Error control and propagation in Adaptive Mesh Refinement applied to elliptic equations on quadtree/octree grids

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ABSTRACT

In this work we propose a new adaptive mesh refinement (AMR) method applied on isotropic octree/quadtree meshes. The new AMR approach uses a metric-based linear interpolation error estimation [2] extended to square/cubic elements. The analysis of various examples shows that the minimization of the total numerical error can lead to a suboptimal mesh in terms of pure interpolation error. The grids that minimize the error for different values of N (the number of elements imposed) is related to a fixed ratio between the minimal and mean cell size named the compression ratio. Above a certain value, a clear proportionality between the interpolation and the total error allows us to use the former as a criterion to adapt the grid. However, below a certain critical value of the compression ratio, no correlation between both errors is observed and the interpolation error is no longer representative of the total error contained in the solution. Based on these results, we propose to add a model to estimate the discrete minimum grid size and to impose it as an additional constrain to the error minimization problem. The proposed minimum grid size depends on (i) the structure of the solution, (ii) the number of grid points specified and (iii) a security coefficient defined such that it controls the distance between the optimal pure interpolation error and the targeted performance. By increasing this user defined parameter we show that we effectively restrict the range of the minimization problem to regions where we can safely use the local estimation of the interpolation error to drive the mesh adaptation and reduce the total numerical error.

The method is implemented in our in-house open-source solver Basilisk [1, 3] and the performance of our new approach is validated on a Poisson-Helmholtz solver and an incompressible Euler solver through academic problems with known solutions. These problems, with diverse levels of complexity, are shown to be challenging test cases to evaluate the efficiency of AMR methods.

REFERENCES

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