

SIMULATING THE INFLUENCE OF PARTICLE GEOMETRY AND ARRANGEMENT ON THE COLLAPSE OF A SUBMEGRED GRANULAR STEP

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Abstract. To study the behaviour of granular media immersed in fluid, we have developed a two-dimensional simulation code that couples the Discrete Element Method (DEM) for the simulation of the granular particles with the Finite Element Method (FEM) for the simulation of the fluid. With this code, we simulated the collapse of a submerged granular step for various parameters of the granular material. In this paper, we investigate how these parameters influence the behaviour of the granular step during the collapse and the configuration of the granular medium afterwards.

1 INTRODUCTION

Granular media saturated with fluids give rise to a number of liquefaction phenomena from mud slides to manholes being lifted out of sandy ground after an earthquake. Many of such events result in tremendous damage to people and infrastructure, but none can be described by purely granular or purely fluid mechanical processes. While those phenomena are regularly observed in nature and have been studied on a macroscopic level, the underlying interactions between granules and fluid in the porespace as well as the behaviour of the fluid inside the porespace itself are not yet fully understood on the microscopic level.

By recreating these phenomena in micromechanic simulations, information about the full flow field becomes accessible and offers deeper insights into the related mechanisms, provided the processes and interactions in the simulation are not modeled in an unphysical way. For now, we limit ourselves to two dimensions, as a firm understanding of the two-dimensional situation is required before three-dimensional simulations should be attempted.

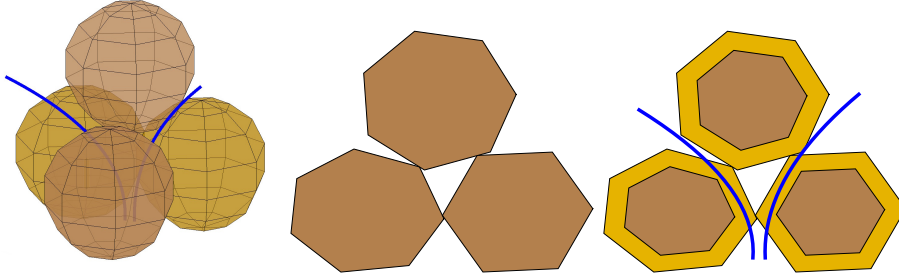


Figure 1: Contacting granular particles in three dimensions will leave openings in the porespace for fluid to flow (left). However in two dimensions, contacting particles would isolate portions of the porespace (center). Using two separate outlines (right), from the outer shadow (bright) the particle interaction is computed while the smaller core (dark) is the boundary for the fluid field (blue lines).

1.1 Granular Particles

The granular material can effectively be modeled by treating each individual granule as a discrete element. We use polygonal particles for the granules as they are able to form stable heaps with realistic angles of repose [1], whereas commonly used circular particles would just roll away without unphysical tweaks to the simulation. The polygons are created by inscribing them into an ellipse and randomizing edge lengths and orientations [2].

1.2 Fluid

Discretizing the fluid via particles may sound appealing at the first glance but would be plagued by shot noise in the system which can induce mechanical resonance and therefore cause unphysical vibrations in the whole granular assembly. A grid method is more suitable to our needs to obtain continuous flow amplitudes. To exactly discretize the pore space between the particles we need a triangular mesh and the complex geometry prohibits the use of structured grids. Accordingly, FEM is the most reasonable choice here, as it works effectively on this type of grid [3].

1.3 Fluid-Solid-Interaction

In a two-dimensional simulation, two contacting particles would block any fluid from passing between them, which would be in conflict with the real three-dimensional case. Therefore we create the particles with two outlines (see Fig. 1): The larger shadow is only seen by the DEM for particle-particle interactions while the smaller core exists only in the FEM and defines the boundary of the fluid. This way, the flow can never be blocked by contacting particles and the fluid inside the porespace stays interconnected. The outline of the particle core defines the geometry of the FEM mesh and imposes a no-slip boundary condition for the fluid velocity. In turn, velocity u gradients and pressure p at the particle surface Γ are used to compute the drag forces acting upon the particle

$$F_{drag} = \int_{\Gamma} \{ -p\delta_{ij} + \eta ((\nabla u) + (\nabla u)^T) \} \cdot \hat{n} dl \quad (1)$$

