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S-N BASED FATIGUE DAMAGE MODELLING OF OFFSHORE STRUCTURES: RECENT DAMAGE ACCUMULATION MODELS AND THE WAY FORWARD

F. BJØRHEIM[*], SUDATH C. SIRIWARDANE[†] AND DIMITRIOS G. PAVLOU[†]

[*] Department of Mechanical and Structural Engineering and Materials Science
University of Stavanger, Stavanger, N-4036 Norway
e-mail: Fredrik.bjorheim@uis.no

[†] Department of Mechanical and Structural Engineering and Materials Science
University of Stavanger, Stavanger, N-4036 Norway
e-mail: sasc.siriwardane@uis.no
e-mail: dimitrios.g.pavlou@uis.no

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Abstract. *Recently proposed damage models are presented and compared. In addition, a deterministic algorithm for nonlinear fatigue damage monitoring is presented and discussed. Furthermore, the commonly adopted functional forms for damage modelling is proposed and the adequacy of their functional form is directly investigated by the help of experimental data. Thereafter, the accuracy and whether the presented models give conservative estimates in comparison to Miner's rule are checked with experimental data. It is found that the proposed models generally perform better for various materials commonly subjected to fatigue. Finally, it is discussed how both the theory and deterministic algorithms now exist for both adopting nonlinear functions in design if the expected loading sequence can be determined, whereas it can always be adopted for fatigue based structural health monitoring.*

1 INTRODUCTION

Fatigue is a major cause of failure for onshore and offshore structures subjected to variable amplitude loading [1]. These structures are commonly exposed to variable amplitude loading due to wind, wave, traffic loads and etc. The induced stresses due to such a loading cycle will commonly be low in comparison to the ultimate tensile strength of the material. Thus, failure will not occur under a single or several cycles, whereas after 10^5or 10^6 cycles, it might result in catastrophic failure due to the accumulated fatigue damage.

Accumulated fatigue damage is today commonly assessed by the theory first proposed by Palmgren [2] and later popularized by Miner [3], commonly known as the Palmgren-Miner's rule or just Miner's rule. The model or rule hypothesizes that the accumulated fatigue damage is a linear summation of the exhausted number of cycles divided by the capacity or resistance for each stress amplitude. This can mathematically be expressed as follows in Eq. (1), where failure is expected to occur when the mathematical expression reaches unity.

$$D = \sum_{i=1}^{k} \frac{n_i}{N_{fi}} = \frac{n_1}{N_{f1}} + \frac{n_2}{N_{f2}} + \cdots + \frac{n_k}{N_k} \quad (1)$$

From the presented equation, it can be noted that the expression does not have any correction/consideration to take into account the damage effect due to the loading sequence effect. Consequently the model ignores this significant effect, where it is known that low-high transition might result in a significantly underestimation of the remaining fatigue capacity, whereas a high-low transition might result in an overestimation of the remaining fatigue capacity subjected to variable amplitude loading [4, 5].

The fact that Miner's rule still remains, despite its negligence towards the well-known phenomenon of loading sequence effect, highlights the need for further research towards fatigue damage accumulation modelling. In fact, it has resulted in a significant number of models, which can be found in review papers presented by such as Fatemi [6] and recently Hectors [7].

Quantification/modelling of the accumulated fatigue damage can commonly be separated into two different categories. The first category is related to theoretical reflections of fatigue damage and its relation to the damage accumulation for various stress amplitudes [8-13]. The other category can be defined based on the relation between changes in mechanical properties to the consumed and remaining fatigue capacity [14-21].

Despite the vast number of theories which are presented, generally being verified for both two-block loading and multiblock loadings, with good results. It remains that the damage accumulation theory adopted within the standards and codes is still based on the linear damage hypothesis which is commonly known as Palmgren-Miner's rule [2, 3, 22, 23].

Herein, the recently proposed high cycle fatigue damage models and a deterministic algorithm for nonlinear, fatigue-based structural health monitoring, which have been proposed at the University of Stavanger will be presented, discussed and compared. It will be highlighted that a full framework now exists to be able to adopt nonlinear damage functions in the standards, whereas the remaining requirement is for the standardization organizations to adopt the theory and models. In addition, it will be highlighted that nonlinear models generally perform better than Miner's rule. Furthermore, the functional forms commonly being adopted in fatigue damage models will be investigated and compared with experimental data, subsequently highlighting the potential for further improvement of the models.

