EVALUATION OF THE DAMAGE DEGREE IN BUILDINGS STRUCTURES SUBJECTED TO EARTHQUAKES

by A. H. BARBAT *, A. HANGANU *, E. OÑATE * and S. OLLER *

This paper represents an overview of the investigation pursued by the authors in the field of damage evaluation of concrete structures [1, 2, 3]. A procedure to evaluate the damage of reinforced concrete structures subjected to static and dynamic actions has been developed, which is based on a local damage constitutive model using Kachanov's theory [4]. The constitutive model is used within a finite element frame, a non-linear Newmark-type tangent incremental time integration scheme is employed and finally, an overall global damage index, based on potential energy criteria, is developed and justified.

1. INTRODUCTION

The damage of reinforced concrete structures can be defined as the degree of structural degradation that allows conclusions about the future capacity of the structure to withstand other important loadings. It is quantified through a "damage index", defined as the value of damage normalized to the failure level of the structure, so that a value equal to 1 will reflect complete failure [5, 6]. Different definitions of the damage index have been given in the literature. Recent works [7] define a damage index for structural members using a linear combination between a ductility factor and an energy factor. For complex structures the definition of a global structural damage index is generally based on a weighted average of the indices corresponding to the different members of the structure. In this paper, an overall global damage index based on potential energy considerations is proposed.

2. CONSTITUTIVE MODELS

The analysis of framed structures subjected to seismic actions beyond the linear behaviour has been traditionally treated in the following ways :

(a) Theories based on plastic hinge formation [8]. This approach has the inconvenience of admitting that the damage of a structure point is dominated by bending criteria, what is true only for some particular structures.

* Universidad Politécnica de Cataluña, Escuela Técnica Superior de Caminos, Canales y Puertos, Departamento de Resistencia de Materiales y Estructuras en Ingenieria, Calle Gran Capitán s/n, Edificio Gran Capitán C1, Campus Norte UPC, 08034 Barcelona, Spain.

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(b) Simulation of beam structures based on the concept of plastification bending moment. This procedure is based on formulating simplified curvature-bending moment constitutive laws [9, 10].

The last formulations started from representing the behaviour of materials in an approximate form, based mainly on experimental studies. Today, it is required that these formulations be thermodynamically sustainable. Among those which meet this requirement, the so-called continuous damage theory is generally accepted as an alternative in the most complex constitutive formulations [6, 11]. Such a model can be seen in [12] where a column discretized in plane finite elements, subjected to seismic action, is calculated. The damage models have a rigorous but relatively simple formulation based strictly on thermodynamics. They deal with the non-linear behaviour by means of one or more internal variables, called damage variables, which weight the losing of secant stiffness of the material and are normalized to an unit value which corresponds to maximum damage. Figure 1 shows an unidimensional representation of the behaviour of a point of a damaged material.

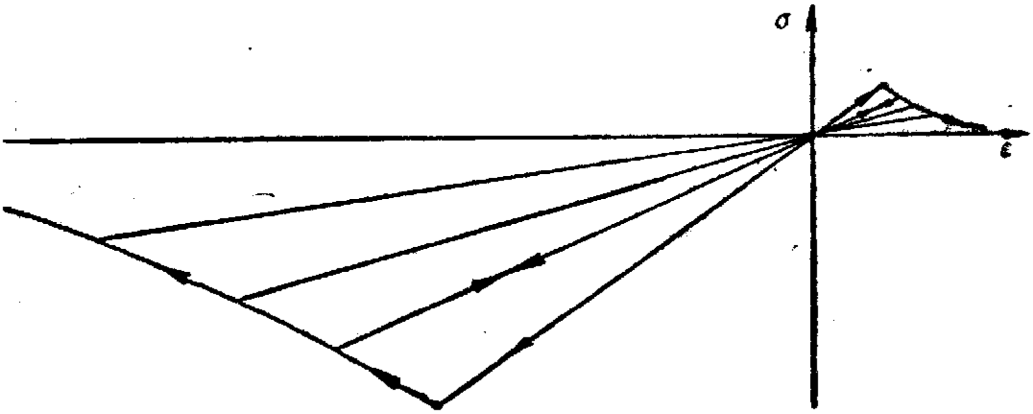

Fig. 1. —Local damage behaviour.

The model presented herein is a 3D damage constitutive model with a single internal variable. Therefore it is a local isotropic damage model, based on Kachanov's theory and also includes later works [11, 13, 14]. This formulation is a compromise between the complexity implied in modeling concrete behaviour [3] and the versatility needed when dealing with dynamic problems, which insures accurate results and low cost solutions.

