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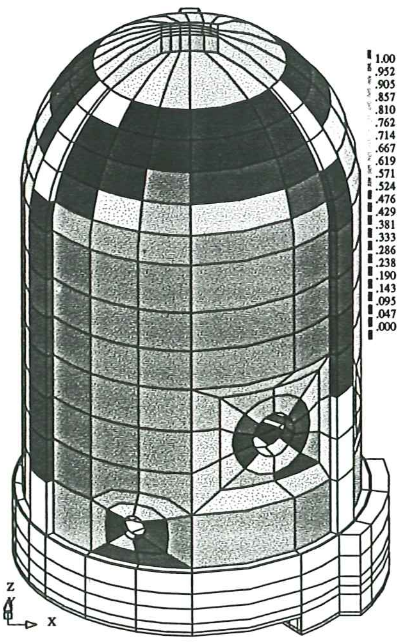

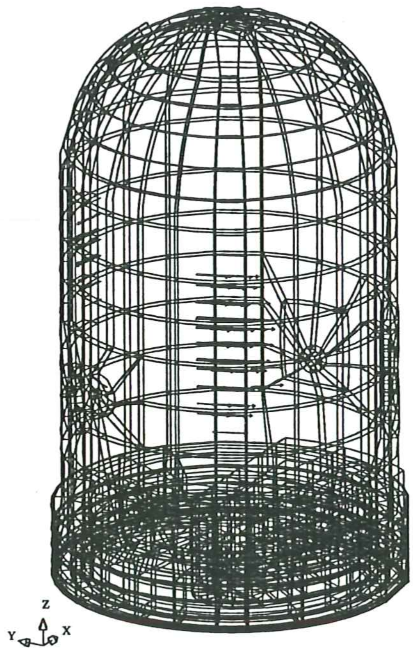

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PREDICTION OF DAMAGE AND FAILURE IN CIVIL ENGINEERING STRUCTURES USING A FINITE ELEMENT MODEL

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SUMMARY: The paper describes a finite element damage model for non linear analysis of concrete or reinforced concrete structures. It is shown how the model can be effectively used to predict local and global damage up to structural failure. Examples of applications of the model to the analysis of different structures such as a nuclear containment shell, a housing building and the domes of St.Mark Basilica are presented.

KEYWORDS: local and global damage, damage indices, structural failure, non linear analysis, concrete, masonry, reinforced concrete, nuclear containment shells, buildings, historical constructions.

INTRODUCTION

The design of concrete structures requires an accurate evaluation of the structural response both at service and ultimate load levels. Traditional methods for structural analysis generally provide safe designs, but they frequently contain inherent inconsistencies and often do not reflect a clear understanding of the actual composite behaviour of the material. Present-day design codes continue, in many respects, to be based on empirical approaches and they rely heavily on the results of considerable amount of experimental data. This situation is largely attributable to the complex behaviour of concrete. Thus, concrete cracking, tension stiffening, non linear multi-axial material properties and complex interface behaviours were previously ignored or treated in a very approximate manner. Numerical methods, and particularly the finite element techniques, now permit a more rational analysis of these complexities, thus allowing the assessment of the "safety" of a concrete structure both at local and global levels.

[13,17-22]. It is worth noting, that in this case cracks at a microscopic point need not to have not particular direction and a macroscopic crack is then defined as the locus of damage points as previously mentioned.

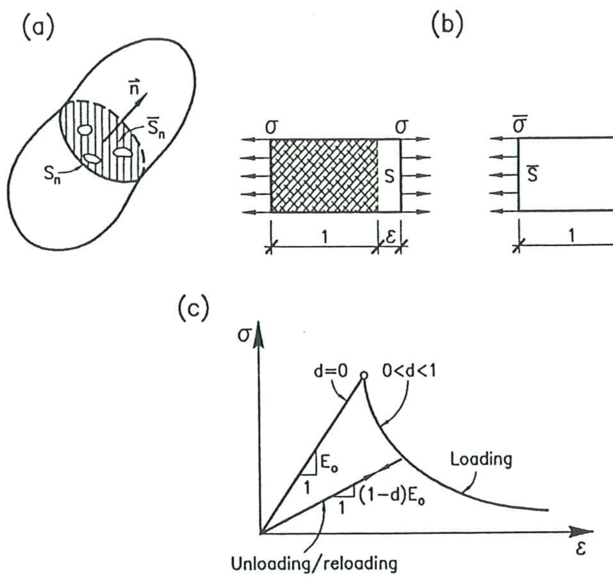

Fig. 1: (a) Damaged surface; (b) Cauchy stress σ and effective stress $\bar{\sigma}$; (c) Evolution of uniaxial stress-strain curve.

An useful concept for understanding the effect of damage is that of *effective stress.* The equilibrium relationship between the standard Cauchy stress σ and the "effective" stress, $\bar{\sigma}$, in the damaged bar specimen of Fig. 1 is

$$\sigma S = \bar{\sigma}\bar{S} \tag{2}$$

and from (1) and (2)

$$\sigma = (1-d)\bar{\sigma} = (1-d)E\varepsilon \tag{3}$$

When a damaging process is occurring, the external loading is resisted by the effective stress area and, therefore, $\bar{\sigma}$ is a more physically representative parameter than σ.

CONTINUUM DAMAGE MODEL FOR CONCRETE

In this work a single parameter damage model will be used. Examples of different tensor-valued models can be found in [19,20,22]. The constitutive equation will therefore be simply written in vector form as an extension of eq. (3) as

$$\boldsymbol{\sigma} = (1-d)\bar{\boldsymbol{\sigma}} = (1-d)\mathbf{D}\boldsymbol{\varepsilon} \tag{4}$$

where $\mathbf{D}$ is the elastic constitutive matrix and $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the standard stress and strain vectors. Fig. 1(c) shows the one dimensional representation of the stiffness evolution of the material.

The model defined by eq.(4) requires the knowledge of the damage variable d at every stage of the deformation process. For this purpose one must define:

a) A suitable scalar norm τ of the strain tensor (or alternatively of the undamaged stress tensor). Here, several possibilities exist and a suitable option for concrete and masonry is [10,23]

$g_f \geq (f'_t)^2/2E$ (the material must dissipate at least the energy stored when the elastic limit is reached), parameter A must be positive [28]. Defining $g'_f = g_f - (f'_t)^2/2E$, the second expression in (10) is obtained.

The damage model presented above is extremely simple in comparison with more sophisticated constitutive models for concrete. A flow chart summarising the steps required in the practical implementation of the model within a standard non linear finite element solution scheme are shown in Box 1.

The experimental characterisation of the model is also simple and the following material parameters are only required: Young modulus, tension and compression limit strengths and specific fracture energy obtained from uniaxial tests.

$\boxed{n\text{th load increment, } i\text{th iteration}}$

$$\boldsymbol{a}_i^n = \boldsymbol{a}^{n-1}$$

Compute displacement increment

$$\Delta \boldsymbol{a}_i^n = -[\boldsymbol{H}_i^n]^{-1} \boldsymbol{\Psi}_i^n$$

$\boldsymbol{\Psi}_i^n$: residual force vector $(= \int_V \boldsymbol{B}^T \boldsymbol{\sigma} \, dV - \boldsymbol{f})$

$\boldsymbol{H}_i^n$: iteration matrix (i.e. tangent stiffness matrix)

Update displacements and strains

$$\boldsymbol{a}_{i+1}^n = \boldsymbol{a}_i^n + \Delta \boldsymbol{a}_i^n, \qquad \boldsymbol{\varepsilon}_{i+1}^n = \boldsymbol{\varepsilon}_i^n + \boldsymbol{B} \Delta \boldsymbol{a}_i^n$$

Evaluate stresses

(1) Compute undamaged stresses: $\bar{\boldsymbol{\sigma}}_{i+1}^n = \boldsymbol{D} \boldsymbol{\varepsilon}_{i+1}^n$

(2) Evaluate r_{i+1}^n using Eq.(5): $r_{i+1}^n = \tau_{i+1}^n$

(3) Update r and d

$$r_{i+1}^n = \max(r^{n-1}, r_{i+1}^n), \qquad d_{i+1}^n = G(r_{i+1}^n)$$

(4) Update stresses: $\boldsymbol{\sigma}_{i+1}^n = (1 - d_{i+1}^n) \bar{\boldsymbol{\sigma}}_{i+1}^n$

Evaluate residual force vector: $\boldsymbol{\Psi}_{i+1}^n$

Check convergence: $\|\boldsymbol{\Psi}_{i+1}^n\| \leq \epsilon \|\boldsymbol{f}\|$?

No: Continue iterations: $i = i + 1$

Yes: $\boldsymbol{a}^n = \boldsymbol{a}_{i+1}^n$, next load increment: $n = n + 1$

Box 1: Quasi-static non linear finite element solution using the damage model

families (N-S, E-W) anchored in a perimetrical gallery located in the lower part of the foundation slab.

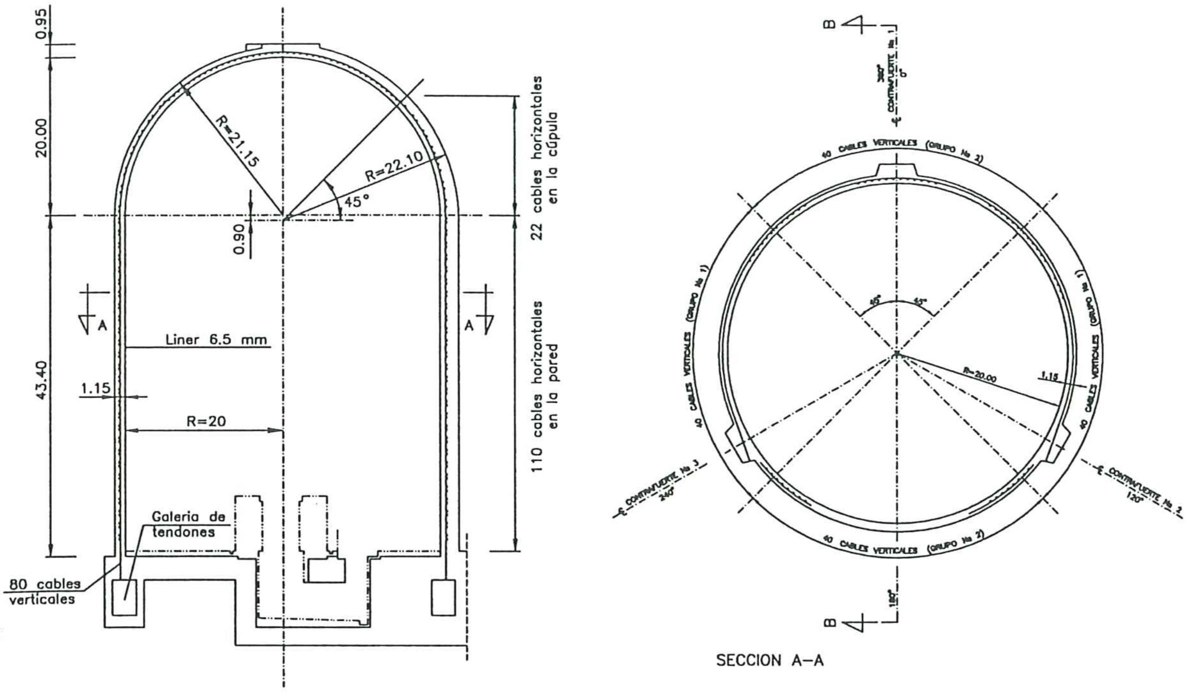

Fig. 3: Containment shell. (a) vertical section; (b) horizontal section.

Strategy of analysis

The failure pressure is defined as the inner pressure corresponding to the structural material exhaustion, that is, to a certain strain limit of the reinforcement steel, prestressing tendons and liner. The failure criterion assumes that local steel rupture occurs when the mentioned strain limit reaches 0.8% for the reinforcement and 1% for the tendons. The straining up to failure limit of the reinforcement is made possible by the damage-induced stress loss in concrete leading to stress redistribution towards the steel components. The global damage indices describe the state of entire structural parts, summing up both concrete and steel data.

The loads considered in the analysis were the self-weight, the external pressures generated by the prestressing system and the internal pressure corresponding to a specified accident. The distribution of the pressures equivalent to those produced by the prestressing system has been evaluated analytically for all the nodes of the mesh. All the possible sources of prestressing losses have been included in this evaluation, i.e. friction, wobble, anchor set, instantaneous and long term, etc. The internal pressure was incremented gradually until the structural collapse occurred.

Failure pressure evaluation

The influence of including the foundation slab in the structural model on the global structural behaviour and especially on the failure pressure was first examined. The results show that the influence of including the slab is quite small for low levels of internal pressure; it decreases further as the pressure increases and it is negligible near the failure pressure, which is 1.11 MPa in both cases. Furthermore, the cylindrical wall behaves better when the slab is present, due to the fact that the displacements of the slab slightly reduce the circumferential displacements of the wall; this allows to conclude that by not including the slab, one stays on the safety side during the complete load history. The comparison was based on an extensive survey of displacements, cracking

was divided in three disjoint rings of finite elements, thus: the first ring is made of the inferior row of elements which join the slab, the second ring contains the following three rows and the last three rows ending where the dome begins belong to the third ring. Separately, GDIs for the cylinder as a whole, the dome, the slab and the entire structure were also calculated. In both figures, zooms of the final instants are given in order to show in detail what happens just before structural failure.