2 DAMAGE MODELS AND ALGORITHMS

Aeran et al. [9, 10] highlighted in 2017 that several damage models had been proposed by that time. However, the models would generally depend on additional parameters, consequently resulting in that further fatigue testing would be needed for the models to be applicable. Thus, none of the proposed models can be found in the existing codes and standards, which still use Miner's rule. To overcome this, Aeran et al. proposed a new model, which only depend on the readily available S-N curves in the standards. The new model can be seen in Eq. (2), where the expression for the exponent δ_i can be found in Eq. (3).

$$D_i = 1 - \left[1 - \frac{n_i}{N_i}\right]^{\delta_i} \quad (2)$$

$$\delta_i = \frac{-1.25}{\ln N_i} \quad (3)$$

The parameter n_i is the number of cycles at a specific stress amplitude, whereas N_i is the fatigue capacity at the same stress amplitude. Furthermore, Aeran et al. also proposed a new damage transfer concept, where the effective number of applied cycles, $n_{(i+1),eff}$ are adjusted by the use of Eq. (4), where the new exponential term μ_i is defined as presented in Eq. (5). The term effective number of applied cycles or equivalent number of cycles is herein defined as the number of cycles which would result in the same value for damage D at the next stress amplitude.

$$n_{(i+1),eff} = \left[1 - (1 - D_i)^{\frac{\mu_{i+1}}{\delta_{i+1}}}\right] N_{i+1} \tag{4}$$

$$\mu_i = \left(\frac{\sigma_i}{\sigma_{i+1}}\right)^2 \tag{5}$$

The model is essentially based on the use of fatigue damage evolution curves, but also adopts a load interaction factor. For a material or component/part subjected to one constant amplitude fatigue loading, the presented formula in Eq. (2) can be used with the aid of Eq. (3). However, if the same material or component/part is thereafter subjected to a different stress amplitude, the transfer concept should also be adopted. As an example, to determine the damage after the second loading block, the number of cycles experienced at the first stress amplitude must be transferred to the second stress amplitude, which is performed by adopting Eq. (4). Thereafter, the effective cycles from the previous loading, and the number of cycles at the present stress amplitude must be added, as presented in Eq. (6).

$$n_{(i+1),total} = n_{(i+1),eff} + n_{(i+1)} \tag{6}$$

Subsequently, the initial damage function can be adopted with the number of cycles being $n_{(i+1),total}$ as presented in Eq.(7) and (8)

$$D_{i+1} = 1 - \left[1 - \frac{n_{(i+1),total}}{N_{i+1}}\right]^{\delta_i} \tag{7}$$

$$D = Abs(D_{i+1}) \tag{8}$$

The model was verified and exhibited very good results for two-stage and multilevel block loading, for six materials.

The theory of isodamage lines assumes that the *S-N* curve has a damage state of 100%, whereas the damage state for a given number of cycles and stress amplitudes falls on straight lines and can therefore be related to the next stress amplitude. Two well-known examples regarding isodamage lines are Subramanyan [11] and Hashin and Rotem [12]. Subramanyan proposed a set of straight isodamage lines, which all converge at the knee point of the S-N curve, whereas Hashin and Rotem proposed a set of straight isodamage lines, which all converge at the intersection of the S-N curve with the stress axis. Consequently, both theories implies that the number of cycles required to induce a certain level of damage increases with a reduction in stress amplitude and vice versa. However, it is generally accepted that the isodamage lines proposed by Subramanyan are more favourable [7, 24].

Rege and Pavlou [24] highlighted the fact that Subramanyan's model is somewhat non-conservative. Therefore, they proposed nonlinear isodamage curves in contrast to the commonly straight lines proposed by Subramanyan. The proposal was based on a combination of the commonly accepted damage function as presented in Eq. (9) and Eq. (10), the latter being known as the damage formulation proposed by Subramanyan [11]. The combined function can be seen in Eq. (11), where the exponent $q(\sigma_i)$ was found to be as presented in Eq. (13).

$$D = \left(\frac{n}{N_f}\right)^{q(\sigma)} \tag{9}$$

$$D = \frac{\log N_e - \log N_i}{\log N_e - \log n_i} \tag{10}$$

The parameter $N_i$is the constant amplitude fatigue life at the specific stress amplitude, N_e is the fatigue limit, whereas the parameter n_i is the number of applied cycles. The combined function for continued loading can be seen in Eq. (11), where the expression $\log n_{(i-1),(i)}$ is the equivalent number of cycles and can be calculated as presented in Eq. (12). Furthermore, the exponent $q(\sigma_i)$ was found to be as presented in Eq. (13).

$$D_i = \left(\frac{\log N_e - \log N_i}{\log N_e - \log(n_{(i-1),(i)} + n_i)}\right)^{q(\sigma_i)} \tag{11}$$

$$\log n_{(i-1),(i)} = \log N_e - \frac{\log N_e - \log N_i}{D_{i-1}^{\frac{1}{q(\sigma_i)}}} \tag{12}$$

$$q(\sigma_i) = (a\sigma_i)^b = \left(2\frac{\sigma_i}{\sigma_s}\right)^{-0.75} \tag{13}$$

σ_i and σ_s are the applied stress amplitude and stress at which the straight S-N curve intersects the stress axis respectively, whereas the parameters a and b can either be constants or expressions, highlighted by the author at which can be calculated by taking the local evolution of material parameters like ductility or hardness during the fatigue damage accumulation into account. However, to estimate the remaining fatigue capacity, the parameter a is not necessary as can be seen in Eq. (14).

$$\frac{q(\sigma_1)}{q(\sigma_2)} = \frac{(a\sigma_1)^b}{(a\sigma_2)^b} = \left(\frac{\sigma_1}{\sigma_2}\right)^b \tag{14}$$

Although parameter a is not needed to estimate the remaining fatigue capacity, parameter b is still needed, and was proposed by Rege and Pavlou to be -0.75 for most steels [24]. With this parameter, it was found that their proposed model is more conservative for high to low loading tests, whereas less conservative for low to high loading tests.

