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ELASTIC BEHAVIOUR OF LINEAR STRUCTURES USING MODAL SUPERPOSITION AND LAGRANGIAN DIFFERENCING DYNAMICS

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Abstract: Elastic deformation and dynamics response of the linear structures due to fluid loads are studied to understand the Fluid Structure Interaction (FSI). A modal coupling solver is developed by solving dynamic equation of motion with external loads, using the mode superposition method with the help of relevant mode shapes and natural frequencies associated with the structure. Natural frequencies and mode shapes have been pre-calculated and provided as input for the simulation. Modal coupling is integrated into the Lagrangian Differencing Dynamics (LDD) method, utilizes finite differences within the framework of Lagrangian context, and strong and implicit formulation of Navier-Stokes equations to model the incompressible free-surface fluid. Elastic deformation of the structure due to fluid force obtained from the flow solver is calculated in the modal coupling algorithm using direct numerical integration. Then the elastic deformation is imposed in the flow solver to account for change of the geometry and obtain new flow pressure and velocity fields. The two-way coupling of fluid and structure is successfully validated by simulating dam-break through an elastic gate. Since the LDD method works directly on surface meshes, the simulation is quickly setup and direct coupling of structural deformation eliminated the usual step of mapping of fluid results on the structural mesh and vice-versa.

Keywords. FSI, LDD, Natural Frequencies, Mode Shapes

1 INTRODUCTION

Fluid-Structure Interaction (FSI) stands out as a prevalent physical phenomenon in engineering problems. However, effectively simulating FSI is intricate, prompting the need for certain assumptions in both structural and fluid simulations. In Computational Fluid Dynamics (CFD) simulations, for instance, the consideration of elastic deformation at boundaries is omitted. Similarly, in Structural simulations, a consistent pressure is applied at the interior and exterior boundaries. Modal analysis comes into play to ascertain vibration characteristics, primarily the mode shapes and natural frequencies of a mechanical system or component. Natural frequency, or eigen frequency, denotes the frequency at which a system naturally oscillates without any external driving force. On the other hand, mode shapes, also referred to as eigenvectors, depict the inherent behavior of the component at its natural frequency. Both these parameters hold significance in the structural design process, especially for scenarios involving dynamic loads. They serve as the foundational elements for subsequent dynamic analyses like transient dynamic responses, harmonic analyses, and spectrum analyses.

FSI analysis plays a pivotal role in refining structural designs for optimal performance under fluid loads, ensuring both efficiency and reliability. Identifying FSI-related issues at an early stage enables cost-effective design adjustments, reducing the necessity for costly modifications during manufacturing or operation. Damping in FSI, representing energy dissipation within vibration cycles, emerges as a key factor in resonance phenomena, impacting harmonic vibration amplitudes and the count of noteworthy vibrations in time-dependent scenarios. While damping's role can be negligible in slightly damped vibrations when identifying natural frequencies, its influence becomes pronounced around these frequencies, especially in resonant conditions where excitation is balanced solely through damping. Though damping in the structure is generally low, except when nearing resonance and vibrational cycle maintains a substantial level of independence [1].

Within the current corpus of literature, a variety of approaches have been formulated to tackle the intricacies for coupling of fluid structure interaction (FSI). A prominent approach involves fully coupled (monolithic) methods that integrates both structural and flow calculation in one solver. Conventional Computational Fluid Dynamics solvers are predominantly uses eulerian based approach. However, coupling of structural formulation which mostly uses lagrangian based approach and it leads to stiffer computation for the structural component compared to the fluid component. Consequently, employing a unified scheme for extensive scenarios becomes computationally intensive. Partitioned methods offer an alternative by tackling of both flow and structural formulation on two different meshes utilizing distinct solvers. These methods necessitate the establishment of a communication protocol at the interface between grids to appropriately transfer fluid loads to structural mesh, and conversely, to map the deformation onto the fluid mesh. Effective adjustment on the boundaries of the fluid mesh requires precise manipulation of adjacent mesh nodes to prevent mesh entanglement or deformation. Notably, recent advances have demonstrated successful application of partitioned methods, such as coupling thin-walled girder theory with potential flow theory [4, 1] and linking modal structure solvers with RANS-VOF solvers [8], Boundary-Integral Equation Methods [8, 4], and potential flow theories [4, 1]. A method to forecast fluid-structure interaction (FSI) by employing a reduced-order structural model. This innovative approach stands out for its effectiveness and simplicity in predicting FSI, proving its applicability even in complex scenarios such as compressor stages. However, it's recommended to carefully manage integration time step sizes to avert potential stability challenges that might arise due to disparities with the frequency of the highest mode used for structural calculations [5].

In this paper, we focus on determining the elastic behavior of linear structures using modal coupling integrated into the Lagrangian Differencing Dynamics (LDD) method.

