus, the update method is different, but it can be shown that they can find the structure the minimum strain energy.

Next, we discuss about the results in Cases 3 and 4, where the initial density average $\overline{\rho_0}$ is set 0.3. As well as Cases 1 and 2, Fig. 5 shows the density distribution at final iteration, and Fig. 6 shows the history of normalized performance function. From the density distributions shown in Fig. 5, their density distributions are not similar. And, from the results of Cases 3 and 4 in Table 3, the maximum and minimum normalized performance functions in Case 3 are larger than in Case 4. In other words, an appropriate structure is obtained at the final iteration when the modified OC mehtod is employed. The reason for this result is that the settings of weighting factor η and move limit $\overline{\rho_0}$ are important for the updating process, when using the OC method. On the other hand, it can be seen that appropriate updates are made for each element when the modified OC mehtod is used. The smaller the total volume, the greater the degree of freedom in design, thus increasing the need for the move limit in the OC method. However, the cantilever beam model is responsible for increase in the performance function during the update.



(a) Case 1



(b) Case 2

Fig. 3 Density distributions at final iteration in Cases 1 and 2.

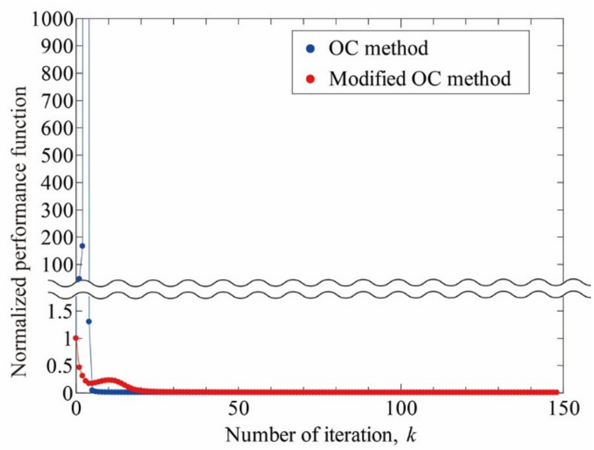
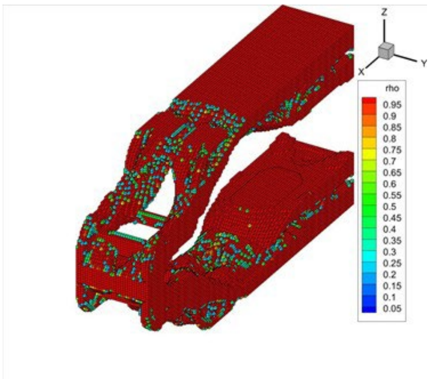
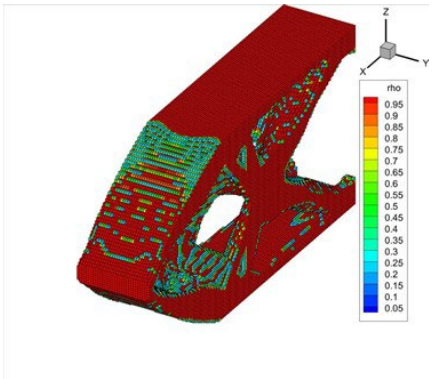


Fig. 4 History of performance function in Cases 1 and 2.



(a) Case 3



(b) Case 4

Fig. 3 Density distributions at final iteration in Cases 3 and 4.

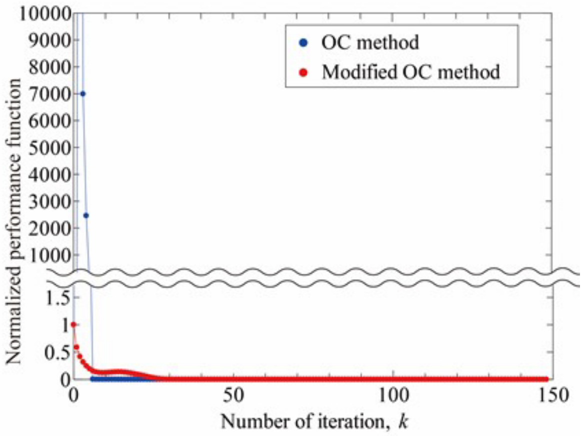


Fig. 4 History of performance function in Cases 3 and 4.

Table 3 Maximum/minimum normalized performance function in Cases 1 to 4.

	Maximum normalized performance function	Minimum normalized performance function
Case 1	26121.89	6.51×10^{-3}
Case 2	1.00	6.57×10^{-3}
Case 3	47051.59	49.73×10^{-5}
Case 4	1.00	5.51×10^{-5}

6. Conclution

In this study, density-based topology optimization analysis was performed for the cantilever beam model. As the optimization problem, the performance function is to minimize strain energy under a volume constraint. The updating process of the OC method and the modified OC method were compared. The conclutions in this study are as follows.

- The modified OC method is not needed to set the weighting factor because the method is derived by the concept of Newton's method.
- When the OC method is employed (Cases 1 and 3), the performance function increases significantly in the updating process.
- When the modified OC method is employed (Cases 2 and 4), there is a slight increase in the performance function during the updating process.
- When the total volume is small (Cases 3 and 4), the move limit is important in the OC method. However, it is not so necessary in the modified OC method.

In future work, we would like to examine different performance function such as minimization of stress.

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