Pavlou [8] proposed a different concept in comparison to the commonly adopted straight isodamage lines. In fact, the proposed theory was called the S-N fatigue damage envelope, which is a generalization of most of the existing fatigue damage models. It should already here be mentioned that the stress-axis (ordinate) and the cycles-axis (abscissa) are normalized and as presented in Eq. (15) and (16) respectively.

$$\sigma_i^* = \frac{\sigma_i - S_e}{S_u - S_e} \quad 0 \le \sigma_i^* \le 1 \tag{15}$$

$$n_i^* = \frac{n_i}{N_e} \quad 0 \le n_i^* \le 1 \tag{16}$$

σ_i is the applied stress amplitude, S_e is the fatigue endurance limit, S_u is the ultimate stress and N_e is the fatigue capacity at the knee-point stress, resulting in the normalized parameters σ_i^* and n_i^* both obtaining values in the range of 0 to 1. The model assumes as with isodamage lines, that the S-N curve itself has a damage state of 1, whereas both the ordinate and abscissa have a damage state of zero. This from the perspective that no damage can have been accumulated, unless stress amplitude cycles above the knee point stress has been applied. However, as both the ordinate and the abscissa have the damage state of zero, it will not result in straight lines which only converge at one location, such as with the lines proposed by Subramanyan and Hashin & Rotem. In fact, the damage envelope will converge both at the knee point, and at the intersection of the stress axis by the S-N curve, consequently resulting in isodamage curves instead of straight lines. Pavlou showed that the isodamage curves as presented can be developed through a heat transfer analysis. Thereafter, he would use the commercial finite element package ANSYS to develop the isodamage curves for two different materials and compare with experimental results found in the literature for two stage block loading, which showed good results.

Bjørheim et al. [25] adopted the theory of the S-N damage envelope to develop a damage model through interpolation of the results. The interpolation function adopted was similar as presented in Eq. (9), whereas the exponent was presented as $q(\sigma, m)$, where the notation or parameter "m" is used to highlight that there is a necessity of one or more material parameters to determine the exponent for this equation in addition to information regarding the applied stress amplitude. It is highlighted that both the functional form of the exponent and the number of material parameters "m" with their respective values, depends upon the theoretical or experimental method which is adopted to develop the expression. This from the perspective that both theoretical and experimental methods of defining damage exist. Furthermore, it was highlighted that the governing parameters for the resulting expression with its material parameters is related to the nature of the S-N curve. According to the theoretical background of the model, the S-N damage envelope demonstrates the inherent fatigue mechanisms during damage accumulation based on macroscopic parameters (i.e. stress or strain amplitude). The adopted S-N curve was a general, simplified representation of the S-N curve, where it is assumed that the curve is a straight line passing through the ultimate tensile strength and the knee-point stress. However, it should be noted that the ordinate and abscissa were defined as presented in Eqs. (15) and (16), consequently resulting in that the curve itself is a straight line from the point where the abscissa and ordinate both reaches unity. The resulting damage function from the analysis was as presented in Eq. (17).

$$D = \left(\frac{n}{N_f}\right)^{\frac{a(S_u - S_e)}{\sigma_i - S_e}} \tag{17}$$

The parameters σ_i, S_e and S_u are as previously explained, whereas the parameter N_f is the fatigue capacity for the given stress amplitude, and a is the parameter of interpolation for the adopted S-N curve, found to be equal to six in this case. The damage transfer concept of this equation also exists, where the expression to determine the equivalent cycle ratio can be seen in Eq. (18), whereas the continued damage evolution is expressed in Eq. (19).

$$D_1 = \left(\frac{n_1}{N_{f1}}\right)^{\frac{a(S_u - S_e)}{\sigma_1 - S_e}} = \left(\frac{n_2}{N_{f2}}\right)_{eq}^{\frac{a(S_u - S_e)}{\sigma_2 - S_e}} \rightarrow \left(\frac{n_2}{N_{f2}}\right)_{eq} = \left(\frac{n_1}{N_{f1}}\right)^{\frac{\sigma_2 - S_e}{\sigma_1 - S_e}} \tag{18}$$

$$D_{cont} = \left(\frac{n_2}{N_{f2}} + \left(\frac{n_1}{N_{f1}} \right)^{\frac{\sigma_2 - S_e}{\sigma_1 - S_e}} \right)^{\frac{a(S_u - S_e)}{\sigma_2 - S_e}} \tag{19}$$