The model is formulated in the material configuration, for thermodynamically stable problems. For this specific case the subsequent mathematical form for the free energy Ψ is supposed, where the non-damaged elastic energy factor Ψ_0 is expressed as a scalar quadratic function of tensorial arguments [1, 2, 3]

$$\Psi(\varepsilon\,;d) = (1-d)\,\Psi_0(\varepsilon) = (1-d)\left(\frac{1}{2m_0}\,\varepsilon : \mathbf{C}^\circ : \varepsilon\right) \tag{1}$$

In this equation, the strain tensor ε is the free variable of the problem, $d(0 \leqslant d \leqslant 1)$ the internal damage variable, m_0 the density in the material (lagrangean) configuration and $\mathbf{C}^0$ the stiffness tensor of the material in the initial undamaged state.

The constitutive law and its incremental form for a Kelvin type viscous material are [2]

$$\boldsymbol{\sigma} = (1 - d)\mathbf{C}^0 : \boldsymbol{\varepsilon} + \alpha(1 - d)\ \mathbf{C}^0 : \dot{\boldsymbol{\varepsilon}} = \mathbf{C}^s : \boldsymbol{\varepsilon} + \alpha\mathbf{C}^s : \dot{\boldsymbol{\varepsilon}} \tag{2}$$

$$\delta\boldsymbol{\sigma} = \mathbf{C}_v^p : \delta\boldsymbol{\varepsilon} + \alpha\mathbf{C}^s : \delta\dot{\boldsymbol{\varepsilon}} = (\mathbf{I} - \mathbf{D}_v) : \mathbf{C}^0 : \delta\boldsymbol{\varepsilon} + \alpha\mathbf{C}^s : \delta\dot{\boldsymbol{\varepsilon}} \tag{3}$$

where $\mathbf{C}^s$ is the secant constitutive matrix of the damage model, $\alpha = \dfrac{\eta}{E^0}$ is the retardation time, defined as the time needed by the structure-damper system to reach a stable configuration in the undamaged state, η being the unidimensional viscous parameter. Matrix $\mathbf{D}_v$ takes the following value :

$$\mathbf{D}_v = d\mathbf{I} + \frac{\mathrm{d}G(\bar{\sigma})}{\mathrm{d}\bar{\sigma}} (\boldsymbol{\sigma}^0 + \boldsymbol{\sigma}_v^0) \otimes \frac{\partial\bar{\sigma}}{\partial\boldsymbol{\sigma}^0} \tag{4}$$

with $\boldsymbol{\sigma}^0 = \mathbf{C}^0 : \boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}_v^0 = \alpha\mathbf{C}^0 : \dot{\boldsymbol{\varepsilon}}$. $d = G(\bar{\sigma}) = 1 - \dfrac{\bar{\sigma}^*}{\bar{\sigma}}\, e^{A\left(1 - \frac{\bar{\sigma}}{\bar{\sigma}^*}\right)}$ is the function which links the damage level d to the equivalent stress $\bar{\sigma}$, $\bar{\sigma}^*$ being the first damage yield limit and A a parameter depending on the specific fracture energy of the material. The equivalent stress $\bar{\sigma} = [1 + r(n - 1)] \sqrt{\sum_{i=1}^{3} (\sigma_i^0)^2}$ (see figure 2) depends on the non-damaged

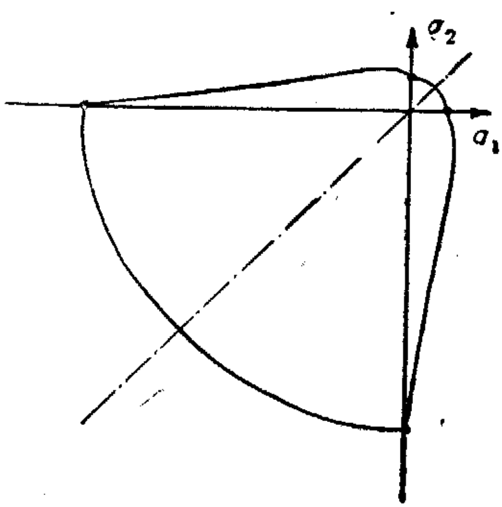

Fig. 2. — Damage Yield in the principal plane $\sigma_1 - \sigma_2$.

principal stresses σ_i^0 and also on a traction/compression index, $r = \sum_{i=1}^{3} \langle \sigma_i^0 \rangle / \sum_{i=1}^{3} |\sigma_i^0|$; $\langle \pm x \rangle = \frac{1}{2} (|x| \pm x)$, so that $r=1$ would correspond to a tridimensional traction state and $r = 0$ to a tridimensional compression state; finally, variable $n = f_c/f_t$ gives the ratio of the compression strength to the traction strength of the material. A complete description of the damage model can be found in references [1, 2, 3].

3. DYNAMIC STRUCTURAL MODEL

The structure is modelized using the finite element method. The element used is based on Timoshenko's beam theory completed with a layered formulation. The sectional stresses are solved through C^0 lagrangean finite elements with three degrees of freedom per node in the plane case. The cross-sectional generalized stresses obtained are decomposed point by point, layer by layer, in stress tensors which are treated and corrected by the damage model and afterwards recomposed in the resultant sectional stresses. These last stresses are used afterwards to compute the residual forces, in order to iterate for equilibrium if necessary.