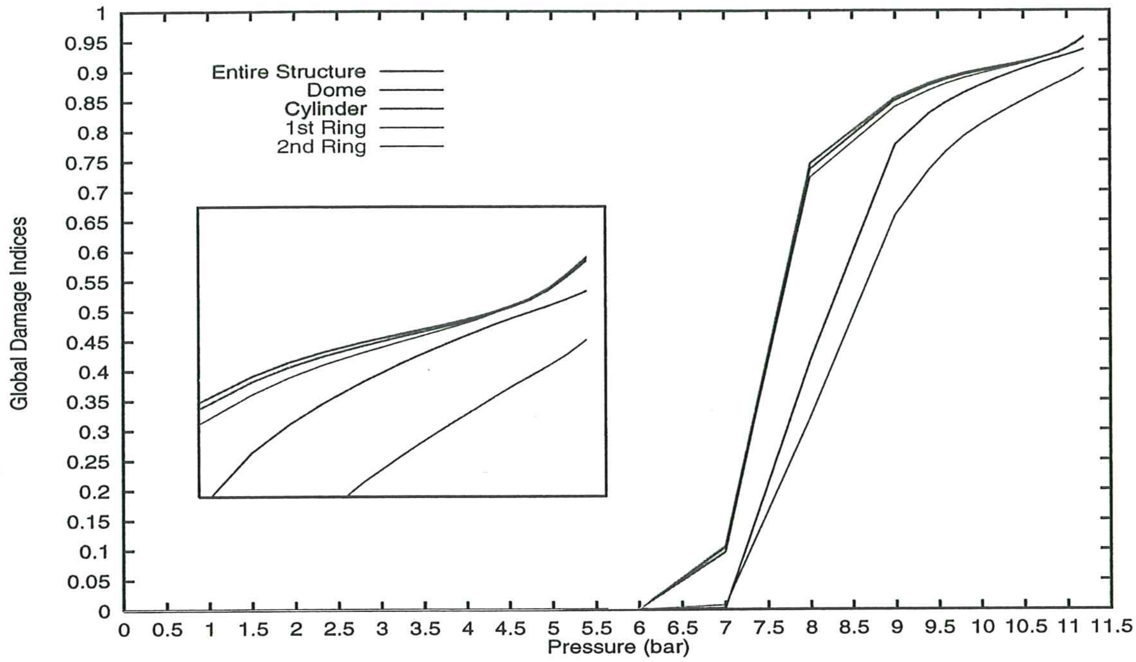

Fig. 5: Model without slab: Global Damage Indices evolution.

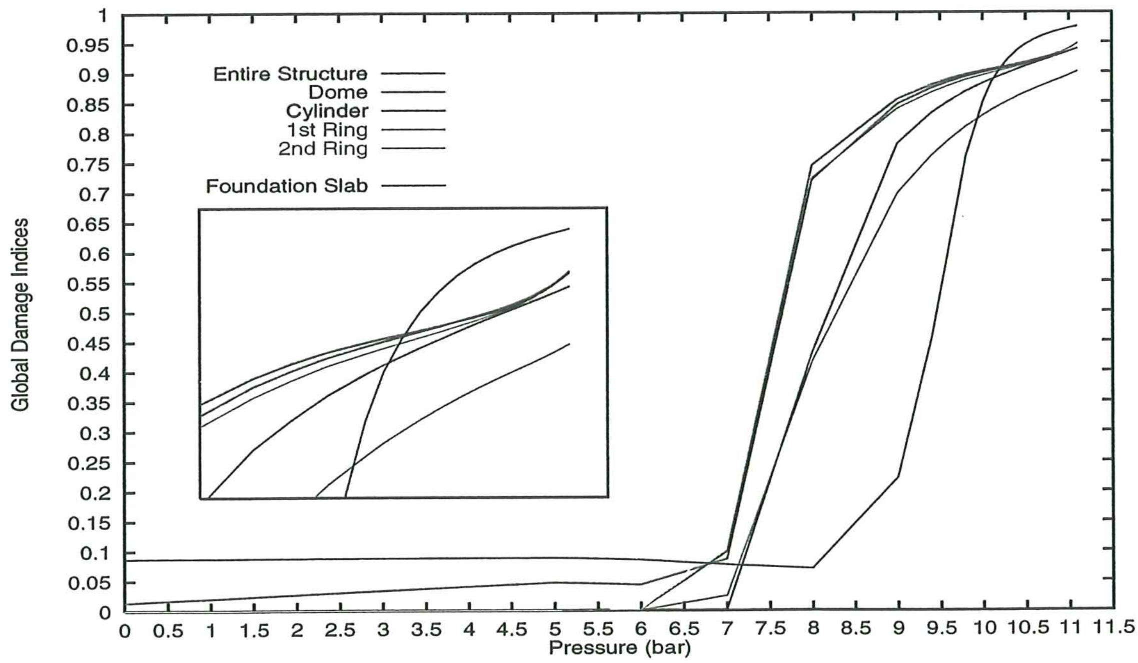

Fig. 6: Model with slab: Global Damage Indices evolution.

The first observation is that the GDIs have almost identical behaviours in both cases, which confirms that the presence of the foundation slab does not influence in the overall degradation patterns which develop at mid-cylinder. The overall GDI and the GDIs for cylinder, the 2nd and 3rd rings take very close values, which means the overall damage takes into account exclusively what happens in those rings and what happens measures which were developed in two stages: survey of the actual state of the building and numerical modelling in order to simulate its behaviour.

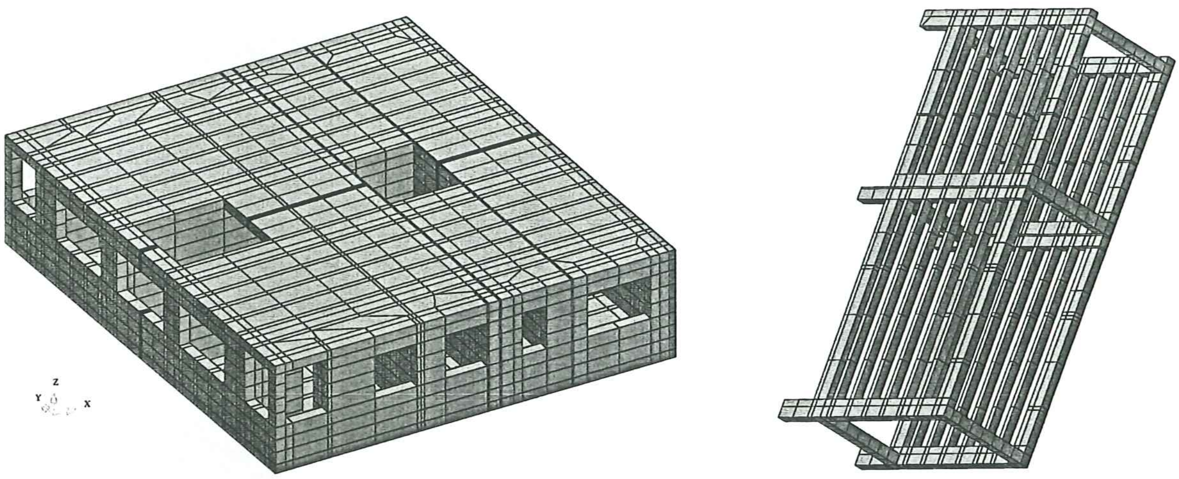

Fig. 6: 3rd floor mesh. Reinforcement bearing members.

Strategy of analysis

The applied load in the numerical simulation consisted in own weight plus an incremental pressure on both upper and lower floors so scaled so that nominal service load correspond to a load factor of 1.

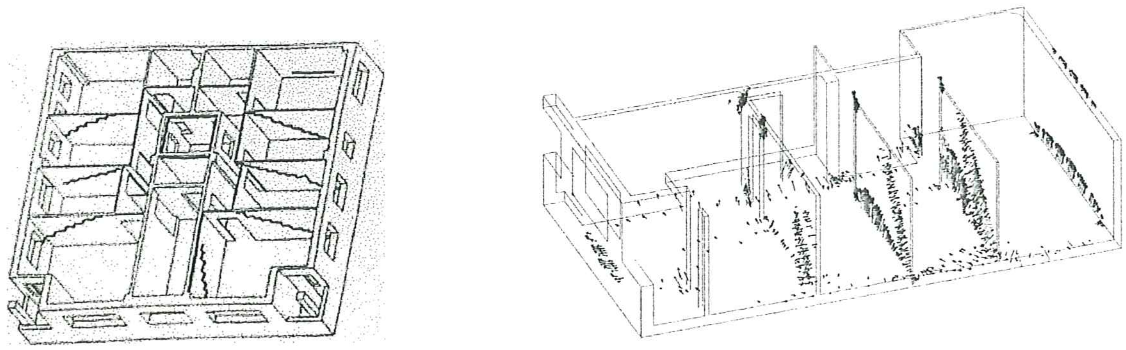

Fig. 7: Third floor cracks: (a) Visual survey; (b) Numerical Simulation.

Fig. 7 shows the cracks observed on the 3rd storey during the survey and those same cracks as obtained from the analysis results. The most important cracks were detected in the partition walls and excellent overall correlation with the analysis was accomplished, with exactly the same localisations as in the real case, but for a load factor of 4. This suggests that the trussed joists in the floors are much more flexible than expected given that the partition walls are not supported by any column. However, as complete failure occurred for a load factor of 5.9, may be concluded that the actual safety factor is only 1.45 and that the actual state of the structure under service load is that corresponding to the computed configuration subjected to 4 times the service load.

In view of this surprising result, a verification campaign was initiated and it was discovered that the reinforcement bars of trussed joists and some of the beams showed important drift from design specifications about distance between bars that leads to important stiffness alterations.

Fig. 8 shows elemental damage distribution and confirms that the cracks appear in the most damaged zones. Fig. 9 presents the evolution of GDIs for the complete storey and its constitutive parts. The dominant GDI is that of the partition walls as

HISTORICAL CONSTRUCTIONS

Introduction

Nowadays there is a general concern for preservation of historical constructions [1-3]. It is obvious that as time goes by, the number of historical constructions increases. In particular, many existing bridges, towers and dams as well as some relevant buildings can now be considered in every respect historical constructions [3].

The philosophy of conservation has the goal to preserve the original architectural message of any monument. The first step to reach such objective is the accurate prediction of the actual safety level of the structure. This includes the knowledge of both the mechanism of deterioration of the material and of the structural components and the evolution of degradation with time. The causes of deterioration usually can be subdivided in two main groups : physical-chemical-biological aggressive agents and mechanical problems. The latter can be nowadays accurately studied by means of coupled numerical- experimental procedures, where the structural behaviour is analysed by advanced non-linear finite element models which are adequately calibrated using experimental data [5-28].

The objective of this section is to describe a methodology which can be effectively used for assessing the structural conditions and durability of historical masonry and concrete constructions. This includes the prediction of local and global behaviour up to structural failure.

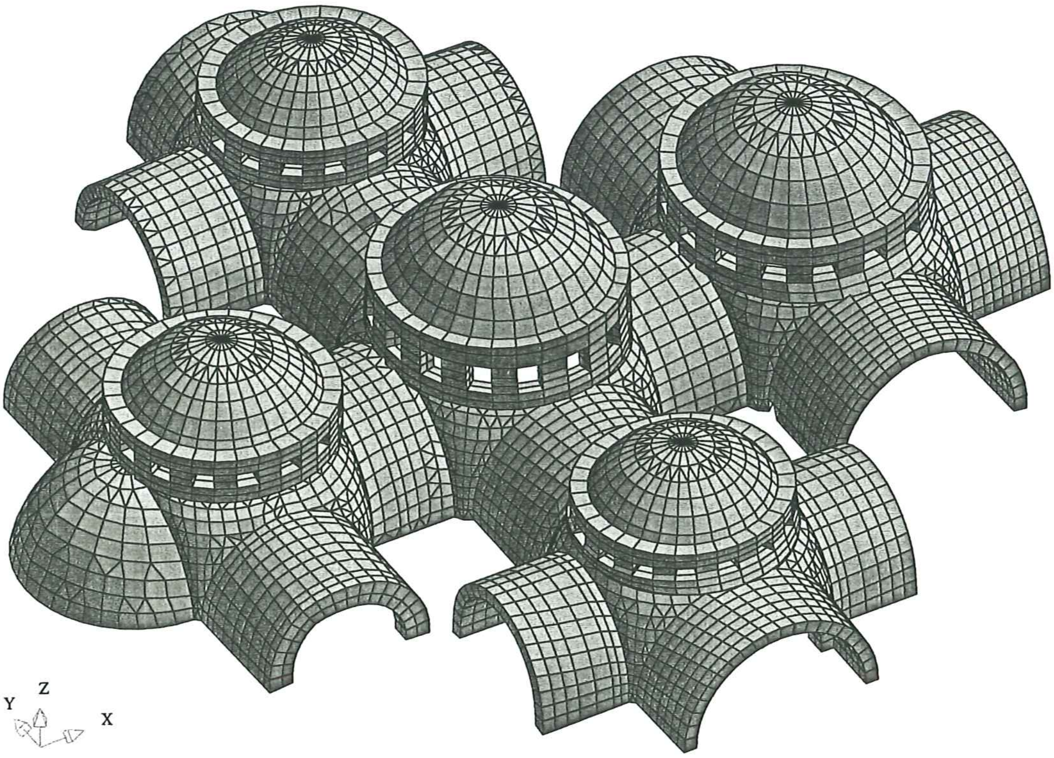

Fig. 10: FE mesh for the complete five-dome 3D analysis of St. Mark Basilica.

Description of the structure

Saint Mark Basilica is one of the most impressive cathedrals in the world and one of the oldest. The structure was erected between 1063 and 1073 but may not be considered concluded until the 15th century. Along that time work went on almost continously, in successive ages, and brought about a huge variety of materials and construction techniques left almost undocumented.