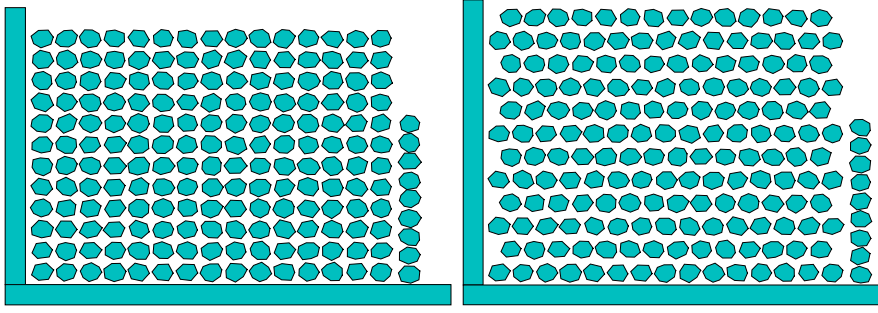


Figure 2: Particles are initialized with their centroids either on a square (left image) or a staggered (right image) grid and then sedimented (without fluid to save computer time) to form the granular step. A column of fixed particles (at the right of each image) prevents the collapse of the step during this stage of the simulation.

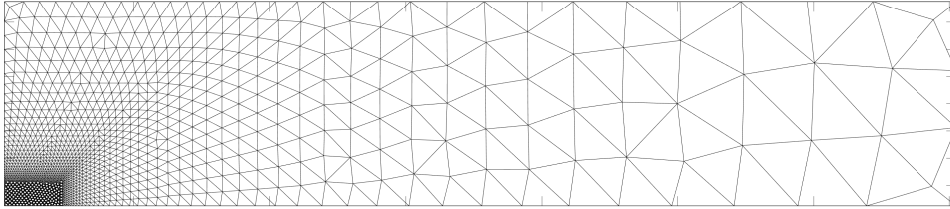


Figure 3: Complete fluid simulation domain. The granular step occupies only the bottom left corner. A graded mesh is required to effectively discretize the big empty buffer regions that stretch from the step towards the top and right walls.

with η as the dynamic viscosity and \hat{n} as the vector normal to the particle surface [3].

2 CREATING THE GRANULAR STEP

We test our simulation against the collapse of a submerged granular step by Rondon et al. [4], which uses a relatively simple setup. Accordingly, we have modeled our simulation in line with the parameters: The fluid domain is 70 cm wide and 15 cm high, with the granular step in the bottom left corner of the domain 4 cm wide and 2 cm tall. The fluid is a mixture of water and Ucon oil with a density of $\rho_F = 1000 \frac{\text{kg}}{\text{m}^3}$ and a dynamic viscosity of $\eta_F = 23 \cdot 10^{-3}$ Pa s.

The granular particles have a density of $\rho_P = 2500 \frac{\text{kg}}{\text{m}^3}$ and in our simulations have an average diameter of $d_P = 2$ mm, which makes them about five times larger than the ones used in the experiments by Rondon et al. This results in the step containing between 180 and 190 particles depending on the initial configuration, which is a considerably larger system size than was possible with earlier iterations of the simulation code [2]. Simulations with smaller granular particles are possible due to the improvements in the meshing algorithm introduced in [5] and [6] but require considerably higher amounts of CPU time.

Particles are initialized with plenty of space between them (see Fig. 2). In a pure DEM simulation they are then sedimented into the granular step. To avoid the step collapsing before the fluid is added, the particles in the rightmost column are fixed and do not move during this stage. Once sedimentation is completed, the particles are confined in