Kelvin's model together with the proposed damage model allow the decomposition of the total stress acting on a structure point in a hyperelastic part and a viscous part. These terms introduced in a finite element scheme produce the classical damping and stiffness matrices. If an incremental time integration algorithm is chosen, the corresponding tangent constitutive matrices should be used instead of the secant ones thus obtaining tangent stiffness and damping matrices.

The dynamic equilibrium equation, corresponding to this visco-damage model, written at time $t = t_{i+1}$, takes the following form:

$$\mathbf{M} : \ddot{\mathbf{a}}_{i+1} + \int_l \mathbf{B} : \hat{\boldsymbol{\sigma}}_{i+1} \, \mathrm{d}V = \mathbf{f}(t_{i+1}) \tag{5}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{B}$ is the classical finite element method derivative matrix, $\mathbf{f}$ is the dynamic load vector and $\mathbf{a}$ is the nodal displacement vector. In this and the coming formulae, the lower index represents the time step number. $\hat{\boldsymbol{\sigma}}_{i+1}$ is the total sectional stresses vector which, according to equations (3) and (4), allows the following incremental decomposition:

$$\hat{\boldsymbol{\sigma}}_i = \hat{\boldsymbol{\sigma}}_i^{str} + \hat{\boldsymbol{\sigma}}_i^{vis} = \hat{\mathbf{C}}^S : \mathbf{B} : \mathbf{a}_i + \hat{\boldsymbol{\eta}}^S : \mathbf{B} : \dot{\mathbf{a}}_i \tag{6}$$

$$\Delta\hat{\boldsymbol{\sigma}}_{i+1} = \hat{\mathbf{C}}_v^D : \mathbf{B} : \Delta\mathbf{a}_{i+1} + \hat{\boldsymbol{\eta}}^S : \mathbf{B} : \Delta\,\dot{\mathbf{a}}_{i+1} \tag{7}$$

where $\boldsymbol{\eta}^S = \alpha\mathbf{C}^S$ and $\hat{\mathbf{X}} = \int_A \mathbf{S} : \mathbf{X} : \mathbf{S}\mathrm{d}A$ are sectional matrices obtained by integrating corresponding local matrices over the finite element section. As well, $\hat{\boldsymbol{\sigma}} = \int_A \mathbf{S} : \boldsymbol{\sigma} dA$ and $\boldsymbol{\varepsilon} = \mathbf{S} : \hat{\boldsymbol{\varepsilon}}$ are the relations linking sectional stresses and strains to the local ones, by means of the correspondence matrix S.

If the Newmark time integration scheme is used, the following relations express the evolution of displacements, velocities and accelerations for the time step $(i + 1)$ function of their known values for the precedent time step (i):

$$\ddot{\mathbf{a}}_{i+1} = \frac{1}{\beta \Delta t^2} \Delta \mathbf{a}_{i+1} - \frac{1}{\beta \Delta t} \dot{\mathbf{a}}_i - \left(\frac{1}{2\beta} - 1 \right) \ddot{\mathbf{a}}_i \tag{8}$$

$$\dot{\mathbf{a}}_{i+1} = \frac{\gamma}{\beta \Delta t} \Delta \mathbf{a}_{i+1} + \left(1 - \frac{\gamma}{\beta} \right) \dot{\mathbf{a}}_i + \left(1 - \frac{\gamma}{2\beta} \right) \Delta t \ddot{\mathbf{a}}_i \tag{9}$$

$$\mathbf{a}_{i+1} = \Delta \mathbf{a}_{i+1} + \mathbf{a}_i \tag{10}$$

If relation $\hat{\boldsymbol{\sigma}}_{i+1} = \hat{\boldsymbol{\sigma}}_i + \Delta \hat{\boldsymbol{\sigma}}_{i+1}$ together with relations (6—10) are replaced in equation (5), the incremental equilibrium equation becomes

$$\hat{\mathbf{K}} : \Delta \mathbf{a}_{i+1}^{(1)} = \hat{\mathbf{f}}_{i+1}^{(1)} \tag{11}$$

where the upper index is the current iteration number. The following notations have been introduced:

$$\hat{\mathbf{K}} = \frac{1}{\beta \Delta t^2} \mathbf{M} + \frac{\gamma}{\beta \Delta t} \mathbf{A} + \mathbf{K}^T \tag{12}$$

$$\hat{\mathbf{f}}_{i+1}^{(1)} = \mathbf{f}(t_{i+1}) + \mathbf{M} : \left[\frac{1}{\beta \Delta t} \dot{\mathbf{a}}_i + \left(\frac{1}{2\beta} - 1 \right) \ddot{\mathbf{a}}_i \right]$$

$$-\mathbf{A} : \left[\left(1 - \frac{\gamma}{\beta} \right) \dot{\mathbf{a}}_i + \left(1 - \frac{\gamma}{2\beta} \right) \Delta t \, \ddot{\mathbf{a}}_i \right] - \mathbf{K}^S : \mathbf{a}_i \tag{13}$$