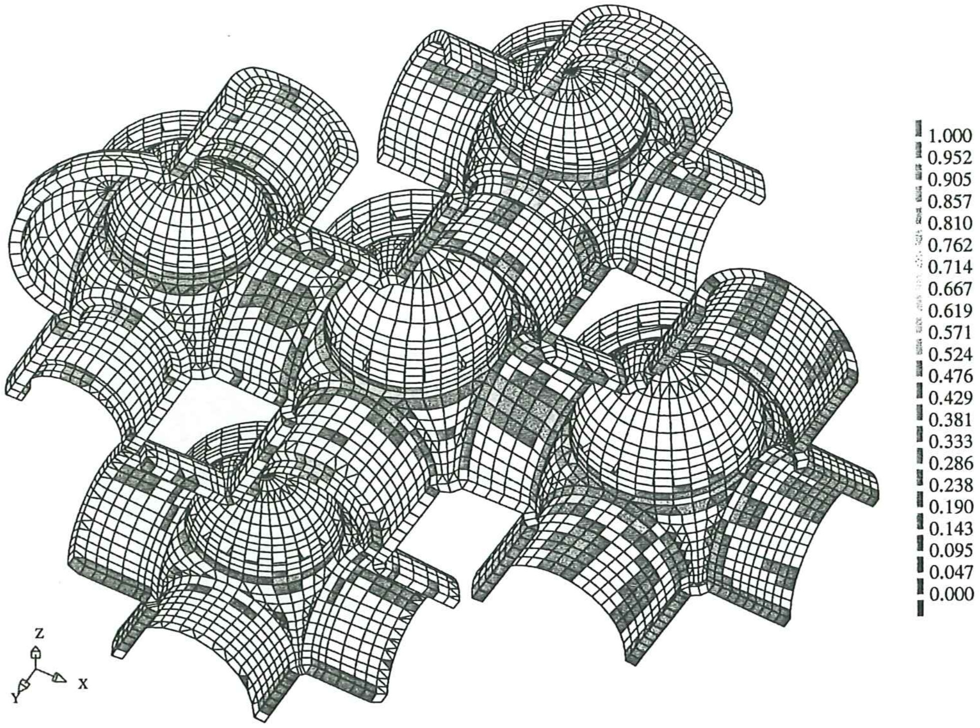

Fig. 12: Elemental damage map for a load factor of 4.

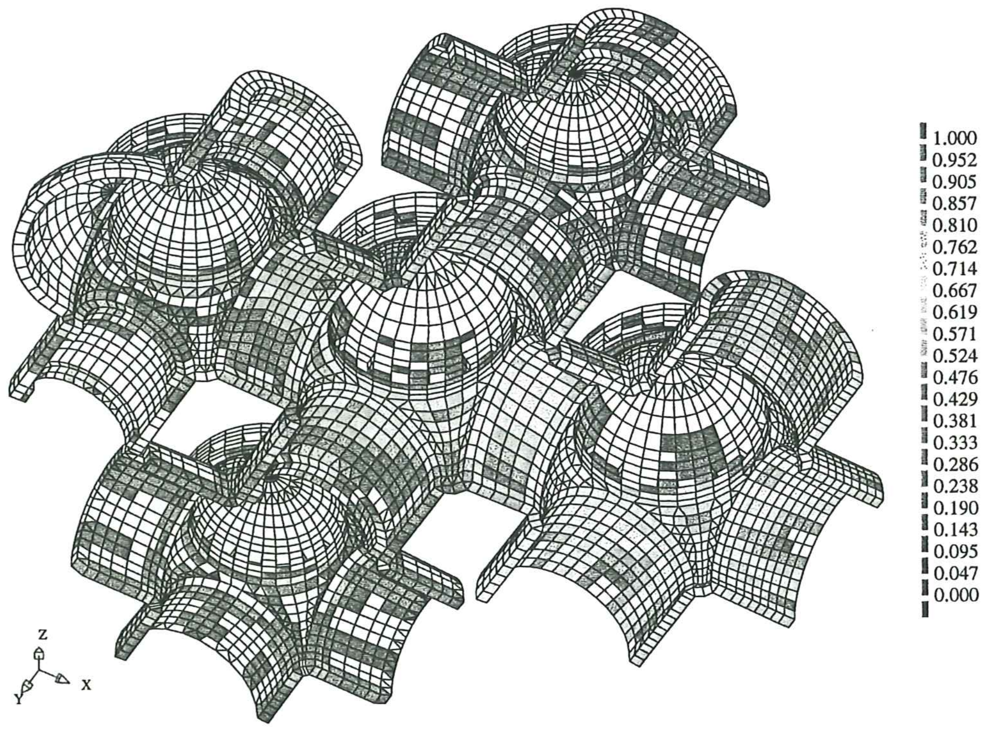

Fig. 13: Elemental damage map for a load factor of 6.45.

observed that first damage occurs at the base of the masonry walls and at the headstone of stone vaults which is exactly the surveyed damage.

Finally, the crack pattern for a load factor of 4 corresponds to the surveyed configuration. This leads to an actual safety factor value of 1.78. Cracks have been assumed to appear at each integration point in the orthogonal direction to the maximum principal strain. For comparison with reality, the size of each crack has been defined as proportional to the damage level at that point.

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In this paper a methodology for assessing the "reliability" of concrete structures (i.e. the study of local and global performance up to failure) is presented. The approach combines the use of a simple damage model to represent the non linear behaviour of concrete with the finite element method. The resulting methodology can be effectively tuned through experimental testing in order to estimate and monitor the structural health of existing structures and also as a useful computer-based tool for defining rehabilitation policies for damaged structures and for the design of new constructions.

The paper is organised as follows: In the next sections the theoretical bases of the damage model are given. The finite element implementation is briefly outlined and the concept of the global damage parameter is presented. Examples of application of the model to the non linear analysis of a nuclear containment shell, a plant of a block of flats and, finally, the domes of Saint Mark Basilica are finally presented.

THE CONCEPT OF DAMAGE

It is now well accepted the non linear behaviour of concrete and masonry can be modelled using concepts of damage theory only [10, 12, 13, 17-21] provided an adequate damage function is defined for taking into account the different response of concrete under tension and compression states. Cracking can, therefore, be interpreted as a local damage effect, defined by the evolution of known material parameters and by one or several functions which control the onset and evolution of damage. One of the advantages of such a model is the independence of the analysis with respect to cracking directions which can be simply identified *a posteriori* once the non-linear solution is obtained [10,12,13].

In this paper a model developed in recent years by the authors group [10-16,22-28] for non-linear analysis of concrete based on the concepts of *damage* mentioned above is extended for structural analysis of historical constructions. The model takes into account all the important aspects which should be considered in the non-linear analysis of concrete and masonry structures such as the different response under tension and compression, the effect of stiffness degradation due to mechanical and physical effects and the problem of objectivity of the results with respect to the finite element mesh.

SCALAR DAMAGE MODEL

In order to clarify the concept of damage consider a surface element in a damaged material volume. This surface has an area large enough to contain a representative number of defects, but still enabling to be referred as pertaining to a particular material point. Thus, if S_n denotes the overall section and $\bar{S}_n$ the effective resisting area ($S_n - \bar{S}_n$ is the area occupied by the voids), the *damage variable* d_n associated to this surface is

$$d_n = \frac{S_n - \bar{S}_n}{S_n} = 1 - \frac{\bar{S}_n}{S_n} \tag{1}$$

Clearly, d_n represents the surface density of material defects and it will have a zero value when the material is in the undamaged virgin state. Conversely, the reduction of the effective resisting area will lead to an increase of damage until rupture defined by some critical value of d_n (bounded by the unreachable value of $d_n = 1$). Note that this is a directional definition of damage. In many cases a single scalar representation of damage is adopted (i.e. $d_n = d$) which suffices to ensure realistic material model

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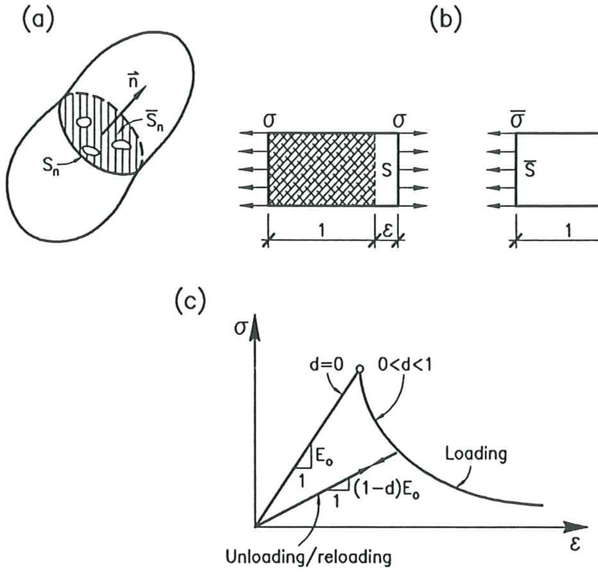

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$$\tau = \left(\theta + \frac{1-\theta}{n}\right)\left[\bar{\sigma}^T D^{-1} \bar{\sigma}\right]^{1/2} \tag{5}$$

where $n = f'_c/f'_t$ is the ratio between the compression and tension limit strengths,

$$\theta = \frac{\sum_{i=1}^{3} <\bar{\sigma}_i>}{\sum_{i=1}^{3} |\bar{\sigma}_i|} \quad \text{with} <\pm\bar{\sigma}_i>= \frac{1}{2}(|\sigma_i| \pm \sigma_i) \tag{6}$$

Expression (6) accounts for the different limit behaviour of the material under compression and tension states.

b) A damage criterion formulated in the strain of the undamaged stress spaces. The simplest form of this can be written as

$$F(\tau, r) = \tau - r \leq 0 \tag{7}$$

where τ is the norm defined in (5) and r is the damage threshold value. Damage grows when the norm τ exceeds the current threshold value. In particular, damage is initiated when τ exceeds for the first time the value r° (typically $r^\circ = f'_t/\sqrt{E}$ is taken [10,13]). Fig. 2 shows the form of the limit surface $\tau^\circ - r^\circ$ defining the onset of damage for the expression of τ given by eq.(5).

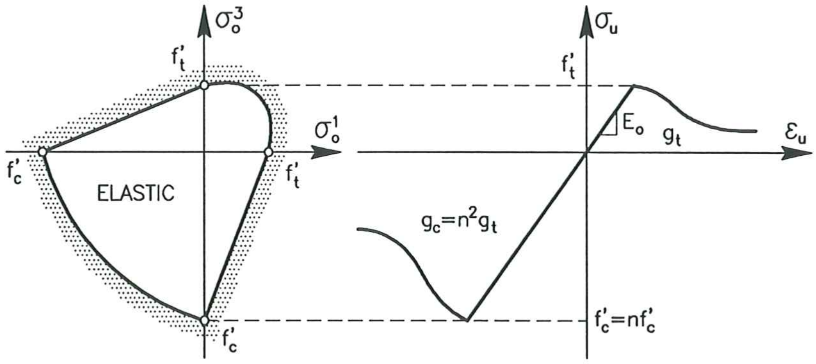

Fig. 2: Limit damage surface and uniaxial stress-strain curve for the model of eq. (5)

c)Evolution laws for the damage variable d and the damage threshold value r. These can be written as [28], [31].

$$d = G(r) \quad , \quad r = \max\{r^\circ, \tau\} \tag{8}$$

where G is a suitable monotonic scalar function taken as

$$G(r) = 1 - \frac{r^\circ}{r} \exp\left\{A\left(1 - \frac{r}{r^\circ}\right)\right\} \tag{9}$$

Note that $G(r^\circ) = 0$ and $G(\infty) = 1$ as expected. The parameter A is determined from the energy dissipated in an uniaxial tension test as [10,23]

$$\frac{1}{A} = \frac{g_f E}{(f'_t)^2} - \frac{1}{2} \quad \text{or, alternatively} \quad \frac{1}{A} = \frac{g'_f E}{(f'_t)^2} \tag{10}$$

where $g_f = G_f/l^*$, G_f being the specific fracture energy per unit area (taken as a material property), l^* is the characteristic length of the fractured domain. As always

$g_f \geq (f'_t)^2/2E$ (the material must dissipate at least the energy stored when the elastic limit is reached), parameter A must be positive [28]. Defining $g'_f = g_f - (f'_t)^2/2E$, the second expression in (10) is obtained.

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Compute displacement increment

$$\Delta \boldsymbol{a}_i^n = -[\boldsymbol{H}_i^n]^{-1} \boldsymbol{\Psi}_i^n$$

$\boldsymbol{\Psi}_i^n$: residual force vector $(= \int_V \boldsymbol{B}^{\mathrm{T}} \boldsymbol{\sigma}\, dV - \boldsymbol{f})$

$\boldsymbol{H}_i^n$: iteration matrix (i.e. tangent stiffness matrix)

Update displacements and strains

$$\boldsymbol{a}_{i+1}^n = \boldsymbol{a}_i^n + \Delta \boldsymbol{a}_i^n, \qquad \boldsymbol{\varepsilon}_{i+1}^n = \boldsymbol{\varepsilon}_i^n + \boldsymbol{B} \Delta \boldsymbol{a}_i^n$$

Evaluate stresses

(1) Compute undamaged stresses: $\bar{\boldsymbol{\sigma}}_{i+1}^n = \boldsymbol{D} \boldsymbol{\varepsilon}_{i+1}^n$

(2) Evaluate r_{i+1}^n using Eq.(5): $r_{i+1}^n = \tau_{i+1}^n$

(3) Update r and d

$$r_{i+1}^n = \max(r^{n-1}, r_{i+1}^n) \,, \qquad d_{i+1}^n = G(r_{i+1}^n)$$

(4) Update stresses: $\boldsymbol{\sigma}_{i+1}^n = (1 - d_{i+1}^n) \bar{\boldsymbol{\sigma}}_{i+1}^n$

Evaluate residual force vector: $\boldsymbol{\Psi}_{i+1}^n$

Check convergence: $\|\boldsymbol{\Psi}_{i+1}^n\| \leq \epsilon \|\boldsymbol{f}\|$?

