

# Momentum-conserving ROMs for the incompressible Navier-Stokes equations

H. Rosenberger<sup>1,\*</sup>, B. Sanderse<sup>2</sup>

<sup>1</sup> Centrum Wiskunde & Informatica, Science Park 123, Amsterdam, The Netherlands,  
henrik.rosenberger@cwi.nl, www.cwi.nl/people/henrik-rosenberger

<sup>2</sup> Centrum Wiskunde & Informatica, Science Park 123, Amsterdam, The Netherlands,  
b.sanderse@cwi.nl, www.thinkingslow.nl

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An ongoing issue with projection-based model order reduction of finite volume discretizations of conservation laws is that the intrinsic structure of conservation is in general not preserved [1]. A possible solution is to augment the ROM formulation by constraints enforcing conservation over specified subdomains, resulting in a constrained optimization problem for the ROM coefficients [1]. However, this constrained optimization problem is not guaranteed to be feasible and more expensive to compute than the original ROM formulation via an ODE.

In this work we propose to instead use an ODE which equivalently describes the solution of the constrained optimization problem (provided such a solution exists). This ODE is similar to the original ROM formulation but with an additional perturbation term. The main novelty is to choose the ROM basis in such a way that we eliminate this perturbation term and guarantee the feasibility of the constrained optimization problem. As a result, the resulting ROM is conservative over specified subdomains while having the same computational costs as the original non-conservative ROM.

We apply this subdomain-conservative ROM to the incompressible Navier-Stokes equations to derive a new ROM which conserves momentum over arbitrarily chosen subdomains. If we restrict the choice of subdomains, the ROM is also locally mass-conserving and energy-conserving in the inviscid limit, thereby generalizing the ROM proposed in [2].

## REFERENCES

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