In these relations appear both secant and tangent matrices, which are obtained integrating the corresponding constitutive matrices over the length of the finite element. Their values are

$$\mathbf{K}^S = \int_l \mathbf{B} : \hat{\mathbf{C}}^S : \mathbf{B} \, ds ; \ \mathbf{K}^T = \int_l \mathbf{B} : \hat{\mathbf{C}}_v^D : \mathbf{B} \, ds ; \ \mathbf{A} = \int_l \mathbf{B} : \hat{\boldsymbol{\eta}}^S : \mathbf{B} \, ds \tag{14}$$

and all three are damage dependent, hencefore, time variable.

Solving the linearized system (11), the displacement increments $\Delta \mathbf{a}_{i+1}^{(1)}$ are obtained and afterwards all the other variables corresponding to the time step $(i + 1)$. If the damage level has changed, new values for matrices (14) need to be calculated and this implies that an unbalanced force may have appeared. This is the residual force $\boldsymbol{\Psi}^{(1)}_{i+1}$, whose value is governed by the difference between the expected stress state $\hat{\boldsymbol{\sigma}}_i + \Delta\hat{\boldsymbol{\sigma}}^{(1)}_{i+1}$, and the real one $\hat{\boldsymbol{\sigma}}^{(1)}_{i+1}$, calculated using the updated secant stiffness and damping matrices

$$\boldsymbol{\Psi}^{(1)}_{i+1} = \int_V \mathbf{B} : (\hat{\boldsymbol{\sigma}}_i + \Delta\hat{\boldsymbol{\sigma}}^{(1)}_{i+1} - \hat{\boldsymbol{\sigma}}^{(1)}_{i+1}) \, \mathrm{d}V \qquad (15)$$

This residual force is now applied as the right hand term in equation (11) and, after having updated the value of $\hat{\mathbf{K}}$, the system is solved, thus obtaining a new correction for the displacement increment. This iterative process stops when the chosen norm of residual forces falls below the desired tolerance.

4. GLOBAL DAMAGE INDICES

The starting point for deducing a global structural damage index is equation (1), which relates the damaged part of the free energy $\boldsymbol{\Psi}$ with the non-damaged elastic free energy $\boldsymbol{\Psi}_0$. In order to find a global index, a similar expression is deduced by integrating (1) over the entire mass of the structure. The integral over mass is transformed in an integral over volume using the transformation $\mathrm{d}m = m_0 \mathrm{d}V$ as follows:

$$\boldsymbol{\Psi} = (1 - d)\, \boldsymbol{\Psi}_0 \Rightarrow W_p = \int_V m_0 \boldsymbol{\Psi} \, \mathrm{d}V = \int_V (1 - d)\, m_0 \boldsymbol{\Psi}_0 \mathrm{d}V =$$

$$= (1 - D)\, W_p^0 \qquad (16)$$

where D is the global damage index, $W_p^0 = \int_V m_0 \boldsymbol{\Psi}_0 \mathrm{d}V$ is the total potential energy of the structure if it were undamaged and W_p is the total potential energy corresponding to the actual damaged state. Solving equation (16) for D, the following final relation is obtained:

$$D = 1 - \frac{W_p}{W_p^0} = \frac{\int_V m_0 \boldsymbol{\Psi}_0 \mathrm{d}V - \int_V (1 - d)\, m_0 \boldsymbol{\Psi}_0 \mathrm{d}V}{\int_V m_0 \boldsymbol{\Psi}_0 \mathrm{d}V} = \frac{\int_V d m_0 \boldsymbol{\Psi}_0 \mathrm{d}V}{\int_V m_0 \boldsymbol{\Psi}_0 \mathrm{d}V} \qquad (17)$$

If a damage index for a subvolume of the structure is needed (such as a floor, some columns, etc.) the integration is done only over that specifc subvolume.