No: Continue iterations: $i = i + 1$

Yes: $\boldsymbol{a}^n = \boldsymbol{a}_{i+1}^n$, next load increment: $n = n + 1$

Box 1: Quasi-static non linear finite element solution using the damage model

THE CONCEPT OF GLOBAL DAMAGE

The global strength of the structure can be assessed by means of a *global damage* index D. The definition for D in energy terms [28] is the following

$$D = 1 - \frac{\bar{U}}{U} \tag{11}$$

where $\bar{U}$ and U are the internal energies corresponding to the damaged and undamaged states, i.e.

$$\bar{U} = \boldsymbol{a}^T \int_V \boldsymbol{B}^T \boldsymbol{\sigma} dV = \boldsymbol{a}^T \int_V \boldsymbol{B}^T (1-d)\bar{\boldsymbol{\sigma}} dV$$

$$U = \boldsymbol{a}^T \int_V \boldsymbol{B}^T \bar{\boldsymbol{\sigma}} dV \tag{12}$$

In (12) the total energy of the structure is obtained by sum of the element contributions in the standard manner.

Note that global structural failure corresponds to a value of D approaching unity. Thus, the computation of the local and global damage indices provides a useful tool for monitoring in detail the evolution of the non linear response of the structure up to failure.

FAILURE PRESSURE EVALUATION OF THE CONTAINMENT BUILDING OF A LARGE DRY NUCLEAR POWER PLANT

Introduction

The evaluation of the failure pressure of the containment building of a large dry PWR-W three loops nuclear power plant is described in this section. The method considers fully tridimensional finite element models in order to take into account the effect of the most significant structural characteristics (presence of three buttresses, penetrations, additional reinforcement around the penetrations, etc.), the lack of symmetry of the forces generated by the prestressing system, as well as the nonlinear behaviour of the materials and the sensitivity of the results to uncertainties associated to several material parameters.

Description of the structure

The reinforced concrete containment building which hosts the reactor core and its cooling system consists of a massive foundation slab and a vertical cylindrical wall closed on the upper part by a hemispherical dome. The structure has an additional prestressing system for the wall and the dome consisting of non-adherent tendons and its interior is protected with a steel liner having a sealing role. Fig. 3 shows vertical and horizontal cross sections of the structure, including the main geometrical parameters.

There are three vertical buttresses on the outer side of the cylindrical wall spaced at 120°, which serve as support for the horizontal prestressing system. The penetrations in the cylindrical walls having a major impact (being modelled therefore) on the structural behaviour are: the personnel airlock, the equipment hatch, the emergency airlock, the main steam penetration, the fuel transfer penetration and the purge line penetration.

The prestressing system is shown in Fig.3. There are 132 horizontal tendons, comprising an angle of 240° each, anchored in the 3 buttresses and 80 vertical tendons in 2

families (N-S, E-W) anchored in a perimetrical gallery located in the lower part of the foundation slab.

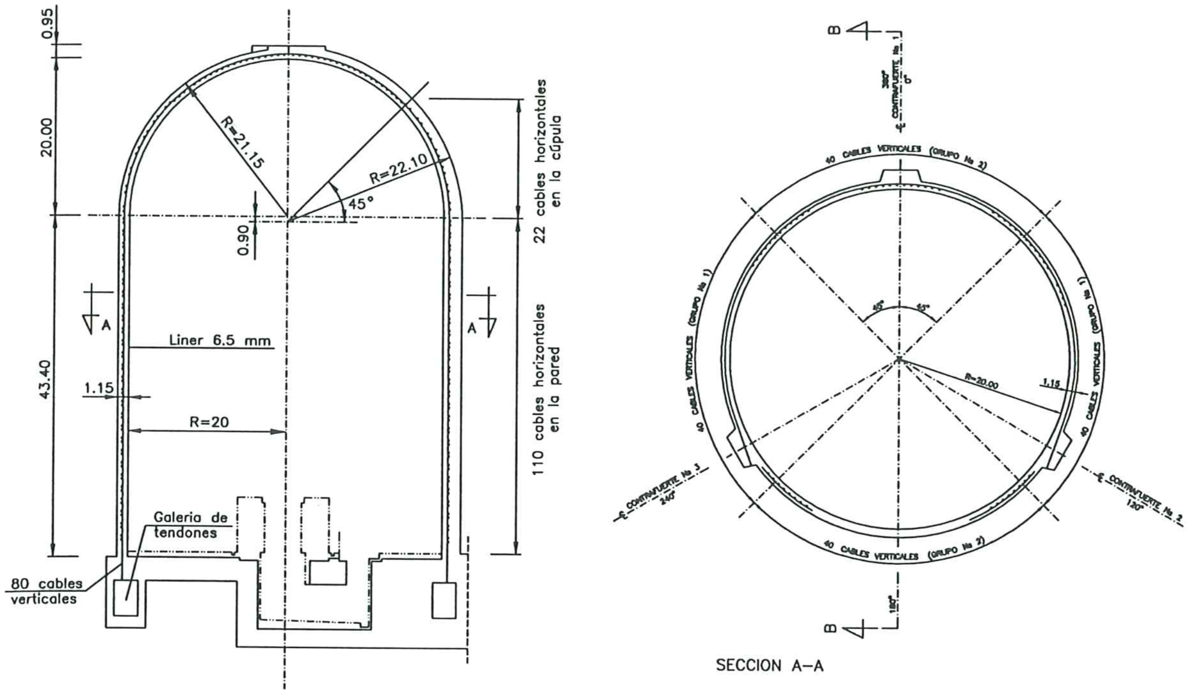

Fig. 3: Containment shell. (a) vertical section; (b) horizontal section.

Strategy of analysis

The failure pressure is defined as the inner pressure corresponding to the structural material exhaustion, that is, to a certain strain limit of the reinforcement steel, prestressing tendons and liner. The failure criterion assumes that local steel rupture occurs when the mentioned strain limit reaches 0.8% for the reinforcement and 1% for the tendons. The straining up to failure limit of the reinforcement is made possible by the damage-induced stress loss in concrete leading to stress redistribution towards the steel components. The global damage indices describe the state of entire structural parts, summing up both concrete and steel data.

The loads considered in the analysis were the self-weight, the external pressures generated by the prestressing system and the internal pressure corresponding to a specified accident. The distribution of the pressures equivalent to those produced by the prestressing system has been evaluated analytically for all the nodes of the mesh. All the possible sources of prestressing losses have been included in this evaluation, i.e. friction, wobble, anchor set, instantaneous and long term, etc. The internal pressure was incremented gradually until the structural collapse occurred.

Failure pressure evaluation

The influence of including the foundation slab in the structural model on the global structural behaviour and especially on the failure pressure was first examined. The results show that the influence of including the slab is quite small for low levels of internal pressure; it decreases further as the pressure increases and it is negligible near the failure pressure, which is 1.11 MPa in both cases. Furthermore, the cylindrical wall behaves better when the slab is present, due to the fact that the displacements of the slab slightly reduce the circumferential displacements of the wall; this allows to conclude that by not including the slab, one stays on the safety side during the complete load history. The comparison was based on an extensive survey of displacements, cracking

patterns and reinforcement stresses along the load path.

A comparison was made of the radial displacement-pressure curves for the models with and without foundation slab, corresponding to the same point of the structure at the cylinder mid-height, where maximum displacement occurs. Slightly different responses were obtained for pressures over 0.7 MPa, due to the fact that cracking appears at the slab-wall junction, thus softening the wall clamping effect in the model which includes the slab. This difference does not affect the failure pressure, and changes only lightly failure displacements.

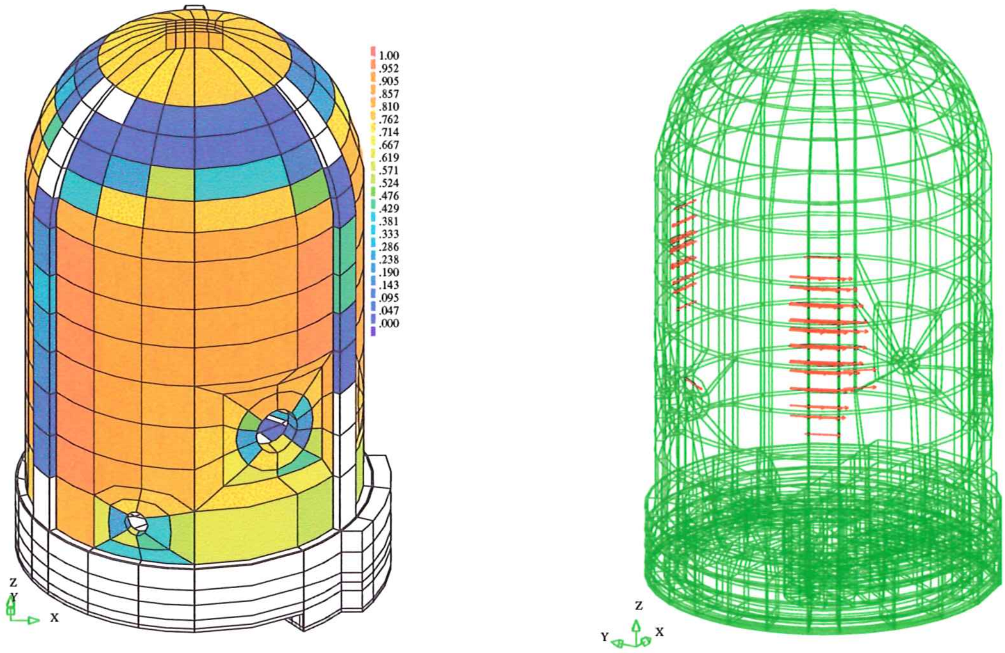

Fig. 4: Structural failure: (a) Damage distribution. (b) Broken steel bars.

Fig. 4 shows results of a typical simulation of the behaviour of the structure under increasing internal pressure until failure. Fig. 4 (a) shows a map of elemental damage of the structure, while Fig. 4 (b) shows the broken reinforcement bars at the moment of failure, corresponding to strains higher than 0.8%.

The model used in the analysis demonstrates an important capacity of localising the deformation when the damage sets in. Once cracking diminishes the stiffness of concrete, the reinforcement remains the only element to withstand the pressure. This reduces heavily the impact of the concrete and therefore of its constitutive behaviour on the failure pressure of the containment and suggests that its complexity may be kept at a minimum.

The analysis on the effect of the foundation slab on the overall behaviour of the structure is completed with the *global damage index* (GDI) study. This consists in examining the evolution of several critical structural parts throughout the loading process. This GDI evaluation is applied to the whole structure and to parts of it. It is obvious that no algebraic relation may be driven between the GDI of a structure and the GDIs of its parts. The smallest entity on which a GDI may be calculated is one finite element.

A GDI value has the significance of the ratio between the potential energy the structure cannot undertake as a result of damage and the potential energy the structure would store had it stayed undamaged.

Fig. 5 (model without slab) and Fig. 6 (model with slab) present the evolution of several GDIs belonging to the most representative (from a failure pressure point of view) zones of the structure. Given that the failure occurs at mid-cylinder, the cylinder was divided in three disjoint rings of finite elements, thus: the first ring is made of the inferior row of elements which join the slab, the second ring contains the following three rows and the last three rows ending where the dome begins belong to the third ring. Separately, GDIs for the cylinder as a whole, the dome, the slab and the entire structure were also calculated. In both figures, zooms of the final instants are given in order to show in detail what happens just before structural failure.

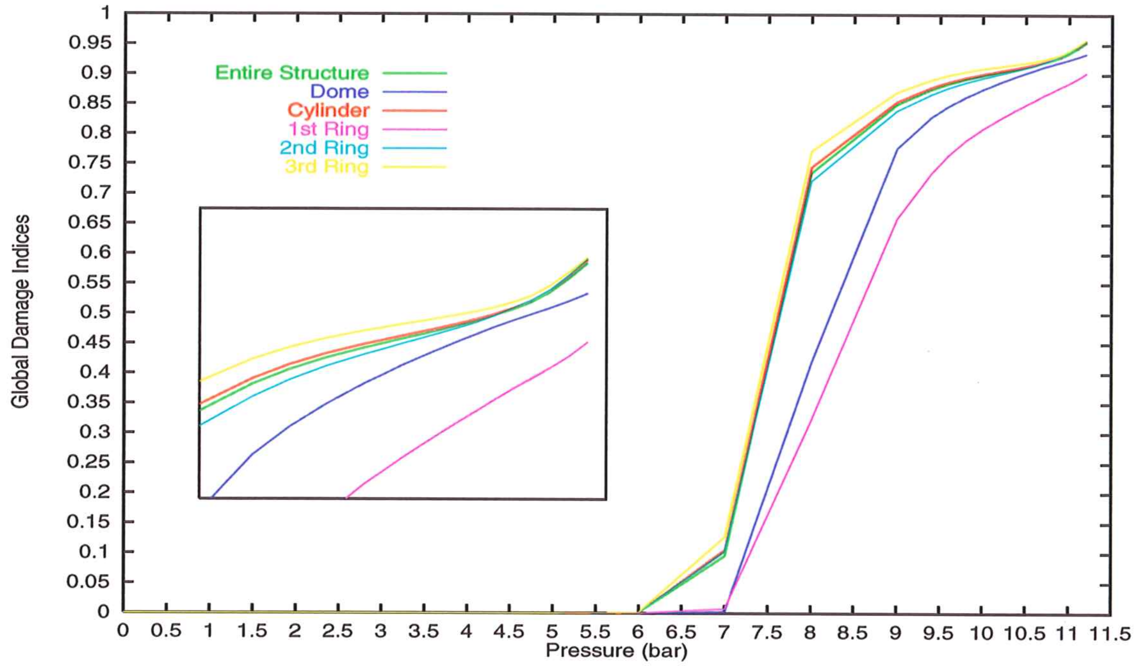

Fig. 5: Model without slab: Global Damage Indices evolution.