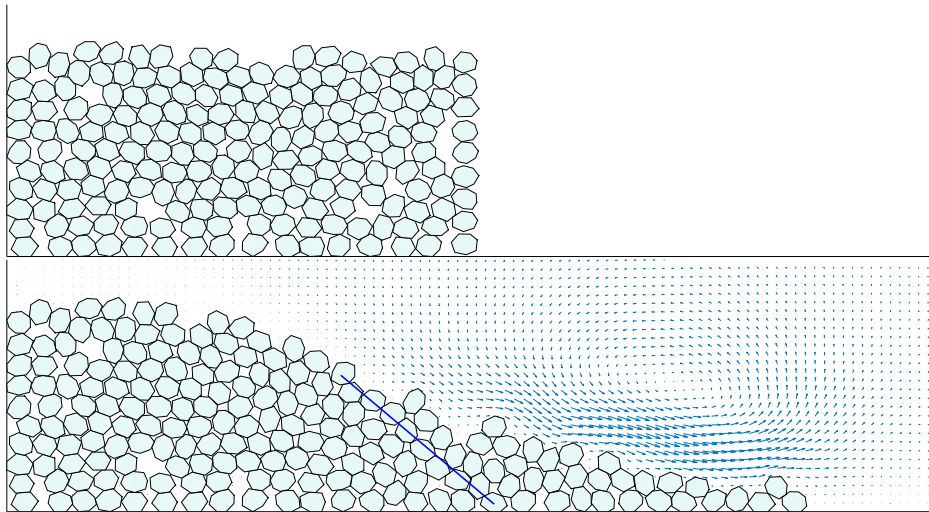


Figure 4: Initial (top) and final (bottom) configuration for a granular step with moderate packing density $b_{shadow} = 76\%$. During the collapse, chains of granular particles form that deflect forces into the ground (one marked with a blue line).

fluid and the simulation continues as a combined FEM-DEM simulation. The particles in the rightmost column now are treated the same way as the other particles, starting the collapse of the granular step.

The fluid domain is considerably larger than the granular step itself (see Fig. 3), giving the step enough horizontal room to collapse into and providing buffer regions that mitigate unwanted influences on the flow field from the top and right walls of the domain. Only by employing techniques like a graded mesh and mesh dragging [5] we are able to provide enough mesh resolution at the particle surfaces while still efficiently discretizing the remaining parts of the domain.

3 SIMULATIONS

In Rondon et al. [4], only two granular packing densities were investigated experimentally, but the detailed density values were not measured. Therefore, we set up various configurations to explore the relation between particle packing and particle shape.

3.1 Moderate Packing Density (Reference Run)

The particles for the initial simulation were set up with the centroids on a square grid before sedimenting into the granular step. This provides us with a granular bulk of medium packing density as seen in Fig. 4 (top). By drawing an alpha shape [7] around the step and relating its area to the combined areas of the individual particles, we obtain as filling ratio $b_{shadow} = 76\%$ if we use the particle shadows and $b_{core} = 43\%$ if we consider the particle cores. Particle shapes are randomized with 6 to 8 corners, have an aspect ratio of $e = 1.2$ and a core-to-shadow (diameter) ratio of $c = 75\%$.

Fig 4 (bottom) shows the granular step after the collapse in a trapezoidal configuration, very similar to the one observed in the experiments by Rondon et al. [4] for high particle

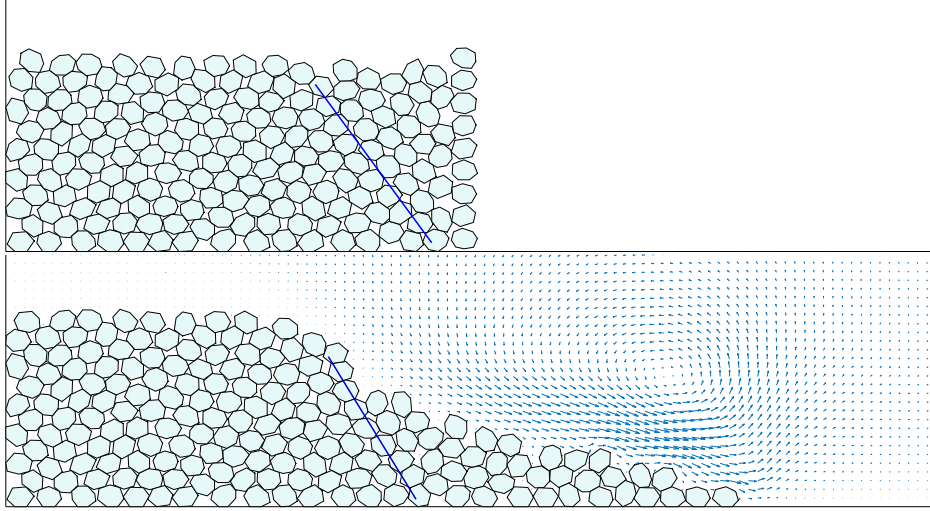


Figure 5: Initial (top) and final (bottom) configuration for a granular step with increased packing density $b_{shadow} = 80\%$. The tight packing results in the formation of particle chains (blue lines) during sedimentation. During the collapse, these chains keep a major part of the granular step stable.