In a finite element scheme, in the case of a structure discretized with layered beams elements, the damage index of a beam point D_p (considering the beam as a unidimensional finite element) is given by a similar expression, obtained by integrating (17) over the cross-section of the beam, with $m_0\Psi_0 = \frac{1}{2}\,\boldsymbol{\varepsilon} : \boldsymbol{\sigma}^0$ and $\boldsymbol{\varepsilon} = \mathbf{S} : \hat{\boldsymbol{\varepsilon}}$

$$D_p = 1 - \frac{\hat{\boldsymbol{\varepsilon}} : \hat{\boldsymbol{\sigma}}}{\hat{\boldsymbol{\varepsilon}} : \hat{\boldsymbol{\sigma}}^0}\,;\ \hat{\boldsymbol{\sigma}} = \int_A \mathbf{S} : \boldsymbol{\sigma}\,\mathrm{d}A = \int_A (1 - d)\,\mathbf{S} : \sigma^0 \mathrm{d}A \qquad (18)$$

where $\hat{\boldsymbol{\varepsilon}}$ and $\hat{\boldsymbol{\sigma}}$ are the generalized strains and stresses in that beam point, respectively.
The global damage index will take the following form, for a finite element scheme:

$$D = 1 - \frac{\sum_e \mathbf{a} : \int_l \mathbf{B} : \hat{\boldsymbol{\sigma}}\,\mathrm{d}s}{\sum_e \mathbf{a} : \int_l \mathbf{B} : \hat{\boldsymbol{\sigma}}^0 \mathrm{d}s} \qquad (19)$$

This stress-oriented formula for the global damage index is a completely all-purpose one and it can be employed in any non-linear formulation, as it is always possible to compare actual non-linear stresses with would-be-linear ones. Thus, the global damage index is defined as the ratio of the potential energy which the structure cannot undertake in the actually damaged state to the potential energy that the structure would undertake if it were undamaged.

5. NUMERICAL EXAMPLES

5.1. STATIC AND CYCLIC TESTS

In order to verify and adjust the whole methodology developed so far, a set of static and cyclic trial simulations have been performed. They aimed at displaying the strongholds as well as the drawbacks of the proposed damage evaluation scheme.

Figure 3 shows the force-displacement response of the reinforced frame described in the next section (see figure 5) subjected to a horizontal displacement load in its upper right extremity. It can be noticed that the damage model allowed to pursue the test almost to the complete failure of the structure, until the complete loss of stiffness of the loaded frame. The descending slope can be controlled entirely by means of the specific material fracture energy. The convergence of the non-linear process is fairly good.

Figure 4 displays the local damage evolution of the midpoint of a beam loaded with cyclic displacements in the same point. The input dis-

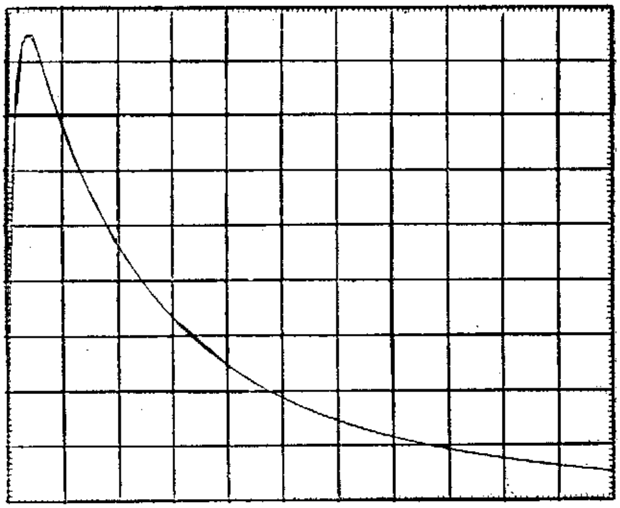

Fig. 3. — Force-displacement curve statically loaded with displacements.

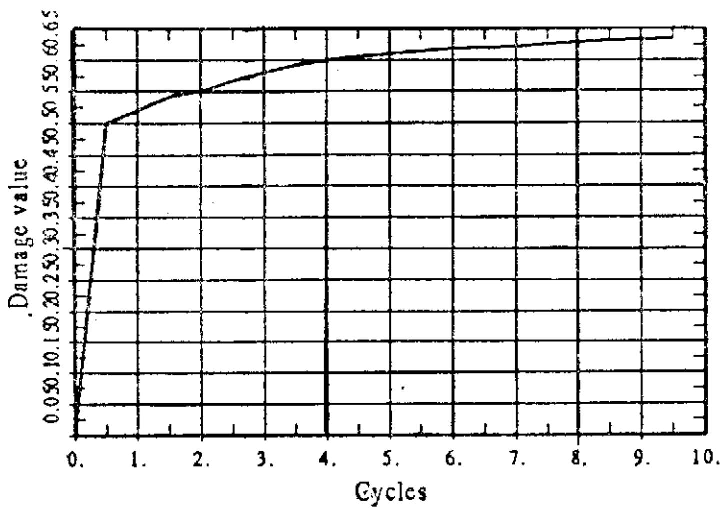

Fig. 4. — Damage evolution of a beam midpoint in a fatigue test.

placement changes sign every load step maintaining the same amplitude and is chosen such as to produce an initial maximum damage index value in the beam mid-span of 1/2. It is evidenced that the damage level increases in time, which is the very result it is looked for as it proves that the damage model and the computer code are able to reproduce material fatigue phenomena. This is the effect of ever changing damage configurations as in every cycle the situation is different from the precedent one, because a damage occurred at a load step induces further damage in the next load step and so on.