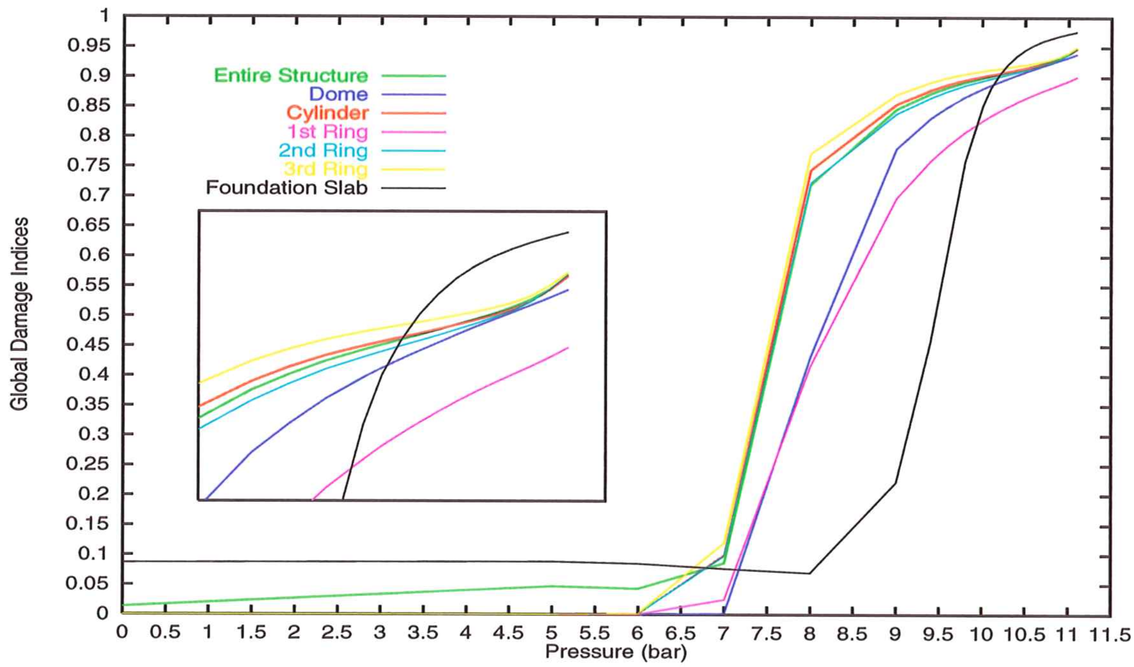

Fig. 6: Model with slab: Global Damage Indices evolution.

The first observation is that the GDIs have almost identical behaviours in both cases, which confirms that the presence of the foundation slab does not influence in the overall degradation patterns which develop at mid-cylinder. The overall GDI and the GDIs for cylinder, the 2nd and 3rd rings take very close values, which means the overall damage takes into account exclusively what happens in those rings and what happens with the rest of the structure has little relevance. The final proof is that although the slab GDI displays important variations and finally takes values well above the overall GDI, this last is never influenced by the state of the slab and the driving influence keeps being that of the above-mentioned rings. Little effect have the states of the 1st ring or the dome, while the cylinder GDI at his turn behaves like the overall GDI.

This behaviour is in line with the known properties of this GDI method [28], to "filter out" the irrelevant parts of the structure and to identify and follow the evolution of its critical zones. The fact that the overall GDI reaches in both cases values close to the unit show that the structure really fails when the pressure reaches 11.1 bars.

Conclusions

This section describes the study of the failure pressure for the containment building of a large dry nuclear power plant, for an accident scenario beyond the design one. The method used in the evaluation of the failure pressure is based on the simulation of the complete damage process of the containment building, by means of a fully tridimensional nonlinear finite element model using a damage model. The model includes the most relevant structural aspects required in an accurate numerical simulation. The results obtained demonstrate that the influence of the foundation slab can be safely neglected in the failure pressure analysis of a structure of this type. The failure mechanism corresponds to the failure of the circumferential reinforcement bars of the mid-cylinder ring of the wall, leading to a wide vertical crack in the structure near to one of the buttresses. A safety coefficient of the structure related to the design pressure of 2.78 is obtained.

PATHOLOGY OF HOUSING BUILDINGS

Introduction

Housing buildings often display structural problems after completion when, due to constructive vices, exceptional loads like earthquakes or later accidents like ground movements, these are rendered unserviceable and rehabilitation decisions need be taken.

The methodology described herein proposes reconstructing through numerical analysis the surveyed damaged state of a structure and in this manner explain the underlying reasons of unaccounted-for structural behaviour while simultaneously quantifying them by means of GDIs. These indices signal the weaker zones and provide the measure of their experimented stiffness loss. When a configuration similar to the real state of the building is found, deductions can be made about the actual structural characteristics using similarity techniques.

Numerical simulations carried out with the damage model can provide assessments of the proposed repair works and help define the optimum intervention, being a valuable tool both for diagnosis and rehabilitation of buildings.

Description of the structure

The studied structure is a five-storey building with two symmetrical flats per storey. The 3rd storey presented extensive damage from unknown reasons and was therefore the object of detailed analysis. Its finite element mesh and reinforced concrete members are shown in Fig. 7.

The trouble with this building was that soon after completion and being already in use, fissures which soon became important cracks appeared. That fact imposed urgent measures which were developed in two stages: survey of the actual state of the building and numerical modelling in order to simulate its behaviour.

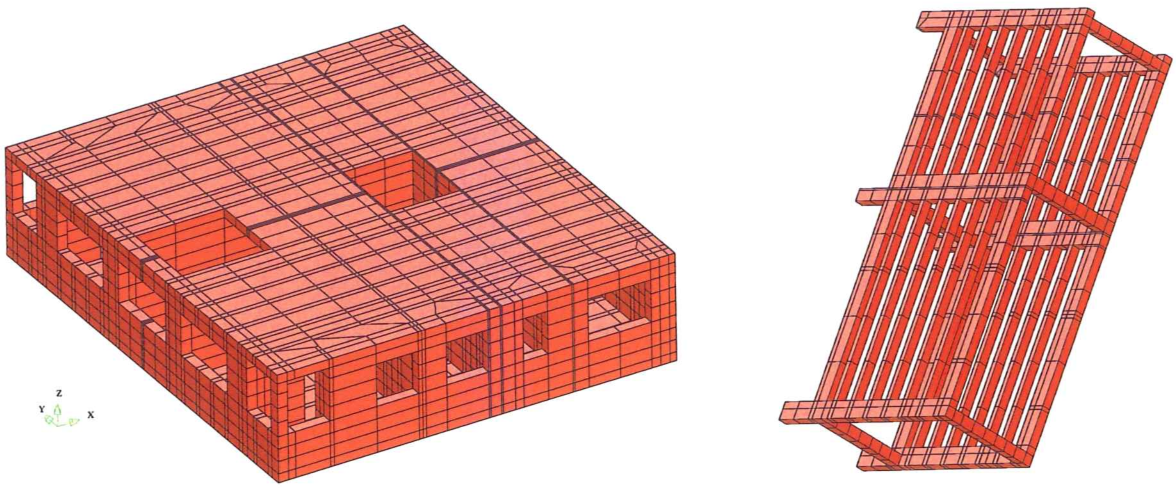

Fig. 6: 3rd floor mesh. Reinforcement bearing members.

Strategy of analysis

The applied load in the numerical simulation consisted in own weight plus an incremental pressure on both upper and lower floors so scaled so that nominal service load correspond to a load factor of 1.

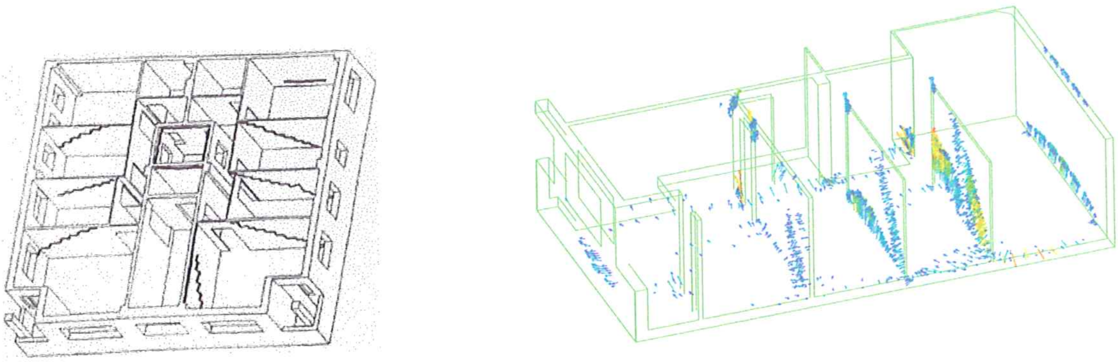

Fig. 7: Third floor cracks: (a) Visual survey; (b) Numerical Simulation.

Fig. 7 shows the cracks observed on the 3rd storey during the survey and those same cracks as obtained from the analysis results. The most important cracks were detected in the partition walls and excellent overall correlation with the analysis was accomplished, with exactly the same localisations as in the real case, but for a load factor of 4. This suggests that the trussed joists in the floors are much more flexible than expected given that the partition walls are not supported by any column. However, as complete failure occurred for a load factor of 5.9, may be concluded that the actual safety factor is only 1.45 and that the actual state of the structure under service load is that corresponding to the computed configuration subjected to 4 times the service load.

In view of this surprising result, a verification campaign was initiated and it was discovered that the reinforcement bars of trussed joists and some of the beams showed important drift from design specifications about distance between bars that leads to important stiffness alterations.

Fig. 8 shows elemental damage distribution and confirms that the cracks appear in the most damaged zones. Fig. 9 presents the evolution of GDIs for the complete storey and its constitutive parts. The dominant GDI is that of the partition walls as

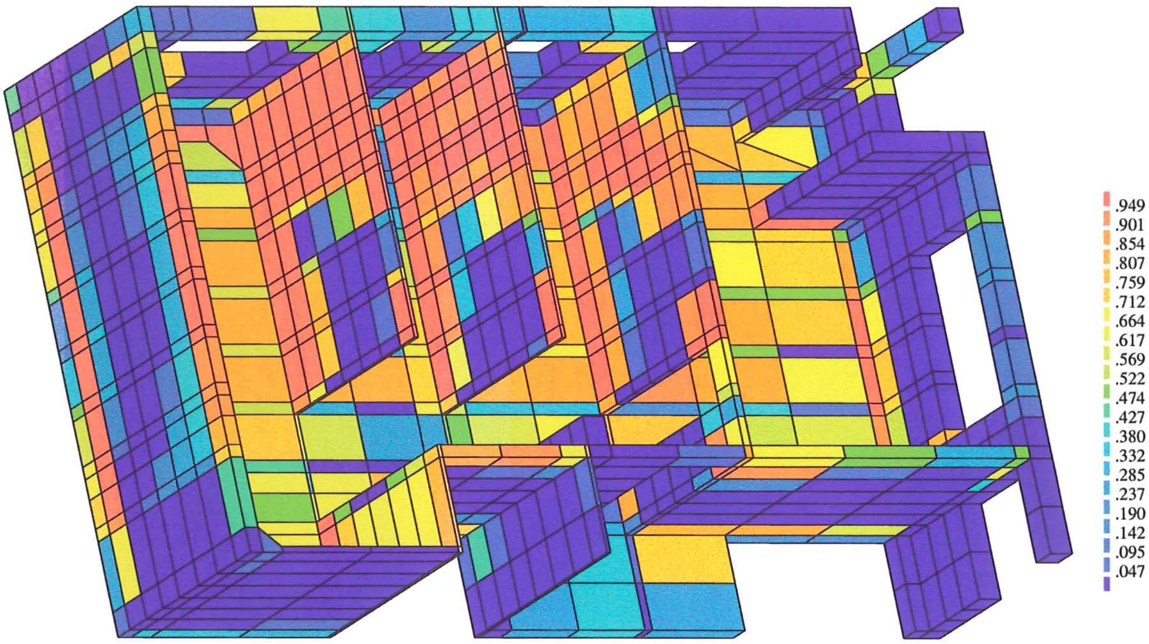

Fig. 8: Damage map at failure.

the overall GDI traces its trajectory since the beginning of the load history. The second most decisive GDI is that corresponding to the beam filling.