The model was thereafter verified and compared with four other models for seven materials, where it generally exhibited good results. Furthermore, it is necessary to highlight the advantage with having a damage function as presented, where the exponential term is defined for each loading block, in contrast to the many functions which propose ratios of the exponent. A detailed description of this is presented in [25] whereas a brief description will be made herein. Damage estimation for two-stage block loading when the exponent is presented as a function can be performed as already presented in Eqs. (17), (18) and (19). However, it is a prerequisite that the exponential term is a defined function. In fact, an example for multistage loading of this is presented in Eq. (20), where the damage for each stage can be defined. In contrast, if ratios are adopted instead of a defined function, it is a requirement to determine the equation in relation to the damage state of unity as presented in Eq. (21).

$$D_{tot} = \left(\left(\cdots \left(\left(\left(\frac{n_1}{N_{f1}} \right)^{\frac{q(\sigma_1,m)}{q(\sigma_2,m)}} + \frac{n_2}{N_{f2}} \right)^{\frac{q(\sigma_2,m)}{q(\sigma_3,m)}} + \frac{n_3}{N_{f3}} \right)^{\frac{q(\sigma_3,m)}{q(\sigma_4,m)}} + \cdots + \frac{n_{k-1}}{N_{f(k-1)}} \right)^{\frac{q(\sigma_{k-1},m)}{q(\sigma_k,m)}} + \frac{n_k}{N_{f(k)}} \right)^{q(\sigma_k,m)} \tag{20}$$

$$\left(\cdots \left(\left(\left(\frac{n_1}{N_{f1}} \right)^{\frac{q(\sigma_1,m)}{q(\sigma_2,m)}} + \frac{n_2}{N_{f2}} \right)^{\frac{q(\sigma_2,m)}{q(\sigma_3,m)}} + \frac{n_3}{N_{f3}} \right)^{\frac{q(\sigma_3,m)}{q(\sigma_4,m)}} + \cdots + \frac{n_{k-1}}{N_{f(k-1)}} \right)^{\frac{q(\sigma_{k-1},m)}{q(\sigma_k,m)}} + \frac{n_k}{N_{f(k)}} = 1 \tag{21}$$

Another worthy and relevant mention is the recently published research by Pavlou [26], where both a new cycle-counting algorithm and a new damage summation algorithm have been proposed. The new counting algorithm is based upon inserting fictitious cycles within the irregular loading history, consequently resulting in that it is not irregular anymore. However, the damage of the new loading history is higher than the real irregular loading history, therefore, the fictitious cycles must subsequently be subtracted. The subtraction rule proposed can mathematically be proposed for two scenarios, presented in Eq. (22) and (23).

$$\left|\sigma_{j+1}^{max} - \sigma_j^v\right| > \left|\sigma_{j-1}^{max} - \sigma_j^v\right| \tag{22}$$

$$\left|\sigma_{j+1}^{max} - \sigma_j^v\right| < \left|\sigma_{j-1}^{max} - \sigma_j^v\right| \tag{23}$$

If the stress σ_j^v for any valley j of the original loading history fulfils the condition presented in Eq. (22), then the fictitious damaging event is deducted from the damaging event of the stress σ_{j-1}^{max}. Thus, the resulting cycle has a maximum stress of σ_{j-1}^{max}, whereas the minimum stress $\sigma_{j-1}^{min} = \sigma_j^v$. However, if the loading history fulfils the condition presented in Eq. (23), then the fictitious damaging event is deducted from the damaging event of the stress σ_{j+1}^{max}. Consequently, resulting in that the cycle has a maximum stress of σ_{j+1}^{max}, whereas the minimum stress $\sigma_{j+1}^{min} = \sigma_j^v$. The proposed counting method was thereafter compared with the commonly adopted Rainflow method, where the model performed much better when it comes to the topic of computational time. Furthermore, it should be mentioned that the proposed counting method holds a strong relation to the known fatigue damaging process, as what is effectively counted, are the stress amplitude for each hysteresis loop which in fact is the fatigue damaging process.

In addition to the counting method, Pavlou also proposed a new damage summation algorithm. It is well known that the function presented in Eq. (9), or any similar function where an equivalent number of cycles must be found by exponential ratios for thousands if not millions of cycles will introduce a high numerical error. Thus, they might give better results for two-level or multilevel block loadings, whereas they are unsuitable for irregular loading. However, Pavlou proposed to discretize the nonlinear damage curves into finite multi-stress damage bands, where the damage curves then can be approximated by linear segments with different slopes. The mathematical expression of the proposed damage summation algorithm is as presented in Eqs. (24) and (25). It should be mentioned that as the curve is divided into linear segments, it follows that as the number of segments increase, the accuracy of the approximation increases.

$$\omega_{ij} = \frac{D_j - D_{j-1}}{D_j^{1/q(\sigma_i)} - D_{j-1}^{1/q(\sigma_i)}} \tag{24}$$

$$\Delta D_{ij} = \omega_{ij} \frac{n_i}{N_i} \tag{25}$$

$D_j - D_{j-1}$ is the damage range for the specific band $D_j^{1/q(\sigma_i)} - D_{j-1}^{1/q(\sigma_i)}$ is effectively the cycle ratio range for the specific damage band (consequently determining the rate of change dD/dn) which is determined for ω_{ij}, where i is the loading cycle/cycles, and j is related to the damage band. Consequently, resulting in that the change of damage can be determined through the linear expression presented in Eq. (25), whereas the total damage is the summation of all the given segments. However, it should be highlighted that the adoption of the rate of change and consequently the function is an iterative process. To determine with certainty what amount of damage the "i-th" cycle will contribute with, it is necessary to have done the summation of damage until the "i-th-1" cycle, as this will subsequently determine which damage band j which should be used.