5.1. SEISMIC NUMERICAL SIMULATION

The simulation of the evolution of the damage process in the reinforced concrete plane frame of figure 5, subjected to a synthetic earthquake accelerogram (figure 6) having a predominant frequency of 4 Hz and a maximum amplitude of 0.175 g, has been performed.

The frame is 9 meters high and 6 meters wide and has three levels. The columns have a 30 cm × 30 cm cross-section of reinforced concrete with a 4.35 % steel ratio. The horizontal beams are 40 cm thick and 30 cm

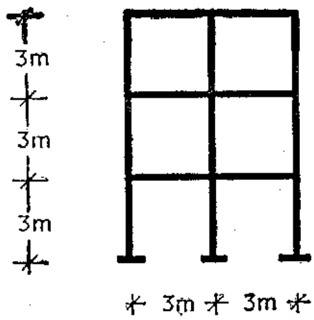

Fig. 5. — Geometry of the frame.

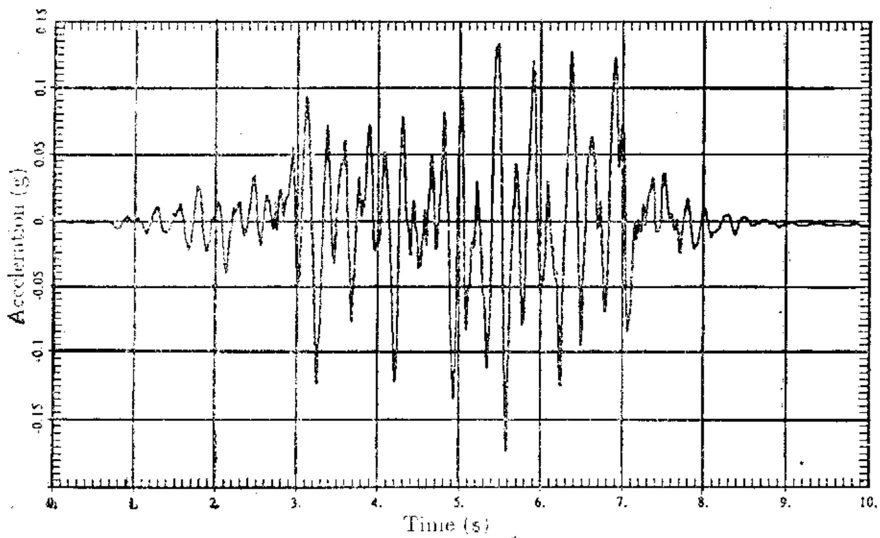

Fig. 6. — Synthetic accelerogram.

wide with a steel ratio of 5.3 %. The structure is discretized in 45 quadratic three-noded beam finite elements having two Gauss points each. Thus, the resulted dynamic model had 87 nodes with three degrees of freedom per node. Each element is one meter long and has the cross-section divided in 20 layers of equal thickess. The 2nd and 19th layers are made of steel and the rest of them of concrete. The steel ratio is controlled by modifying the width of the steel layers. The state of the material is checked at the interface between layers and afterwards interpolated linearly across line), it may be seen that it takes values only slightly smaller than the maximum sectional damage at the bases of the columns [1, 2, 3].

This fact ratifies the choice of the global damage index as the ratio between the potential energy which the structure cannot undertake in the damaged state and the potential energy that the structure should undertake if it were undamaged. The first floor damage is slightly higher than the global damage of the structure as this floor is the most affected, while the second and third floors follow in decreasing order as the damage reduces with height.

Another interesting property of the global damage index is that of allowing the decision of the state of the structure in what regards its failure mechanisms. It permits the identification of the mechanism of collapse by means of observation of the local damage indices and continuous comparison with the global one. When, during a damaging process, the global index gets close to the maximum local damage and the rest of the points of the structure stop degrading, it means we identified the critical points of the structure. The failure of these points leads to the formation of a mechanism, i.e. a collapse of the structure. This fact carries great consequence from an engineering structural retrofitting point of view.

The effect of introducing viscous damping is one of constant "breaking" the motion of the structure as can be seen in figures 8 and 9, thus reducing amplitudes and damage. This phenomenon corresponds to the real behaviour of structures in a dynamic environment, where the materials display increased strengthes and nonconservative energy dissipation.

5. CONCLUSIONS

In this paper a visco-damage constitutive model is developed and tested. It is proven to provide good performances and low-cost solutions and to describe properly the nonlinear behaviour of reinforced concrete structures under dynamic loads. This model is incorporated in a finite element scheme which uses Timoshenko's beam elements, discretized in layers of concrete and steel in order to approximate the behaviour of beam cross-sections. A global damage index is rigorously deduced from the local damage index supplied by the constitutive model. A reinforced concrete building structure, under both non viscous and viscous regimes, subjected to synthetic accelerograms, is solved and satisfactory results were obtained. It is shown that the effect of considering the viscosity is of great importance.