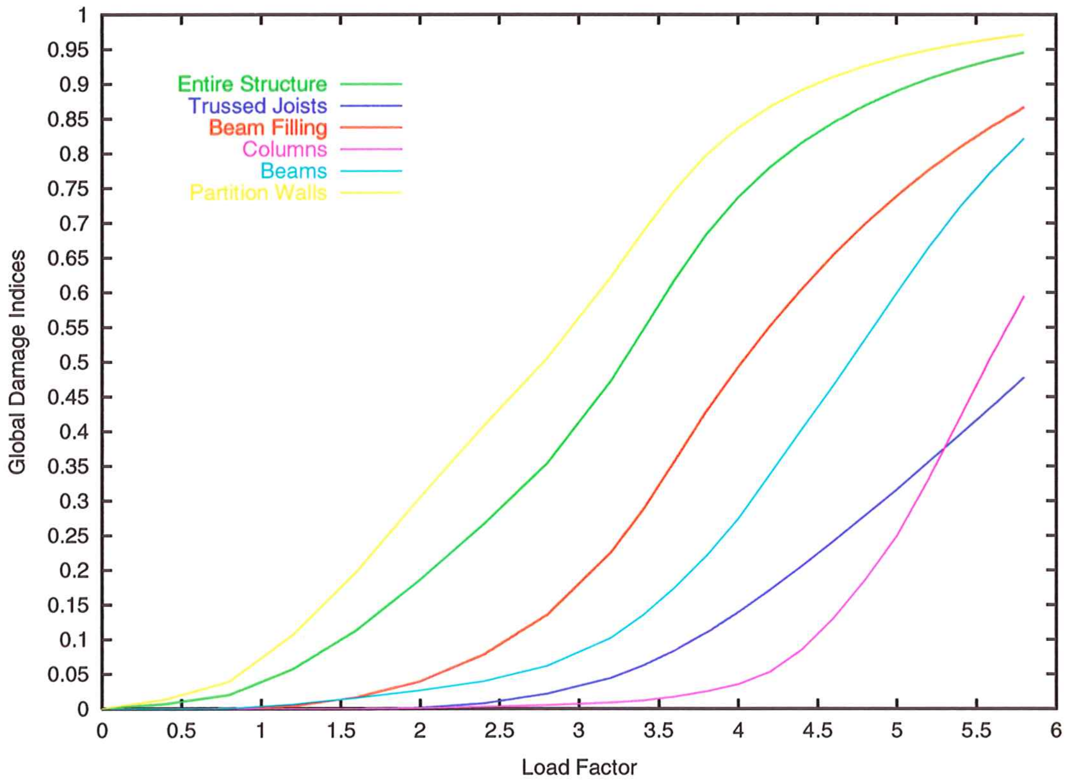

Fig. 9: Damage indices evolution.

This undue protagonist role of plain masonry parts suggests that the intended resistant members are failing their mission. The message this anomalous behaviour sends is that the serviceability of the structure depends on structural parts not supposed to play this role. Thus the Global Damage Index is proven to be a quite resourceful tool for structural health assessment.

HISTORICAL CONSTRUCTIONS

Introduction

Nowadays there is a general concern for preservation of historical constructions [1-3]. It is obvious that as time goes by, the number of historical constructions increases. In particular, many existing bridges, towers and dams as well as some relevant buildings can now be considered in every respect historical constructions [3].

The philosophy of conservation has the goal to preserve the original architectural message of any monument. The first step to reach such objective is the accurate prediction of the actual safety level of the structure. This includes the knowledge of both the mechanism of deterioration of the material and of the structural components and the evolution of degradation with time. The causes of deterioration usually can be subdivided in two main groups : physical-chemical-biological aggressive agents and mechanical problems. The latter can be nowadays accurately studied by means of coupled numerical- experimental procedures, where the structural behaviour is analysed by advanced non-linear finite element models which are adequately calibrated using experimental data [5-28].

The objective of this section is to describe a methodology which can be effectively used for assessing the structural conditions and durability of historical masonry and concrete constructions. This includes the prediction of local and global behaviour up to structural failure.

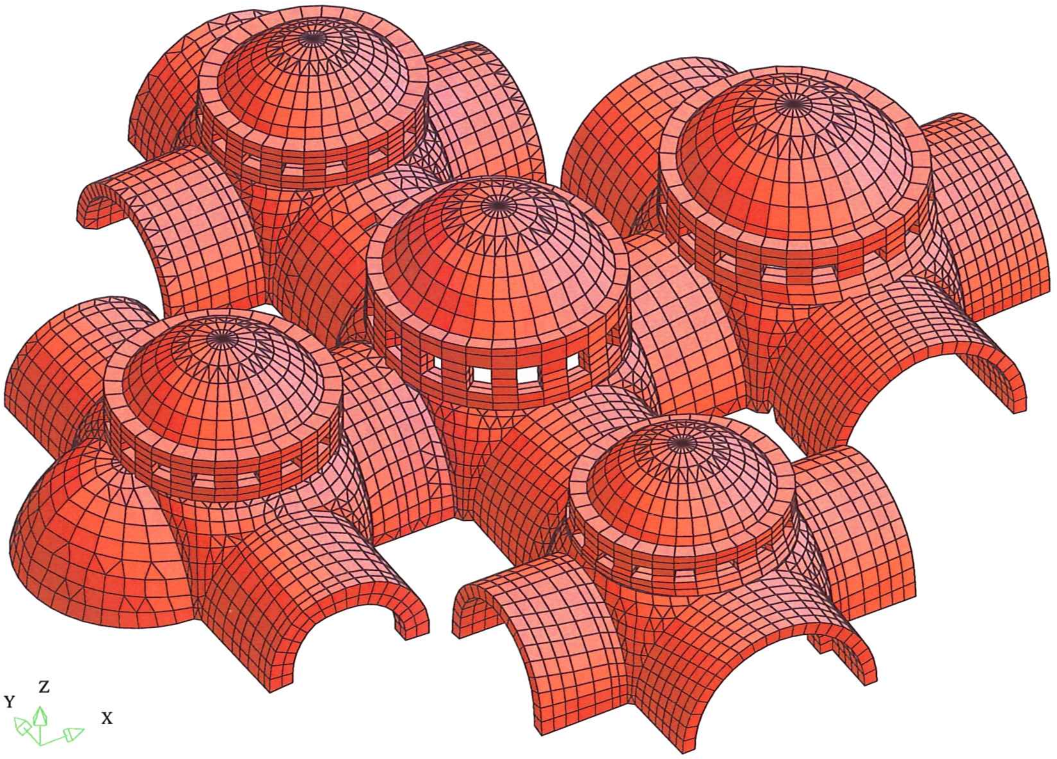

Fig. 10: FE mesh for the complete five-dome 3D analysis of St. Mark Basilica.

Description of the structure

Saint Mark Basilica is one of the most impressive cathedrals in the world and one of the oldest. The structure was erected between 1063 and 1073 but may not be considered concluded until the 15th century. Along that time work went on almost continously, in successive ages, and brought about a huge variety of materials and construction techniques left almost undocumented.

The cathedral has 5 stone domes covered inside by wood domes and outside by lead layers. This circumstance bothers any attempt to structural survey due to the historical value of covering ornaments which cannot be disturbed. In the few visitable zones alarming but not recent cracks have been observed [31]. They affect the lower part of the cylindrical towers which form the support structure of the domes and also the headstone region of the semicylindrical vaults which support the weight of the domes. Fig. 10 shows the mesh of 7676 20-node hexahedra involving 48505 nodes used for the analysis. 2265 15-node triangular prisms were also used as transition elements in some zones.

Strategy of analysis

The objective of the analysis was the mechanical safety assessment of the upper part of the Basilica using the damage model. A 3D static study of structural behaviour under dead load was performed using available material data and the conclusion was that the materials stay inside the elastic range of behaviour. As the real structure displays quite a lot of cracking it may only mean that the measured material properties were unreliable. Therefore, the study was directed toward the simulation of the observed damage by increasing self-weight under the assumption that an unknown decrease in material strength is equivalent to an increase in exterior load. This is true if all structural parts suffer similar environmental damage. As the analysis deals with only the upper part of the cathedral that enjoyed almost uniform atmospheric exposure, the above hypothesis was considered true for the scope of this study.

On this basis, the idea is that considering average material properties and increasing the dead load until failure, a damage record similar to the real one should be obtained. Comparing survey data with computational model data, the real material characteristics may be inferred using similitude techniques.

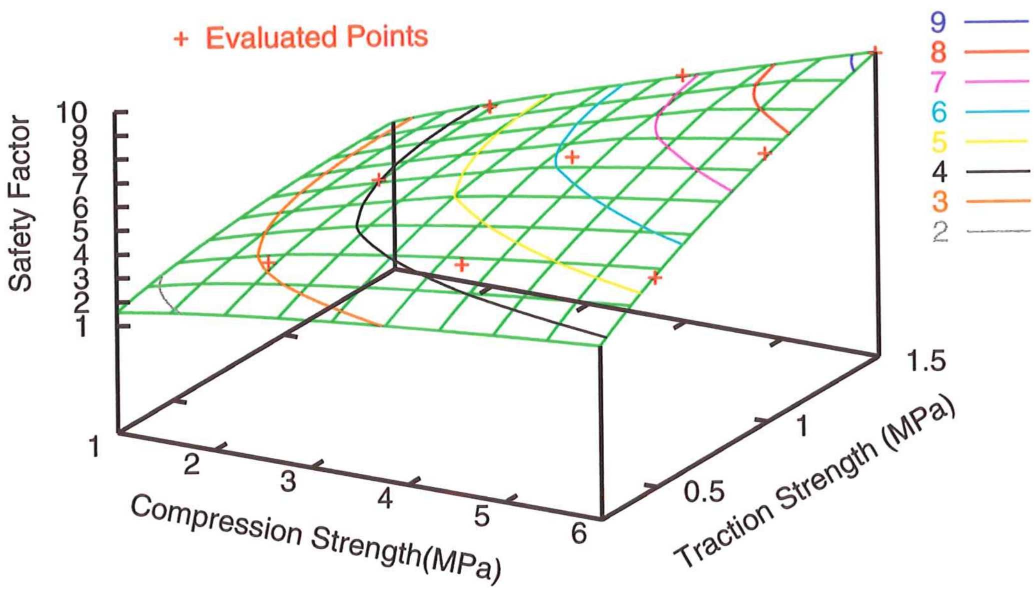

Fig. 11: Safety factor dependence on masonry strengths.

The analysed structure (Fig. 10) was supported on elastic bearings simulating the columns and walls that bear its weight. Fig. 11 presents the result of a parametric study on the influence of masonry characteristics upon the safety factor of the structure under dead load.

Figs. 12, 13 and 14 show maps of elemental global damage for load factors of 4, 6.45 and 7.01, representing damage history in its most representative phases. It may be

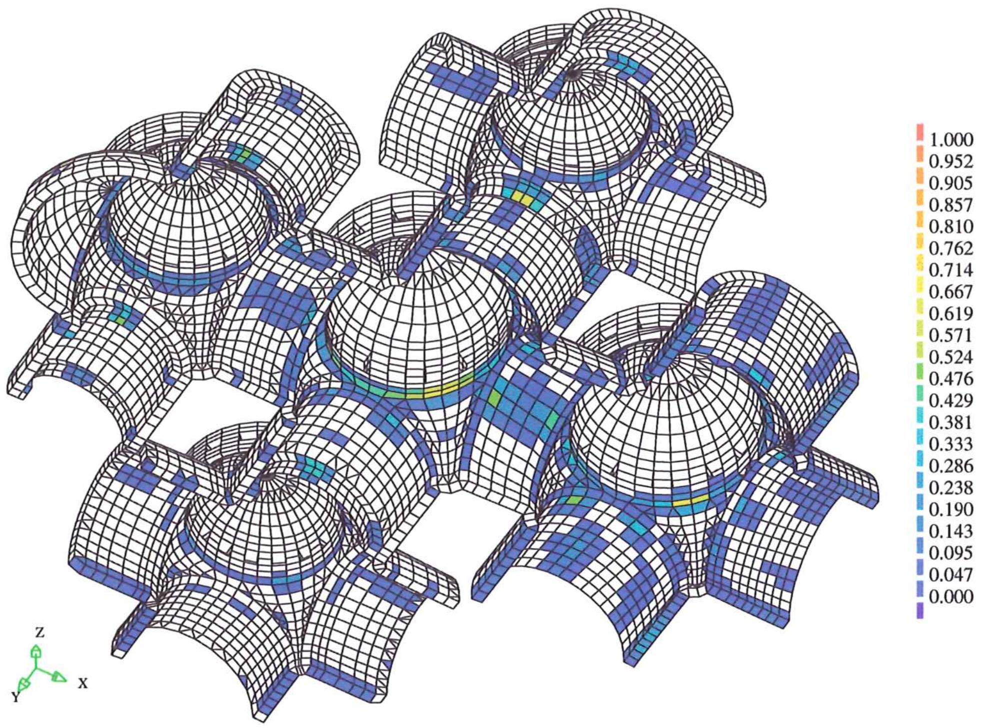

Fig. 12: Elemental damage map for a load factor of 4.

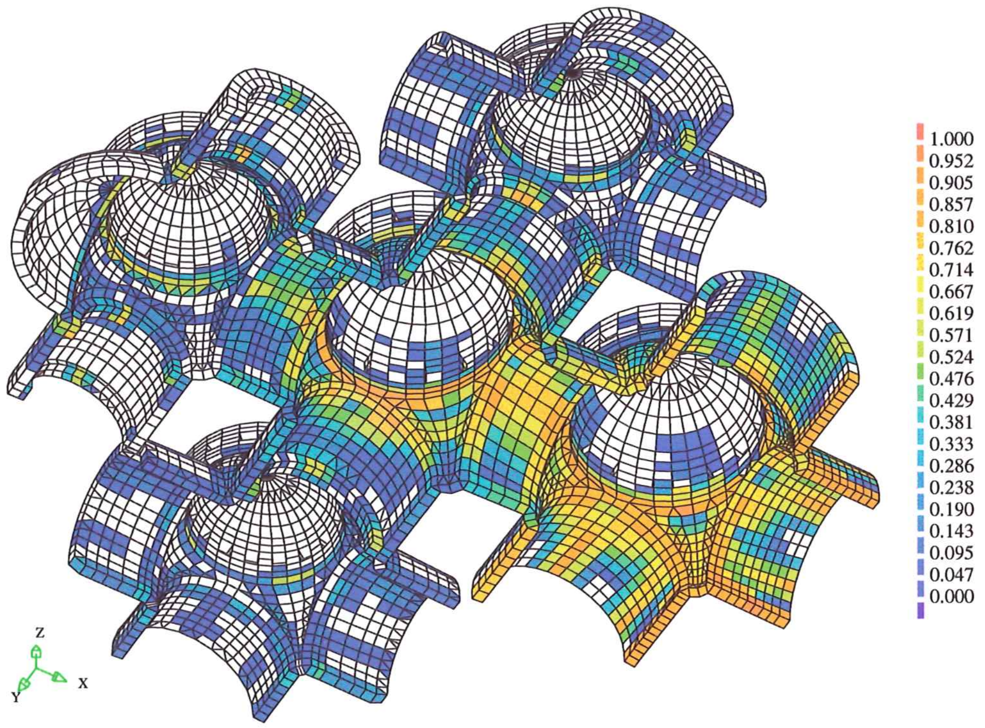

Fig. 13: Elemental damage map for a load factor of 6.45.

observed that first damage occurs at the base of the masonry walls and at the headstone of stone vaults which is exactly the surveyed damage.