3 COMMONLY ADOPTED FUNCTIONAL FORM

Many of the previously proposed damage models in the literature can be generalized by the following functional form.

$$D = 1 - \left(1 - \left(\frac{n}{N_f}\right)^{\alpha}\right)^{\beta} \tag{26}$$

The parameter n is the number of cycles, N_f is the fatigue capacity/strength at the current stress amplitude, whereas the parameters α and β can vary significantly. The parameters α and β can either be material dependent parameters or parameters dependent on the stress amplitude, mean stress, temperature etc. However, it should be noted that it is commonly accepted that at least one of the parameters should be related to the stress amplitude. Two formulations presented herein, namely Eqs. (2) and (9) are special cases of the presented equation. In fact, by defining the parameter α as 1, the functional form of Eq. (2) will be obtained, whereas if β is defined as 1, the functional form of Eq. (9) is obtained, as presented in Eqs. (27) and (28).

$$\alpha = 1 \to D = 1 - \left(1 - \left(\frac{n}{N_f}\right)\right)^{\beta} \tag{27}$$

$$\beta = 1 \to D = \left(\frac{n}{N_f}\right)^{\alpha} \tag{28}$$

Now, there are three functional forms generally being adopted within the field of damage functions as shown in Eqs. (26), (27) and (28), where damage transfer between cycles are defined to be that the damage is equivalent, whereas the equivalent cycle ratio will be adjusted. In the case of a two-stage loading, it is then defined that $D_1 = D_2$ at the instantaneous moment between the stress amplitudes of varying magnitude, where $D_1$is the function containing the current cycle ratio and exponent for the current stress amplitude, whereas D_2 is related to the next stress amplitude. Consequently, resulting in that a different exponent is being adopted if it is stress amplitude dependent, which again results in a change of the equivalent exhausted cycle ratio. The resulting equation can be seen in the expressions presented in Eqs. (29) and (30) after the damage transfer.

$$D_1 = 1 - \left(1 - \left(\frac{n_1}{N_{f1}}\right)^{\alpha_1}\right)^{\beta_1} = 1 - \left(1 - \left(\frac{n_2}{N_{f2}}\right)_{eq}^{\alpha_2}\right)^{\beta_2} \tag{29}$$

$$\left(\frac{n_2}{N_{f2}}\right)_{eq} = \sqrt[\alpha_2]{1 - \left(1 - \left(\frac{n_1}{N_{f1}}\right)^{\alpha_1}\right)^{\frac{\beta_1}{\beta_2}}} \tag{30}$$

For continued loading at the second stress amplitude, the expression for equivalent cycle ratio for the next stress amplitude, presented in Eq. (30) has to be combined with the continued cycle ratio, as presented in Eq. (31), where the term $(n_2/N_{f2})_{cont}$ is the continued cycle ratio, defined as the continued exhausted cycles n_2 divided by the fatigue capacity N_{f2}.

$$\left(\frac{n_2}{N_{f2}}\right)_{tot} = \left(\frac{n_2}{N_{f2}}\right)_{eq} + \left(\frac{n_2}{N_{f2}}\right)_{cont} \tag{31}$$

It should also be mentioned here that it is generally accepted that failure occurs when the accumulated damage reaches unity $D = 1$. Therefore, the expression presented in Eq. (31) can be inserted into the initial functional form as presented in Eq. (26) which will result in the following expression.

$$D = 1 = 1 - \left(1 - \left(\frac{n_2}{N_{f2}} + \sqrt[\alpha_2]{1 - \left(1 - \left(\frac{n_1}{N_{f1}}\right)^{\alpha_1}\right)^{\frac{\beta_1}{\beta_2}}}\right)^{\alpha_2}\right)^{\beta_2} \tag{32}$$

This damage function then represents the functional form of the continued damage accumulation at the second stress amplitude. As the continued accumulated fatigue damage expression has here been defined to be equivalent to 1 (i.e., fatigue failure), it can be further simplified as presented in Eq. (33).

$$\left(1-\left(\frac{n_2}{N_{f2}}+\sqrt[\alpha_2]{1-\left(1-\left(\frac{n_1}{N_{f1}}\right)^{\alpha_1}\right)^{\frac{\beta_1}{\beta_2}}}\right)^{\alpha_2}\right)^{\beta_2}=0 \quad (33)$$