2 GOVERNING EQUATION

2.1 Mode superposition

The method of modal superposition is employed to analyze the dynamic behavior of structures. This approach is particularly effective in minimizing computational efforts when evaluating the dynamic response of linear structures [4]. The dynamic response can be estimated through the superposition of a limited number of modal frequencies of the structure. This technique proves especially advantageous when dealing with constrained loading frequencies that are known. However, it is less applicable to the issues that encompass exceedingly high frequencies.

The dynamic equation for a structure can be represented in matrix form as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}\,(t) \tag{1}$$

In this equation, $\boldsymbol{M}$ denotes the mass-normalized matrix, $\boldsymbol{C}$ represents the damping matrix, and $\boldsymbol{K}$ stands for the stiffness matrix. The column vector $\boldsymbol{u}$ corresponds to the degree of freedom, while $\boldsymbol{f}(t)$ represents the applied forces over time. This matrix is obtained through the discretization of the physical domain, resulting in an NxN matrix if N signifies the degrees of freedom.

The foundation of modal superposition is rooted in modal analysis, yielding essential outputs such as eigen frequencies and their corresponding mode shapes. The eigen frequencies are computed via the undamped dynamic equation, treated as an eigenvalue problem:

$$(-\omega^2\boldsymbol{M} + \boldsymbol{K})\boldsymbol{\Phi} = 0, \qquad\qquad \boldsymbol{\Phi} \neq 0 \tag{2}$$

In this context, the symbol $\boldsymbol{\Phi}$ refers to the modal matrix, which contains vector of mode shape corresponding to every natural frequency of the structure with n DOF, $\boldsymbol{\Phi} = \boldsymbol{\Phi}_1, \boldsymbol{\Phi}_2, ..., \boldsymbol{\Phi}_n$

The overall displacement of the structural system for a time step can be represented as a combination of mode shapes:

$$\boldsymbol{u}(t) = \sum_{i=1}^{n} \boldsymbol{\Phi}\boldsymbol{y}(t) \tag{3}$$

where $\boldsymbol{y}(t)$ is the vector of modal coordinates (or generalized displacement). By applying the generalized displacement and the mass normalized modal vector $\boldsymbol{\Phi}$ (equ (3)) into equ (1):

$$\boldsymbol{\Phi}^T\boldsymbol{M}\boldsymbol{\Phi}\ddot{\boldsymbol{y}}(t) + \boldsymbol{\Phi}^T\boldsymbol{C}\boldsymbol{\Phi}\dot{\boldsymbol{y}}(t) + \boldsymbol{\Phi}^T\boldsymbol{K}\boldsymbol{\Phi}\boldsymbol{y}(t) = \boldsymbol{\Phi}^T\boldsymbol{f}(t) \tag{4}$$

To decouple the equation of motion of a Multi-Degree-of-Freedom (MDOF) system into n equations of motion for Single Degree of Freedom (SDOF) systems, it's necessary to diagonalize the damping term. This entails introducing a damping matrix, as proposed by

Lord Rayleigh, that is assumed to exhibit proportionality to both the mass and stiffness matrices.

$$\boldsymbol{C} = \beta \boldsymbol{K} + \alpha \boldsymbol{M} \tag{5}$$

Final equation of motion will be:

$$\ddot{\boldsymbol{y}}_i(t) + 2\omega_i \xi_i \dot{\boldsymbol{y}}_{\boldsymbol{i}}(t) + \omega_i^2 \boldsymbol{y}_i(t) = \boldsymbol{\Phi}_i^T \boldsymbol{f}(t) \tag{6}$$

Here, ξ_i represents the damping ratio associated with mode i. It signifies the extent of real damping present within a system in comparison to the critical damping.

The equation 6 is solved using the Complementary Function and Particular Integral (CFPI) method [5].

2.2 Incompressible fluid flow

We employed an incompressible fluid flow solver known as LDD, which utilizes a generalized finite difference method with a meshless approach, employing a robust, implicit formulation of the Navier-Stokes equations to simulate incompressible free-surface fluids. This solver is utilized to address initial-boundary value problems, achieving second-order accuracy in its solution. It has been successfully validated across various scenarios, including lid-driven cavity, dam break, sloshing, water entry, and more [2, 3]. The continuity and momentum equations are provided below :

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{g}, \qquad x \in \Omega, \tag{7}$$

$$\nabla . \boldsymbol{u} = 0 \qquad x \in \Omega \cup \Gamma_w \cup \Gamma_{fs}, \tag{8}$$

$$\boldsymbol{u} = \boldsymbol{U} \qquad x \in \Gamma_{w,} \tag{9}$$

$$\boldsymbol{u}(t=0) = \boldsymbol{u_0} \qquad x \in \Omega \tag{10}$$

In this context, $\frac{D}{Dt}$ signifies the time rate of change of a property, $\boldsymbol{u}$ stands for the velocity vector, ρ represents the fluid density, p denotes the dynamic pressure, υ symbolizes the kinematic viscosity, g represents gravity, $\boldsymbol{U}$ corresponds to the wall velocity, and $\boldsymbol{u}_0$ denotes the initial velocity vector.