Finally, the crack pattern for a load factor of 4 corresponds to the surveyed configuration. This leads to an actual safety factor value of 1.78. Cracks have been assumed to appear at each integration point in the orthogonal direction to the maximum principal strain. For comparison with reality, the size of each crack has been defined as proportional to the damage level at that point.

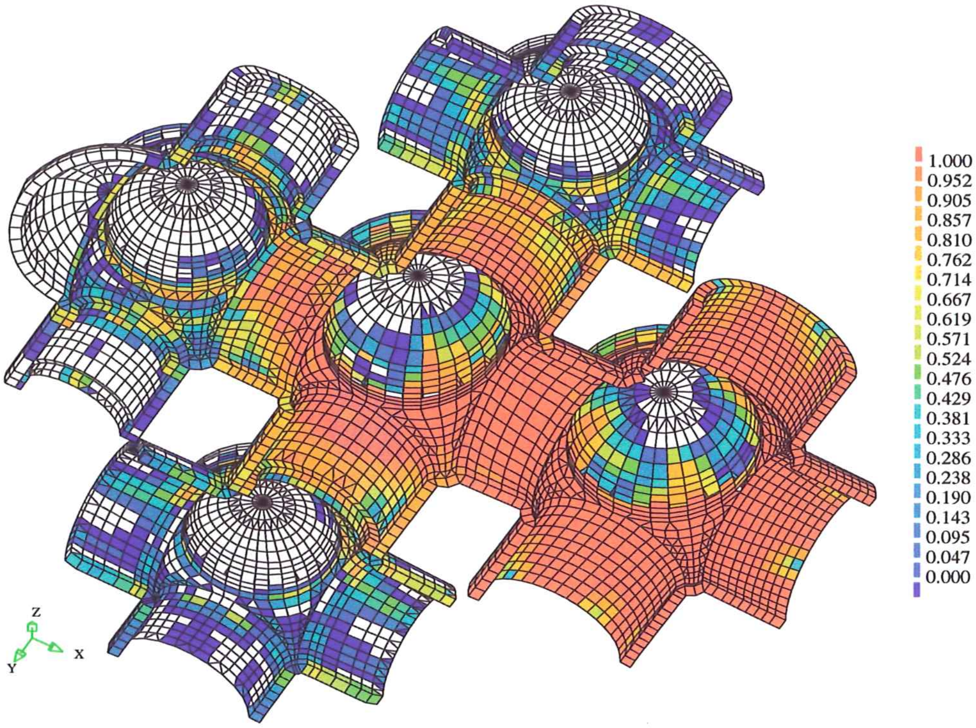

Fig. 14: Elemental damage map for a load factor of 7.11.

Conclusions

The mechanical damage methodology presented can be applied successfully to assess the structural conditions and estimate the safety level and durability of historical constructions. A full coupled solution taking into account physical-chemical-biological degradation and mechanical effects is nowadays possible and this will allow to trace the history of structural pathologies and to design the correct intervention measures.

Safety factors and unknown material characteristics may be inferred from simulations with the proposed methodology which opens new fields of study to civil engineering practitioners.

CONCLUSIONS

The method used in the evaluation of the failure is based on the simulation of the complete damage process of structures as a nuclear containment shell, a housing building and the domes of St.Mark Basilica.

An interesting property of the global damage index is that of allowing the decision of the state of the structure in what regards its failure mechanisms. The model permits the identification of the mechanism of collapse by observing the local damage indices and continuous comparison with the global one. When, during a damaging process, the global index gets governed by the damage index of a subpart and the rest of the parts of the structure stop influencing, the critical points of the structure have been isolated. The failure of these points leads to the formation of a failure mechanism, i.e. collapse of the structure. This is important from an engineering structural retrofitting point of view.

A particularly interesting immediate application of the damage model here proposed is the evaluation of *the structural strength increase* for different intervention measures, thus allowing to optimise any restoration investment.

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Prediction of Damage and Failure in Civil Engineering Structures using a Finite Element Model

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PREDICTION OF DAMAGE AND FAILURE IN CIVIL ENGINEERING STRUCTURES USING A FINITE ELEMENT MODEL

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SUMMARY: The paper describes a finite element damage model for non linear analysis of concrete or reinforced concrete structures. It is shown how the model can be effectively used to predict local and global damage up to structural failure. Examples of applications of the model to the analysis of different structures such as a nuclear containment shell, a housing building and the domes of St.Mark Basilica are presented.

KEYWORDS: local and global damage, damage indices, structural failure, non linear analysis, concrete, masonry, reinforced concrete, nuclear containment shells, buildings, historical constructions.

INTRODUCTION

The design of concrete structures requires an accurate evaluation of the structural response both at service and ultimate load levels. Traditional methods for structural analysis generally provide safe designs, but they frequently contain inherent inconsistencies and often do not reflect a clear understanding of the actual composite behaviour of the material. Present-day design codes continue, in many respects, to be based on empirical approaches and they rely heavily on the results of considerable amount of experimental data. This situation is largely attributable to the complex behaviour of concrete. Thus, concrete cracking, tension stiffening, non linear multi-axial material properties and complex interface behaviours were previously ignored or treated in a very approximate manner. Numerical methods, and particularly the finite element techniques, now permit a more rational analysis of these complexities, thus allowing the assessment of the "safety" of a concrete structure both at local and global levels.

In this paper a methodology for assessing the "reliability" of concrete structures (i.e. the study of local and global performance up to failure) is presented. The approach combines the use of a simple damage model to represent the non linear behaviour of concrete with the finite element method. The resulting methodology can be effectively tuned through experimental testing in order to estimate and monitor the structural health of existing structures and also as a useful computer-based tool for defining rehabilitation policies for damaged structures and for the design of new constructions.

The paper is organised as follows: In the next sections the theoretical bases of the damage model are given. The finite element implementation is briefly outlined and the concept of the global damage parameter is presented. Examples of application of the model to the non linear analysis of a nuclear containment shell, a plant of a block of flats and, finally, the domes of Saint Mark Basilica are finally presented.

THE CONCEPT OF DAMAGE

It is now well accepted the non linear behaviour of concrete and masonry can be modelled using concepts of damage theory only [10, 12, 13, 17-21] provided an adequate damage function is defined for taking into account the different response of concrete under tension and compression states. Cracking can, therefore, be interpreted as a local damage effect, defined by the evolution of known material parameters and by one or several functions which control the onset and evolution of damage. One of the advantages of such a model is the independence of the analysis with respect to cracking directions which can be simply identified *a posteriori* once the non-linear solution is obtained [10,12,13].

In this paper a model developed in recent years by the authors group [10-16,22-28] for non-linear analysis of concrete based on the concepts of *damage* mentioned above is extended for structural analysis of historical constructions. The model takes into account all the important aspects which should be considered in the non-linear analysis of concrete and masonry structures such as the different response under tension and compression, the effect of stiffness degradation due to mechanical and physical effects and the problem of objectivity of the results with respect to the finite element mesh.

SCALAR DAMAGE MODEL

In order to clarify the concept of damage consider a surface element in a damaged material volume. This surface has an area large enough to contain a representative number of defects, but still enabling to be referred as pertaining to a particular material point. Thus, if S_n denotes the overall section and $\bar{S}_n$ the effective resisting area ($S_n - \bar{S}_n$ is the area occupied by the voids), the *damage variable* d_n associated to this surface is

$$d_n = \frac{S_n - \bar{S}_n}{S_n} = 1 - \frac{\bar{S}_n}{S_n} \tag{1}$$

Clearly, d_n represents the surface density of material defects and it will have a zero value when the material is in the undamaged virgin state. Conversely, the reduction of the effective resisting area will lead to an increase of damage until rupture defined by some critical value of d_n (bounded by the unreachable value of $d_n = 1$). Note that this is a directional definition of damage. In many cases a single scalar representation of damage is adopted (i.e. $d_n = d$) which suffices to ensure realistic material model

[13,17-22]. It is worth noting, that in this case cracks at a microscopic point need not to have not particular direction and a macroscopic crack is then defined as the locus of damage points as previously mentioned.

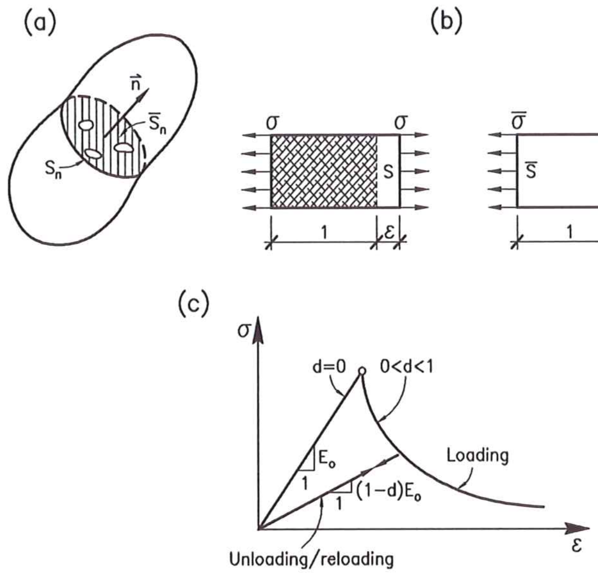

Fig. 1: (a) Damaged surface; (b) Cauchy stress σ and effective stress $\bar{\sigma}$; (c) Evolution of uniaxial stress-strain curve.

An useful concept for understanding the effect of damage is that of *effective stress.* The equilibrium relationship between the standard Cauchy stress σ and the "effective" stress, $\bar{\sigma}$, in the damaged bar specimen of Fig. 1 is

$$\sigma S = \bar{\sigma}\bar{S} \tag{2}$$

and from (1) and (2)

$$\sigma = (1-d)\bar{\sigma} = (1-d)E\varepsilon \tag{3}$$

When a damaging process is occurring, the external loading is resisted by the effective stress area and, therefore, $\bar{\sigma}$ is a more physically representative parameter than σ.

CONTINUUM DAMAGE MODEL FOR CONCRETE

In this work a single parameter damage model will be used. Examples of different tensor-valued models can be found in [19,20,22]. The constitutive equation will therefore be simply written in vector form as an extension of eq. (3) as

$$\boldsymbol{\sigma} = (1-d)\bar{\boldsymbol{\sigma}} = (1-d)\mathbf{D}\boldsymbol{\varepsilon} \tag{4}$$

where $\mathbf{D}$ is the elastic constitutive matrix and $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the standard stress and strain vectors. Fig. 1(c) shows the one dimensional representation of the stiffness evolution of the material.

The model defined by eq.(4) requires the knowledge of the damage variable d at every stage of the deformation process. For this purpose one must define:

a) A suitable scalar norm τ of the strain tensor (or alternatively of the undamaged stress tensor). Here, several possibilities exist and a suitable option for concrete and masonry is [10,23]

$$\tau = \left(\theta + \frac{1-\theta}{n}\right)\left[\bar{\boldsymbol{\sigma}}^T \boldsymbol{D}^{-1} \bar{\boldsymbol{\sigma}}\right]^{1/2} \tag{5}$$

where $n = f'_c/f'_t$ is the ratio between the compression and tension limit strengths,

$$\theta = \frac{\sum_{i=1}^{3} <\bar{\sigma}_i>}{\sum_{i=1}^{3} |\bar{\sigma}_i|} \quad \text{with} <\pm\bar{\sigma}_i> = \frac{1}{2}(|\sigma_i| \pm \sigma_i) \tag{6}$$

Expression (6) accounts for the different limit behaviour of the material under compression and tension states.

b) A damage criterion formulated in the strain of the undamaged stress spaces. The simplest form of this can be written as

$$F(\tau,\ r) = \tau - r \leq 0 \tag{7}$$

where τ is the norm defined in (5) and r is the damage threshold value. Damage grows when the norm τ exceeds the current threshold value. In particular, damage is initiated when τ exceeds for the first time the value r° (typically $r^\circ = f'_t/\sqrt{E}$ is taken [10,13]). Fig. 2 shows the form of the limit surface $\tau^\circ - r^\circ$ defining the onset of damage for the expression of τ given by eq.(5).