Furthermore, it should be mentioned that for this case as well, one can adopt the special cases of $\alpha = 1$ and $\beta = 1$ which can be seen in Eqs. (34) and (35).

$$\alpha = 1 \rightarrow \left(1-\left(\frac{n_2}{N_{f2}}+1-\left(1-\left(\frac{n_1}{N_{f1}}\right)\right)^{\frac{\beta_1}{\beta_2}}\right)\right)^{\beta_2}=0 \quad (34)$$

$$\beta = 1 \rightarrow 1-\left(\frac{n_2}{N_{f2}}+\left(\frac{n_1}{N_{f1}}\right)^{\frac{\alpha_1}{\alpha_2}}\right)^{\alpha_2}=0 \rightarrow \left(\frac{n_2}{N_{f2}}+\left(\frac{n_1}{N_{f1}}\right)^{\frac{\alpha_1}{\alpha_2}}\right)^{\alpha_2}=1 \quad (35)$$

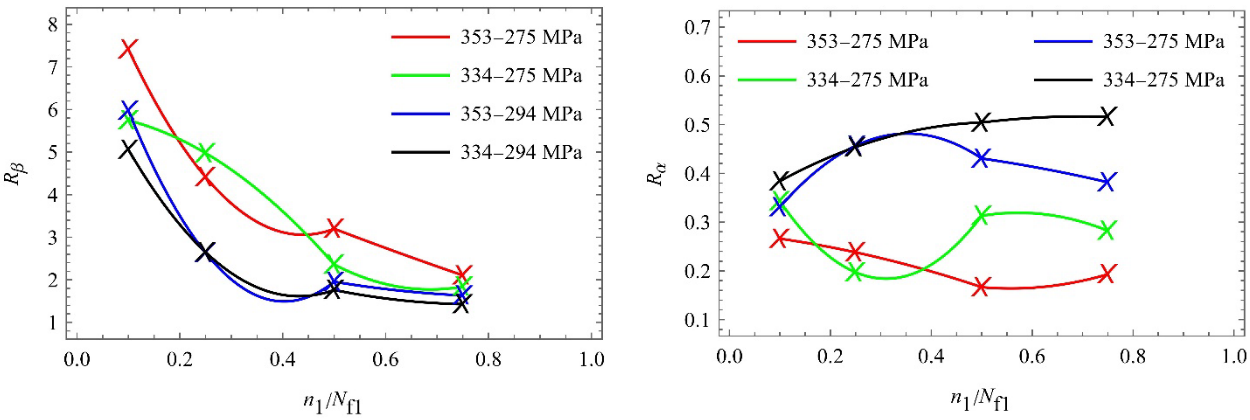
The advantage of the presented three functions in this paper, is that the full equation presented in Eq. (33) can be comparatively studied in a three dimensional contour plot, where the parameters α_1, α_2 and the ratio β_1/β_2 can be studied and compared with experimental results. Given that the ratios n_1/N_{f1} and n_2/N_{f2} are known for specimens which have been fractured under two-stage loading. Furthermore, for the special cases where either $\alpha = 1$ or $\beta = 1$ is true, then the comparison can be further simplified. In fact, it comes down to a single value, namely the ratio β_1/β_2 for Eq. (34) whereas the ratio α_1/α_2 for Eq. (35). This from the perspective that the external exponent can be neglected. Consequently, resulting in the functional forms presented being reduced to the Eqs. (36) and (37), where the expressions are dependent on the first and second known cycle ratios, and the R_α and R_β ratios.

$$\alpha = 1 \rightarrow 1-\left(\frac{n_2}{N_{f2}}+1-\left(1-\left(\frac{n_1}{N_{f1}}\right)\right)^{R_\beta}\right)=0 \quad (36)$$

$$\beta = 1 \rightarrow 1-\left(\frac{n_2}{N_{f2}}+\left(\frac{n_1}{N_{f1}}\right)^{R_\alpha}\right)=0 \rightarrow \frac{n_2}{N_{f2}}+\left(\frac{n_1}{N_{f1}}\right)^{R_\alpha}=1 \quad (37)$$

This is important to note, as these parameters should only be dependent on such as stress amplitude etc. Thus, for two specific stress amplitudes, where the only difference is the cycle ratio of the two loading blocks, the ratios should be equivalent. Consequently, resulting in a plot where the exponential ratio versus the cycle ratio of the first loading block should essentially be a flat line, with the exception of experimental scatter.