densities. The collapse of the step shifts the particles into a triangular packing, very close to the tightest packing configuration. This allows the formation of particle chains that can effectively deflect the forces of the oncoming particles into the ground below, stabilizing the slope and arresting the collapse. The horizontal top of the trapezoid therefore extends up to the onset of one of these particle chains and shows only minimal deviation from the state before the collapse. The material further to the right has collapsed and forms the slanted side of the trapezoid. The slope angle itself is not uniform, which can be attributed to the slope of the particle chain gradually transitioning into the slope of the collapsed heap as well as the particles being relatively large compared to the slope size.

3.2 Increased Packing Density

To increase the packing density in the granular medium, the particles for this simulation were initialized with the centroids on a staggered grid before sedimenting into the step while all other parameters were kept the same. This results in filling ratios of $b_{shadow} = 80\%$ and $b_{core} = 45\%$.

In the collapsed step (see Fig. 5), the trapezoidal shape is now far more pronounced because the particle chains that stabilize the step exist already before the collapse starts. This results in steeper orientations of the particle chains and a smaller tail of collapsed material to the right. The slope also shows a noticeable kink, where the particle chain transitions into the collapsed heap.

3.3 Slightly Increased Packing Density

In an attempt to achieve an even tighter packing within the granular step, we initialized the particles in a staggered formation and dropped them from a greater height (bottom of the bulk at 3 cm height instead of directly above the floor), with the intention that the

impact forces during sedimentation cause more vibrations and the particles settle together with an increased granular density. However, counterintuitively, the impact forces had the opposite effect and caused particles to bounce and end up in a configuration that was less dense with $b_{shadow} = 78\%$ and $b_{core} = 44\%$.

Accordingly, the resulting state in Fig. 6 is situated in between both previous results with a slightly steeper particle chain and a more pronounced transition into the collapsed heap, compared to the reference run in Sec. 3.1.

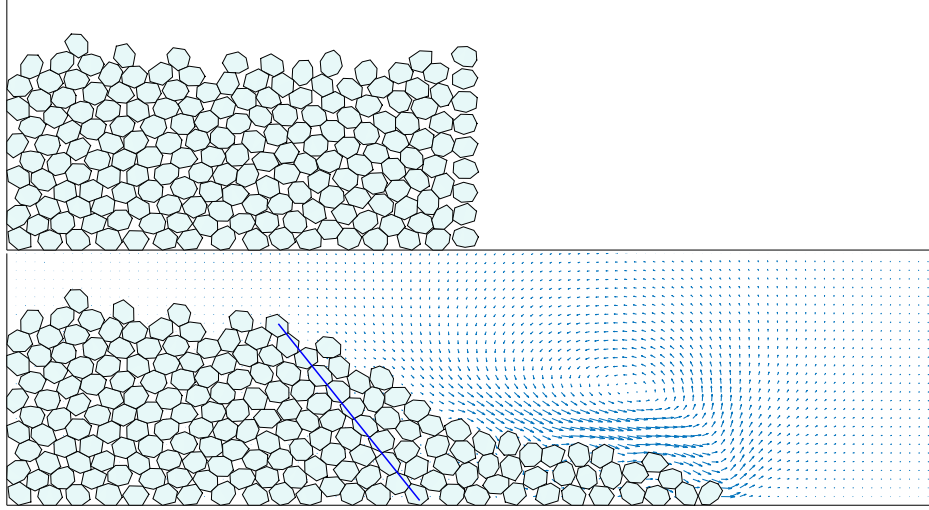


Figure 6: Initial (top) and final (bottom) configuration for a granular step with slightly increased packing density $b_{shadow} = 78\%$. The chain of granular particles marked with a blue line is steeper than the one in Fig. 4, but shallower than the one in Fig. 5.