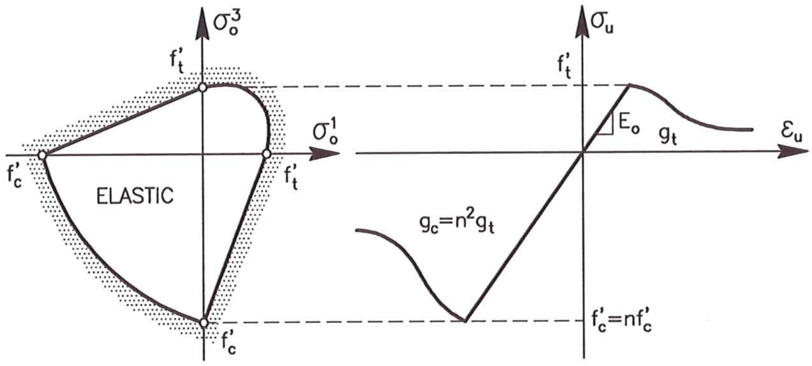

Fig. 2: Limit damage surface and uniaxial stress-strain curve for the model of eq. (5)

c)Evolution laws for the damage variable d and the damage threshold value r. These can be written as [28], [31].

$$d = G(r) \quad , \quad r = \max\{r^\circ,\ \tau\} \tag{8}$$

where G is a suitable monotonic scalar function taken as

$$G(r) = 1 - \frac{r^\circ}{r} \exp\left\{A\left(1 - \frac{r}{r^\circ}\right)\right\} \tag{9}$$

Note that $G(r^\circ) = 0$ and $G(\infty) = 1$ as expected. The parameter A is determined from the energy dissipated in an uniaxial tension test as [10,23]

$$\frac{1}{A} = \frac{g_f E}{(f'_t)^2} - \frac{1}{2} \quad \text{or, alternatively} \quad \frac{1}{A} = \frac{g'_f E}{(f'_t)^2} \tag{10}$$

where $g_f = G_f/l^*$, G_f being the specific fracture energy per unit area (taken as a material property), l^* is the characteristic length of the fractured domain. As always

$g_f \geq (f_t')^2/2E$ (the material must dissipate at least the energy stored when the elastic limit is reached), parameter A must be positive [28]. Defining $g_f' = g_f - (f_t')^2/2E$, the second expression in (10) is obtained.

The damage model presented above is extremely simple in comparison with more sophisticated constitutive models for concrete. A flow chart summarising the steps required in the practical implementation of the model within a standard non linear finite element solution scheme are shown in Box 1.

The experimental characterisation of the model is also simple and the following material parameters are only required: Young modulus, tension and compression limit strengths and specific fracture energy obtained from uniaxial tests.

$$\boxed{n\text{th load increment, } i\text{th iteration}}$$

$$\boldsymbol{a}_i^n = \boldsymbol{a}^{n-1}$$

Compute displacement increment

$$\Delta \boldsymbol{a}_i^n = -[\boldsymbol{H}_i^n]^{-1} \boldsymbol{\Psi}_i^n$$

$$\boldsymbol{\Psi}_i^n : \text{ residual force vector } (= \textstyle\int_V \boldsymbol{B}^{\mathrm{T}} \boldsymbol{\sigma} \, dV - \boldsymbol{f})$$

$$\boldsymbol{H}_i^n : \text{ iteration matrix (i.e. tangent stiffness matrix)}$$

Update displacements and strains

$$\boldsymbol{a}_{i+1}^n = \boldsymbol{a}_i^n + \Delta \boldsymbol{a}_i^n, \qquad \boldsymbol{\varepsilon}_{i+1}^n = \boldsymbol{\varepsilon}_i^n + \boldsymbol{B} \Delta \boldsymbol{a}_i^n$$

Evaluate stresses

(1) Compute undamaged stresses: $\bar{\boldsymbol{\sigma}}_{i+1}^n = \boldsymbol{D} \boldsymbol{\varepsilon}_{i+1}^n$

(2) Evaluate r_{i+1}^n using Eq.(5): $r_{i+1}^n = \tau_{i+1}^n$

(3) Update r and d

$$r_{i+1}^n = \max(r^{n-1}, r_{i+1}^n) \ , \qquad d_{i+1}^n = G(r_{i+1}^n)$$

(4) Update stresses: $\boldsymbol{\sigma}_{i+1}^n = (1 - d_{i+1}^n) \bar{\boldsymbol{\sigma}}_{i+1}^n$

Evaluate residual force vector: $\boldsymbol{\Psi}_{i+1}^n$

Check convergence: $||\boldsymbol{\Psi}_{i+1}^n|| \leq \epsilon ||\boldsymbol{f}||$?

No: Continue iterations: $i = i + 1$

Yes: $\boldsymbol{a}^n = \boldsymbol{a}_{i+1}^n$, next load increment: $n = n + 1$

Box 1: Quasi-static non linear finite element solution using the damage model

THE CONCEPT OF GLOBAL DAMAGE

The global strength of the structure can be assessed by means of a *global damage* index D. The definition for D in energy terms [28] is the following

$$D = 1 - \frac{\bar{U}}{U} \tag{11}$$

where $\bar{U}$ and U are the internal energies corresponding to the damaged and undamaged states, i.e.

$$\bar{U} = \boldsymbol{a}^T \int_V \boldsymbol{B}^T \boldsymbol{\sigma} dV = \boldsymbol{a}^T \int_V \boldsymbol{B}^T (1-d)\bar{\boldsymbol{\sigma}} dV$$

$$U = \boldsymbol{a}^T \int_V \boldsymbol{B}^T \bar{\boldsymbol{\sigma}} dV \tag{12}$$

In (12) the total energy of the structure is obtained by sum of the element contributions in the standard manner.

Note that global structural failure corresponds to a value of D approaching unity. Thus, the computation of the local and global damage indices provides a useful tool for monitoring in detail the evolution of the non linear response of the structure up to failure.

FAILURE PRESSURE EVALUATION OF THE CONTAINMENT BUILDING OF A LARGE DRY NUCLEAR POWER PLANT

Introduction

The evaluation of the failure pressure of the containment building of a large dry PWR-W three loops nuclear power plant is described in this section. The method considers fully tridimensional finite element models in order to take into account the effect of the most significant structural characteristics (presence of three buttresses, penetrations, additional reinforcement around the penetrations, etc.), the lack of symmetry of the forces generated by the prestressing system, as well as the nonlinear behaviour of the materials and the sensitivity of the results to uncertainties associated to several material parameters.

Description of the structure

The reinforced concrete containment building which hosts the reactor core and its cooling system consists of a massive foundation slab and a vertical cylindrical wall closed on the upper part by a hemispherical dome. The structure has an additional prestressing system for the wall and the dome consisting of non-adherent tendons and its interior is protected with a steel liner having a sealing role. Fig. 3 shows vertical and horizontal cross sections of the structure, including the main geometrical parameters.

There are three vertical buttresses on the outer side of the cylindrical wall spaced at 120°, which serve as support for the horizontal prestressing system. The penetrations in the cylindrical walls having a major impact (being modelled therefore) on the structural behaviour are: the personnel airlock, the equipment hatch, the emergency airlock, the main steam penetration, the fuel transfer penetration and the purge line penetration.

The prestressing system is shown in Fig.3. There are 132 horizontal tendons, comprising an angle of 240° each, anchored in the 3 buttresses and 80 vertical tendons in 2

families (N-S, E-W) anchored in a perimetrical gallery located in the lower part of the foundation slab.

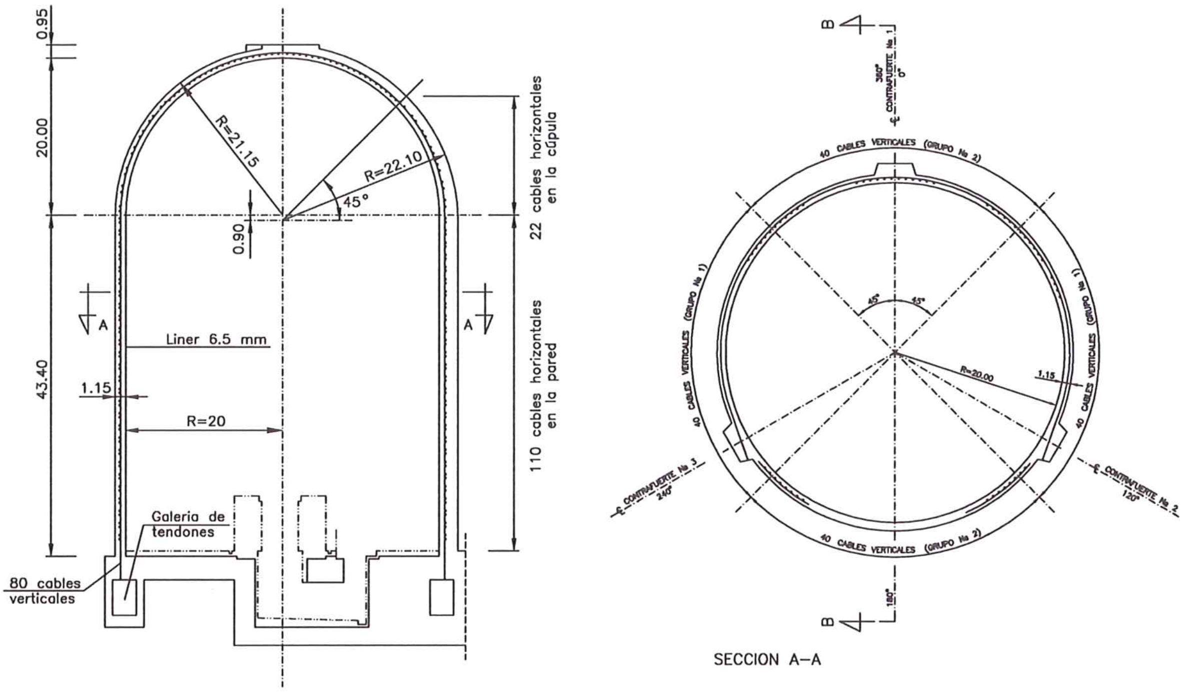

Fig. 3: Containment shell. (a) vertical section; (b) horizontal section.

Strategy of analysis

The failure pressure is defined as the inner pressure corresponding to the structural material exhaustion, that is, to a certain strain limit of the reinforcement steel, prestressing tendons and liner. The failure criterion assumes that local steel rupture occurs when the mentioned strain limit reaches 0.8% for the reinforcement and 1% for the tendons. The straining up to failure limit of the reinforcement is made possible by the damage-induced stress loss in concrete leading to stress redistribution towards the steel components. The global damage indices describe the state of entire structural parts, summing up both concrete and steel data.

The loads considered in the analysis were the self-weight, the external pressures generated by the prestressing system and the internal pressure corresponding to a specified accident. The distribution of the pressures equivalent to those produced by the prestressing system has been evaluated analytically for all the nodes of the mesh. All the possible sources of prestressing losses have been included in this evaluation, i.e. friction, wobble, anchor set, instantaneous and long term, etc. The internal pressure was incremented gradually until the structural collapse occurred.

Failure pressure evaluation

The influence of including the foundation slab in the structural model on the global structural behaviour and especially on the failure pressure was first examined. The results show that the influence of including the slab is quite small for low levels of internal pressure; it decreases further as the pressure increases and it is negligible near the failure pressure, which is 1.11 MPa in both cases. Furthermore, the cylindrical wall behaves better when the slab is present, due to the fact that the displacements of the slab slightly reduce the circumferential displacements of the wall; this allows to conclude that by not including the slab, one stays on the safety side during the complete load history. The comparison was based on an extensive survey of displacements, cracking

with the rest of the structure has little relevance. The final proof is that although the slab GDI displays important variations and finally takes values well above the overall GDI, this last is never influenced by the state of the slab and the driving influence keeps being that of the above-mentioned rings. Little effect have the states of the 1st ring or the dome, while the cylinder GDI at his turn behaves like the overall GDI.

This behaviour is in line with the known properties of this GDI method [28], to "filter out" the irrelevant parts of the structure and to identify and follow the evolution of its critical zones. The fact that the overall GDI reaches in both cases values close to the unit show that the structure really fails when the pressure reaches 11.1 bars.

Conclusions

This section describes the study of the failure pressure for the containment building of a large dry nuclear power plant, for an accident scenario beyond the design one. The method used in the evaluation of the failure pressure is based on the simulation of the complete damage process of the containment building, by means of a fully tridimensional nonlinear finite element model using a damage model. The model includes the most relevant structural aspects required in an accurate numerical simulation. The results obtained demonstrate that the influence of the foundation slab can be safely neglected in the failure pressure analysis of a structure of this type. The failure mechanism corresponds to the failure of the circumferential reinforcement bars of the mid-cylinder ring of the wall, leading to a wide vertical crack in the structure near to one of the buttresses. A safety coefficient of the structure related to the design pressure of 2.78 is obtained.

PATHOLOGY OF HOUSING BUILDINGS

Introduction

Housing buildings often display structural problems after completion when, due to constructive vices, exceptional loads like earthquakes or later accidents like ground movements, these are rendered unserviceable and rehabilitation decisions need be taken.

The methodology described herein proposes reconstructing through numerical analysis the surveyed damaged state of a structure and in this manner explain the underlying reasons of unaccounted-for structural behaviour while simultaneously quantifying them by means of GDIs. These indices signal the weaker zones and provide the measure of their experimented stiffness loss. When a configuration similar to the real state of the building is found, deductions can be made about the actual structural characteristics using similarity techniques.

Numerical simulations carried out with the damage model can provide assessments of the proposed repair works and help define the optimum intervention, being a valuable tool both for diagnosis and rehabilitation of buildings.

Description of the structure

The studied structure is a five-storey building with two symmetrical flats per storey. The 3rd storey presented extensive damage from unknown reasons and was therefore the object of detailed analysis. Its finite element mesh and reinforced concrete members are shown in Fig. 7.

The trouble with this building was that soon after completion and being already in use, fissures which soon became important cracks appeared. That fact imposed urgent

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