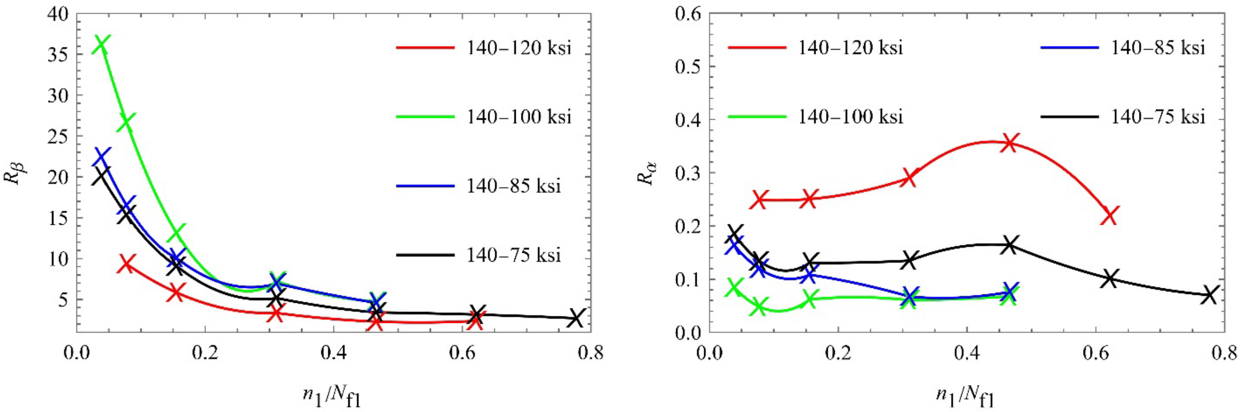
The functional forms presented are herein evaluated using two datasets, which were found in the literature. The first evaluation is based upon the results published by Subramanyan in [11] where specimens made of C35 medium carbon steel was tested for two-stage loading. The material parameters were as follows: yield strength $\sigma_y = 324\ MPa$, ultimate tensile strength $S_u = 458\ MPa$ and the knee-point stress $S_e = 255\ MPa$. The results can be seen in Fig. 1, where the notation 353-275 MPa means that the first and second stress amplitudes were 353 and 275 MPa respectively.



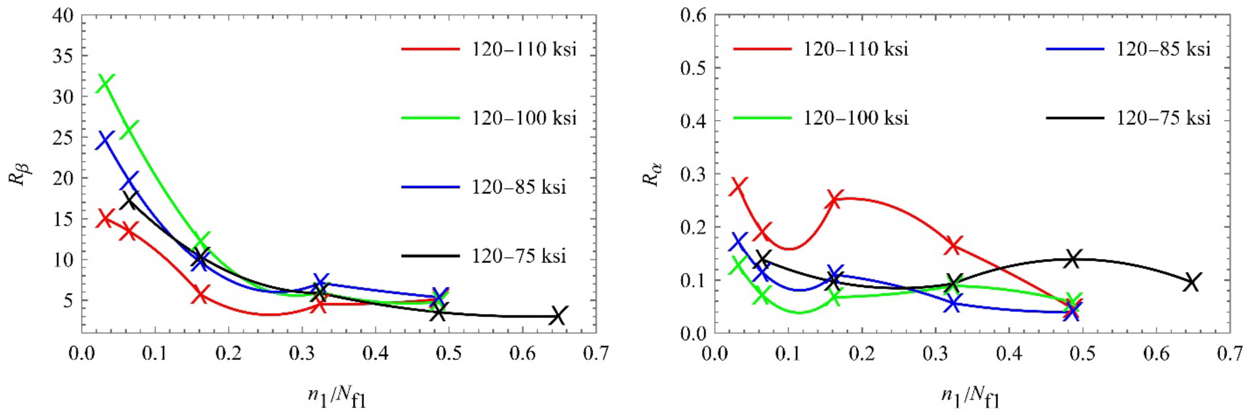
a) b)

Fig. 1: Interpolation of the ratios by (a) Eq. (36) and (b) Eq. (37) based on experimental data for C35 medium carbon steel

The second dataset was published by Manson et al. in [27] where a vast number of SAE 4130 were tested under two-stage loading. The material parameters were yield strength 0.2 percent offset $\sigma_y = 113\ ksi$, ultimate tensile strength $S_u = 130\ ksi$, whereas the knee-point stress is unknown. The results can be seen in Fig. 2, where the notation 140-120 ksi means that the first and second stress amplitudes were 140 and 120 ksi respectively.



a) b)



c) d)

Fig. 2: Interpolation of the ratios by (a) Eq. (36), (b) Eq. (37), (c) Eq. (36) and (d) Eq. (37) based on experimental data for SAE 4130 steel

From the results, it can be seen that especially equation (36), which is dependent on the ratio R_β, exhibits a clear trend with number of cycles, which again indicates that the functional form is not adequate without either the exponential term being related to the cycle ratio or some correction as presented by for instance Aeran et al. [9, 10]. When it comes to equation (37), it cannot clearly be defined that the functional form is inadequate. From the plots, especially in Fig. 1 b), there seems to possibly be a trend, but in relation to Fig. 2 b) and d) it also has to be acknowledged that fatigue is commonly known to have high scatter.