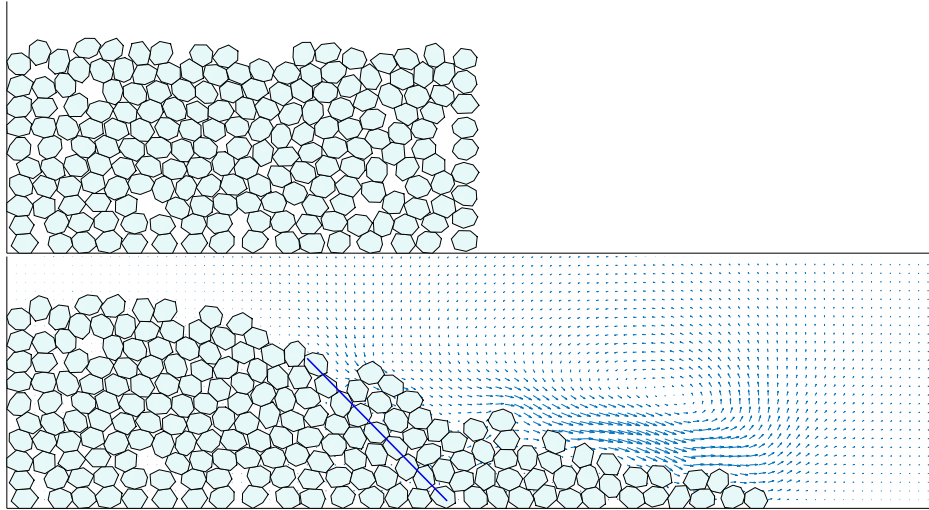


Figure 7: Initial (top) and final (bottom) configuration for a granular step with smaller particle cores $c = 60\%$. The particle chain marked with a blue line is very similar to the one marked in Fig. 4.

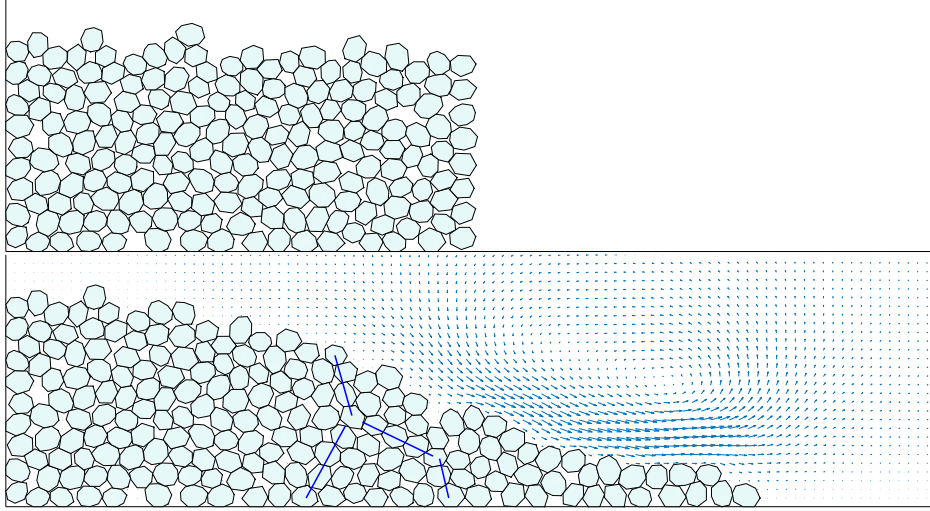


Figure 8: Initial (top) and final (bottom) configuration for a granular step with particles of varying sizes $d_P^{min} = 1.8$ mm, $d_P^{max} = 2.2$ mm. The particle chains (blue lines) do not span over the full height of the granular step but instead form a branching network.

3.4 Smaller Particle Cores

To study the influence of the pore space width, we set up the next simulation in the same way as the reference run, but with a core-to-shadow ratio of $c = 60\%$, resulting in $b_{shadow} = 76\%$ unchanged, but with $b_{core} = 28\%$.

The result in Fig. 7 is very similar to the one seen in the reference run Fig. 4. With the particle chain at almost the same position, it is likely that the fluid-solid-interaction has very little influence on the formation of these structures. This agrees with the fact that during the onset of the collapse in a specific region of the granular step, the flow velocities inside the porespace of this region are still very small.

However, compared to the reference run (Sec. 3.1), the collapsed heap outside of the particle chain is noticeably shorter and the vortex that formed near the surface of the slope is weaker than in Fig. 4. This suggests that the higher permeability of the granular bulk weakened the flow gradients so the particles were not carried as far as in the reference configuration. Furthermore, fluid buried beneath the collapsing heap can seep out easier, providing less of a fluid cushion the particles could ride on.

