A Weighted Shifted Boundary Method for Free Surface Flows

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ABSTRACT

The Shifted Boundary Method (SBM) belongs to the class of unfitted (or immersed, or embedded) finite element methods and was recently introduced for the Poisson, linear advection/diffusion, Stokes, Navier-Stokes, acoustics, and shallow-water equations, and many additional problems [1, 2].

By reformulating the original boundary value problem over a surrogate (approximate) computational domain, the SBM avoids integration over cut cells and the associated problematic issues regarding numerical stability and matrix conditioning. Accuracy is maintained by modifying the original boundary conditions using Taylor expansions. Hence the name of the method, that *shifts* the *location* and *values* of the boundary conditions.

We extend here the SBM to the simulation of incompressible Navier-Stokes flows with moving free-surfaces, by appropriately weighting its variational form with the elemental volume fraction of active fluid [3]. This approach prevents spurious pressure oscillations in time, which would otherwise be produced if the total active fluid volume were to change abruptly over a time step.

In fact, the proposed weighted SBM method induces small mass (i.e., volume) conservation errors, which converge quadratically in the case of piecewise-linear finite element interpolations, as the grid is refined. We present an extensive set of two- and three-dimensional tests to demonstrate the robustness and accuracy of the method.

References

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