4 COMPARISION AND DISCUSSION

Herein, models and algorithms proposed at the University of Stavanger are presented. In fact, three damage models, one computationally effective counting strategy and one numerical algorithm to linearly approximate the nonlinear functions to avoid numerical inaccuracy due to the exponential ratios are presented. The only remaining weakness by one of the formulas is the damage transfer concept which is adopted in Ashish's model. The transfer concept increases accuracy for block-loading, whereas it will both have problems regarding numerical errors and significantly amplifying the reduction or increase of the effective number of cycles $n_{(i+1),eff}$ if it is applied on a cycle-to-cycle basis. If the transfer concept is removed, then the numerical algorithm proposed by Pavlou can again be applied to approximate the nonlinear function. But this model can be directly applied to design details by utilizing appropriate S-N curves given in the design standards. Furthermore, the nonlinear isodamage model proposed by Rege and Pavlou is not easily applicable for such as bilinear S-N curves commonly adopted by the standards. However, the damage theory proposed by Pavlou, which was the theoretical background for the damage function proposed by Bjørheim can be adopted for any S-N curve, including bilinear and trilinear curves, which highlights future work.

However, even though all the necessary theories have been presented, the recommendations in standards and codes are still to apply Miner's rule, which is known to have an inherent flaw. In fact, the rule ignores the perspective of loading sequence, which is known to increase the scatter in fatigue [4-6, 27]. However, it should be acknowledged that if the loading sequence is not known, and cannot be predicted, then again, the Miner's rule might be the best option. Consequently, resulting in that the standard should certainly not remove the possibility to adopt Miner's rule altogether for practicing engineers. However, the fact remains that at the current state of the standards, nonlinear fatigue damage models are not presented as an option. In addition, it should be mentioned that for structural health monitoring (SHM), where the real loading sequence and magnitude can be measured and/or is known, it is again the Miner's rule which has to be adopted for fatigue assessment in relation to the standards and codes.

In the previous paragraph, it was critiqued how nonlinear fatigue damage modelling is not adopted within the industry, whereas the functional form for nonlinear fatigue damage accumulation was also questioned herein. In this regard, it is important to acknowledge that the nonlinear fatigue damage accumulation modelling certainly can be further improved from the perspectives of accuracy and applicability. However, from the accumulation theories presented, it can be noted that the models generally perform better, as in equal or with higher accuracy and obtaining more conservative results than Miner's rule. In fact, by checking the verification recently performed by authors [25], it can be found that each of the models generally perform better than the Miner's rule for a vast number of tests for various materials, including: carbon steels, aluminium alloys and various steels adopted in the electrical power and petroleum chemistry industries subjected to high temperature and pressure. The results are presented in Table 1, where the resulting percentage is the percentage of times the models resulted in a number closer to the numerical correct value of remaining cycles, in comparison to the commonly adopted Miner's rule. The parentheses beneath the various materials display the number of test results which were available for each material subjected to two-stage loading. Furthermore, it should be mentioned that all the results are generally based on two-stage loading, whereas the material Al-6082-T6 is under multistage loading.

Table 1: Comparison of the accuracy of the models and Miner's rule

Models	C35 (22 tests) [11]	C45 (7 tests) [28]	Alloys (6 tests) [29]	Al-2024-T42 (6 tests) [16]	Al-6082-T6 (3 tests) [30]
Bjørheim [25]	100%	85.7%	50%	100%	100%
Rege [24]	100%	85.7%	50%	50%	100%
Aeran [10]	100%	71.4%	83.3%	100%	66.7%

Furthermore, it can be evaluated how often the various models, including the Miner's rule, underestimate the remaining number of cycles for the various materials. This evaluation, based on the same data is presented in, where it should be noted that a higher percentage means that the model is more conservative in comparison to the models with lower percentage. The results can be found in Table 2.

Table 2: Comparison of the underestimation of the results

Models	C35 (22 tests) [11]	C45 (7 tests) [28]	Alloys (6 tests) [29]	Al-2024-T42 (6 tests) [16]	Al-6082-T6 (3 tests) [30]
Miner's rule	27.3%	42.9%	33.3%	50.0%	33.3%
Bjørheim [25]	63.6%	71.4%	33.3%	50.0%	0%
Rege [24]	81.8%	71.4%	33.3%	50.0%	0%
Aeran [10]	36.4%	57.1%	33.3%	50.0%	0%

Consequently, on basis of this dataset, it can be seen that all the presented models generally predict the remaining number of cycles both more accurately and conservatively than the commonly adopted Miner's rule.

5 CONCLUSIONS

- It was identified that the commonly adopted functional forms of the damage accumulation formula can be found to be a special case of an expression with two exponential terms.
- It was found that one of the commonly adopted functional forms might be inadequate for damage modelling based on a comparative study of experimental data, whereas the adequacy of the other functional form remains uncertain.
- The presented models exhibited more accurate and conservative estimates of the remaining fatigue capacity/life with the exception of an aluminium alloy under multiblock loading where the results were more accurate but slightly less conservative.
- It was discussed how there exist a functional framework for adopting nonlinear damage modelling as a means of estimating fatigue damage in the design phase if the future loading sequence can be predicted, whereas the framework is already adequate for SHM.

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