3.5 Particle Size Dispersion

The formation of particle chains which stabilize the bulk is due to the similar size of the particles and their low number of corners. This facilitates the formation of triangular packings which eventually result in the aforementioned chains. By varying the size of the particles we now tried to inhibit this behaviour.

For this simulation we took the setup of the reference run but randomly varied the particle diameter from $d_P^{min} = 1.8$ mm to $d_P^{max} = 2.2$ mm. The filling ratio of the granular step remained roughly the same with $b_{shadow} = 77\%$ and $b_{core} = 43\%$.

As seen in Fig. 8, we still get a trapezoid shape, but with a much shorter horizontal

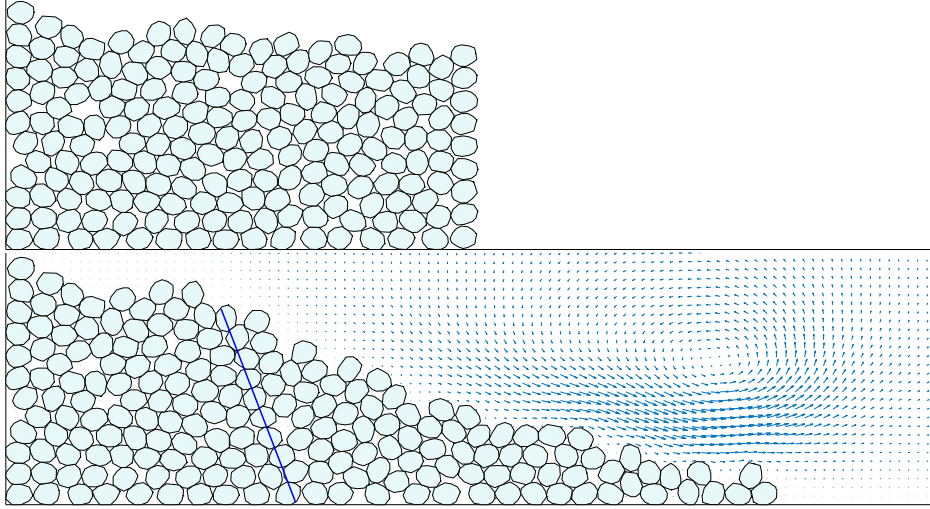


Figure 9: Initial (top) and final (bottom) configuration for a granular step with more rounded particles (9 to 13 corners). Very steep chains of particles (one marked with a blue line) can be seen, but they form more to the left side of the granular step compared to the other simulated cases. Hence, a considerable amount of the step is affected by the collapse.

top part. Furthermore, any chains of particles that deflect forces into the ground are not as pronounced as before, but split up into several parts. This shows how the variation in particle size can reduce the mechanical stability of a granular bulk.

3.6 Rounded Particles

For the final simulation, we prepared a setup similar to the reference run, but with the polygonal particles having between 9 and 13 corners. This resulted in filling ratios of $b_{shadow} = 80\%$ and $b_{core} = 45\%$.

In Fig. 9, it is manifest that a considerable part of the granular step collapses before a particle chain can form, even though the filling ratio is as high as for the increased packing density in Sec. 3.2. As the particle outlines feature less acute angles, it is considerably more difficult for granular layers to interlock, which also results in a more uniform shallow slope.

4 CONCLUSIONS

We have implemented a combined DEM-FEM code that is able to simulate the collapse of a submerged granular step. The simulation results show the same phenomenology as in the experiments by Rondon et al. [4], even though the particles in the current simulation are still much larger. While we were able to reproduce the trapezoid shape of the collapsed step as documented by Rondon et al., our simulations did not result in any collapses into a triangular shape, which Rondon et al. document for less dense granular packings. This may be related to the following causes: The (still relatively) low particle number allows for a faster formation of particle chains, so with decreasing particle size the heap may become less stable. In [4], Rondon et al. describe their granules as "glass beads" without

giving further information on the shape (roundness). Accordingly, polygons with a low corner number (6 to 8) may not be ideal for a comparable simulation as they increase the stability of the granular bulk.

Nonetheless, the simulations presented in this paper provide a useful insight on how parameters like packing density, particle size dispersion and particle roundness influence the collapse of a granular step and this data can now be used to efficiently prepare configurations of the granular step with smaller particle sizes.

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