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LINEAR AND QUADRATIC INVARIANTS PRESERVING DISCRETIZATION OF EULER EQUATIONS

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The design of accurate and reliable numerical methods for the numerical simulation of compressible (low-Mach) flows is an important and challenging research topic. An ideal discretization should be locally conservative of linear invariants (a property which is naturally reproduced in finite-volume methods) and globally conservative of quadratic invariants (for which the so-called split forms are typically used in finite-difference methods). Moreover, the correct choice of the setup for the discretization of the (total) energy equation is of paramount importance, as it influences the correct preservation of additional invariants of the Euler equations (e.g. entropy). Many contributions have been published on this topic (e.g. [1, 2] and references therein), and general criteria for linear and quadratic invariants preservation, valid on arbitrary (Cartesian) meshes, have been recently derived in [3], for both finite-difference and finite-volume methods.

In this work, which constitutes a follow-up investigation of the analysis presented in [3], the effects of the spatial discretization of convective terms are studied in conjunction with the question of the specification of the thermodynamic terms in the total energy equation. Classical formulations already presented in the literature are investigated and reformulated within the matrix-vector approach employed in [3]. The compatibility relations among the discrete versions of the various terms in the Euler equations are analyzed and the additional degrees of freedom identified by the proposed theory are investigated in this context. Numerical simulations on classical and recently identified new test cases are used to validate the theory and to assess the effectiveness of the various formulations.

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