The model, in its present form, has two major drawbacks : first, it does not produce remanent deformation, which is a well-known feature of non-linear materials and second, it does not discriminate between traction damage and compression damage, which makes impossible the simulation of "crack closure". These are the problems which are currently under study and solutions are in sight already.

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the layer. This gives 40 check points per cross-section in each Gauss point.

The materials have the following properties: (a) steel — $E = 2.1 \cdot 10^6$ daN/cm², $\sigma^\circ = 4\,200$ daN/cm², $\nu = 0.25$, $\rho = 8$ g/cm³; (b) concrete — $E = 2.0 \cdot 10^5$ daN/cm², $\sigma^0 = 300$ daN/cm², $\nu = 0.17$, $\rho = 2.5$ g/cm³.

The equations of motion governing the dynamic behaviour of the structure have been solved using the described Newmark step-by-step algorithm for $\beta = 0.25$ and $\gamma = 0.5$ [15]. The time step used is a thirtieth of the fundamental period of the structure, that is 0.14 s.

The structure is calcuated in two load cases: (a) without acounting for damping [1], (figure 7a), and (b) considering damping effects through a value for the retardation time of $\alpha = 0.001$ s, (figure 7b).

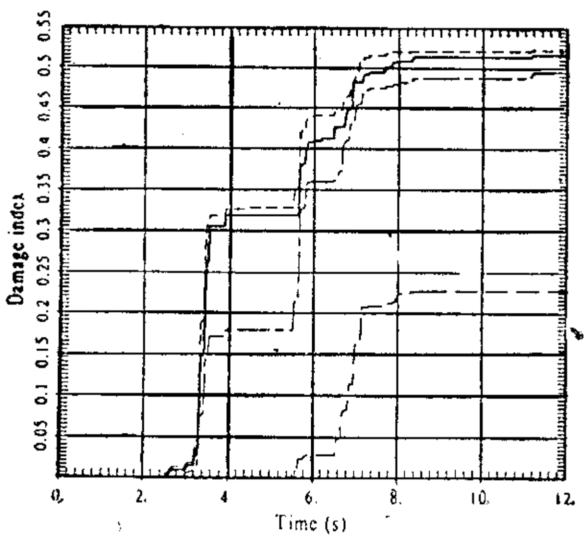
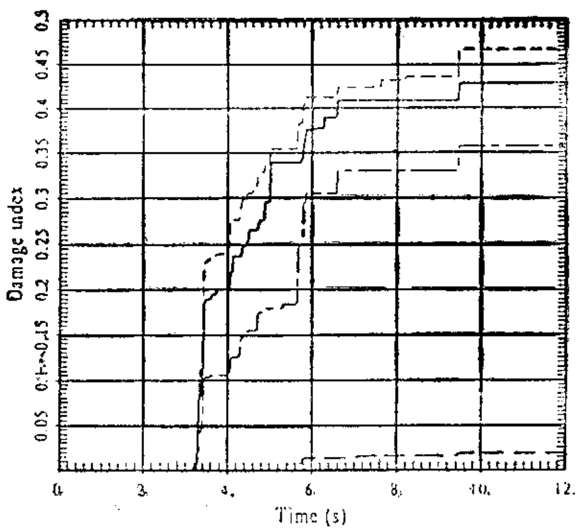


Fig. 7. — Global and floor damage indices — global; ----first floor; • — - second floor; — — — third floor.

a, without damping;



b, with damping.

