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MATRIX PROPERTIES ASSOCIATED WITH DISCRETE CONSERVATION IN FLOW SIMULATIONS

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It has been found advantageous for finite-volume and other discretizations of flow equations to possess additional (secondary) invariants, next to the (primary) invariants from the constituting conservation laws. With this purpose, in the literature extensive studies of so-called split forms of the equations have been carried out, e.g. [1]. The current paper will present a general framework for such supraconservative discretizations, in particular their coefficient matrices will be studied in a volume-consistent scaling.

The analysis is formulated in matrix-vector terminology, assuming exact time integration. This allows a general, abstract study of the conservation properties of the equations, and is independent of the way in which these equations have been derived. In particular we will present properties of the coefficient matrices that govern local and global conservation in compressible mass and momentum transport.

It turns out that secondary energy conservation is guaranteed by a specific relation between the transport terms for mass and momentum, which fixes the discrete mass transport operator once the discrete convective transport operator is given. In fact, it also determines the discretization of the thermodynamic terms [2]. Moreover, it proves that the combination of discrete energy and momentum conservation, implies discrete mass conservation. We will discuss the properties of finite-difference discretizations of the split forms, but many of the results hold for any discretization scheme with a volume-consistent scaling. In particular, it is shown that any conservative discretization method can be written as a finite-volume method.

Because the studied methods do not require any numerical diffusion to remain stable, they are the methods of choice in turbulent-flow simulations, where the subtle balance between turbulence production and viscous dissipation is critical. An example of natural transition around a delta wing will be shown [3].

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