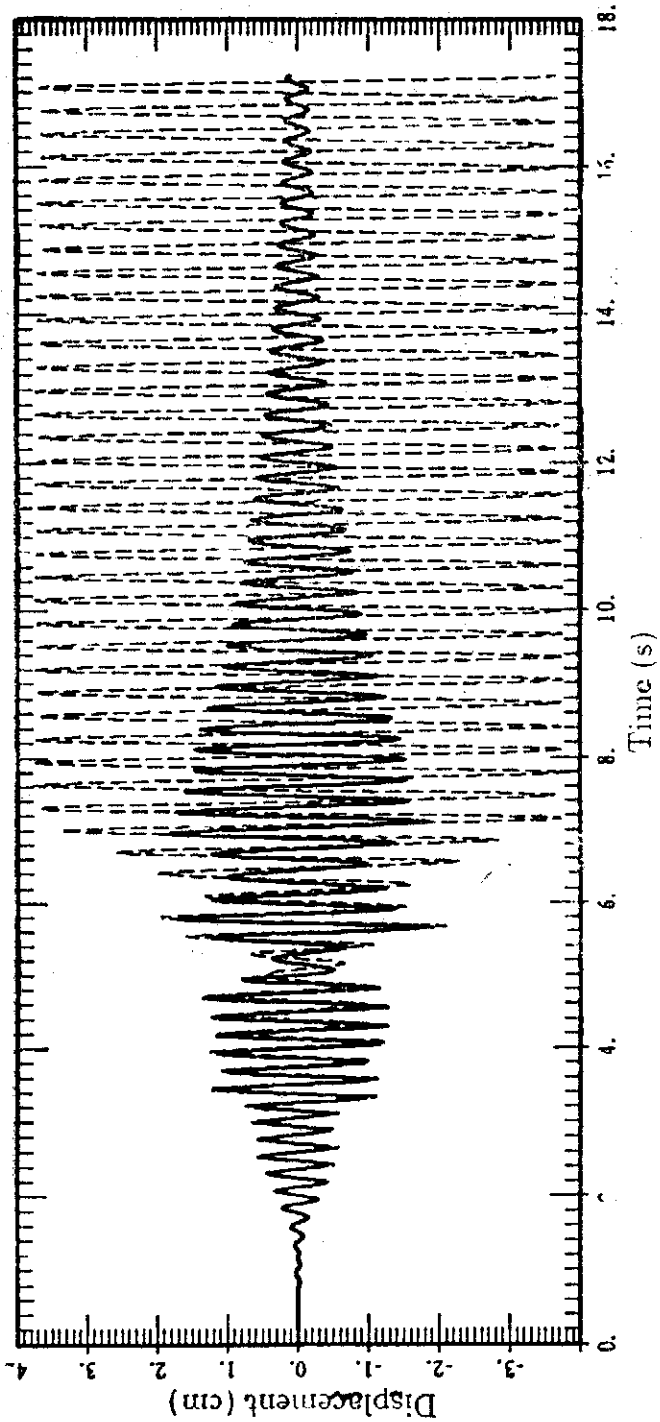


Fig. 8. — 3rd floor displacements

Figures 10 and 11 show the distribution of the sectional damage as given by formula (18). The damage is located at the joints of the columns with the floors, what is precisely the expected damage localization for this type of structure and load. As the structure is to fail mainly by des-

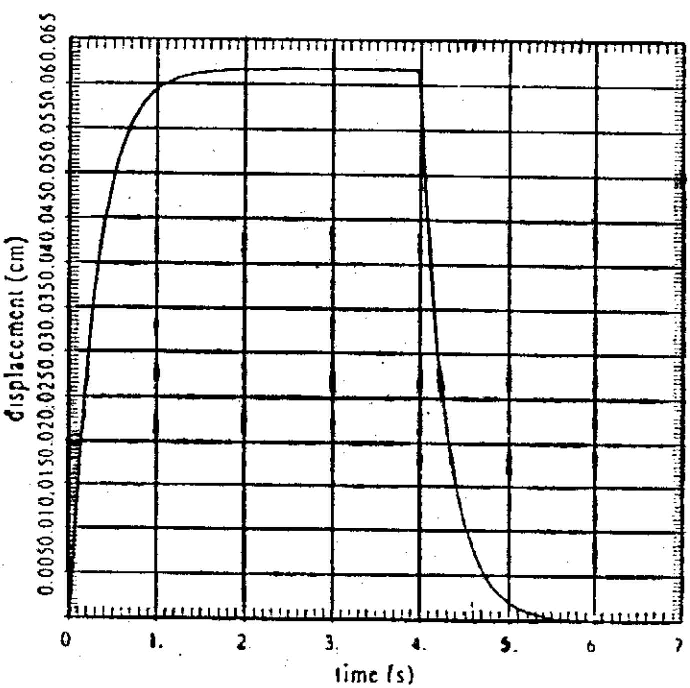


Fig. 9. — Pseudostatic viscous response.

truction of the columns at their joint with the base floor, the mentioned diagrams confirm this prognosed behaviour too. Comparing these diagrams with the global damage given in figures (7(a) and 7(b) (the continuous

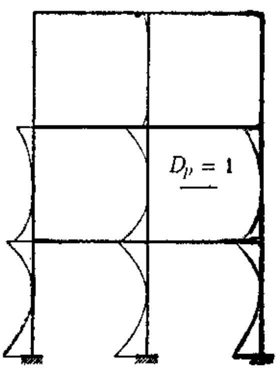


Fig. 10. — Distribution of sectional damage D_p, after mild earthquake.

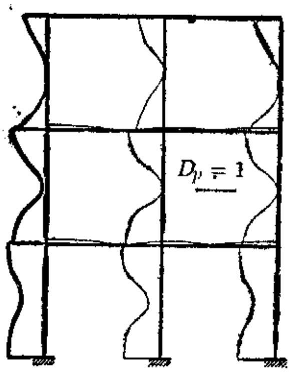


Fig. 11. — Distribution of sectional damage D_p, after a strong earthquake.

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