The pressure and velocity equations are solved in a decoupled manner. The pressure poisson equation and the pressure gradient are presented as follows:

$$\begin{cases} \nabla^2 p = -\rho \nabla \cdot D\boldsymbol{u}/Dt & x \in \Omega, \\ \boldsymbol{n} \cdot \nabla p = \rho \boldsymbol{n} \cdot [-\frac{D\boldsymbol{u}}{Dt} + \boldsymbol{g} + \nu \nabla^2 \boldsymbol{u}] & x \in \Gamma_w \\ p = p_{atm,} & x \in \Gamma_{fs} \end{cases} \tag{11}$$

Here, n stands for the normal vector, p_{atm} represents atmospheric pressure, and $\boldsymbol{g}$ is considered constant, resulting in a divergence of $\boldsymbol{g}$ equal to zero.

3 METHODOLOGY

In this paper, we express the modes using natural frequencies and mass-normalized modal vectors, as detailed in Section 2.1. Equations (1) to (6) encompass the force vector, bridging hydrodynamics, inertial loads, and the structural system. Modal equation (6) is directly solved within the fluid flow solver using the CFPI method, yielding the generalized displacement function, denoted as $\boldsymbol{y}(t)$. This accounts for all known mode shapes at each time step. The global structural deformation is reconciled using equation (3), incorporating the calculated generalized displacement, thereby ensuring that updated structural shapes influence the flow calculations [5]. Natural frequencies and corresponding modal vectors are determined externally before commencing the CFD computations. Vibrating mode shapes are represented through generalized displacements and mode shapes.

Exchange of forces and displacements is essential between the structure and fluid meshes. During each time interval, it is crucial that the fluid mesh undergoes deformation, utilizing the deformations calculated through the mode superposition method at the interface. The workflow of the fluid-structure solver is depicted in the Figure 1. Below steps are followed to establish two way coupling of fluid and structure interaction.

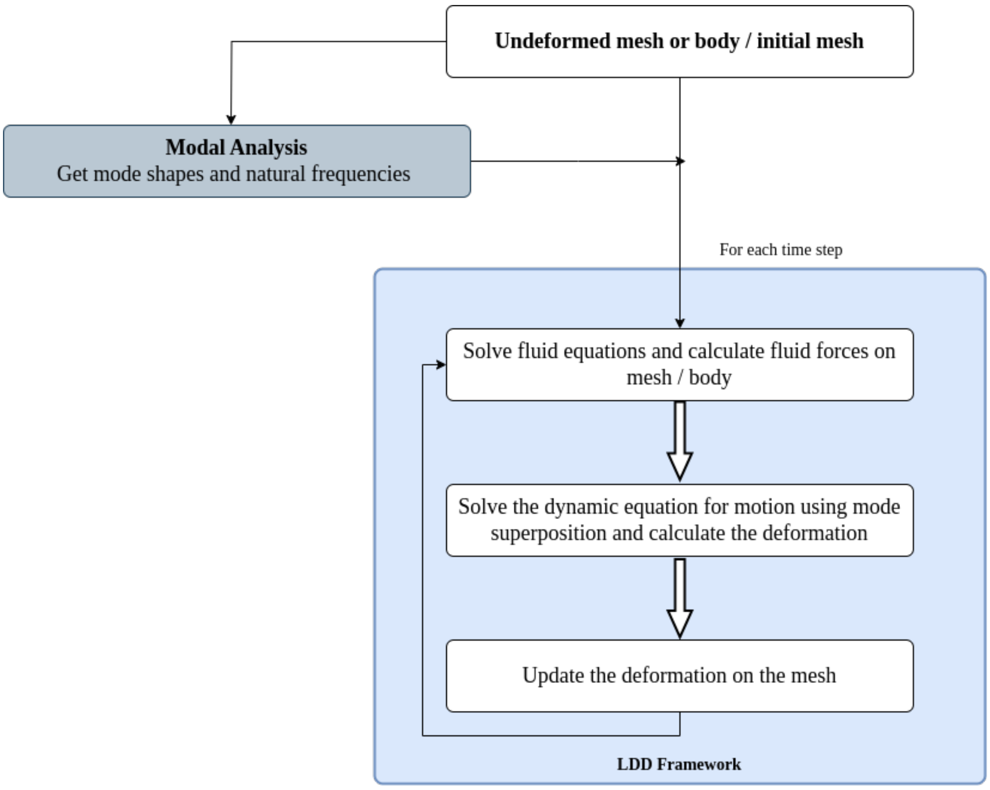

Figure 1: Workflow of FSI with LDD

1. Calculate the mass-normalized mode shapes and corresponding natural frequencies, denoted as $\omega_1, \omega_2, ..., \omega_n$.

2. Define the initial and boundary conditions for the simulation, encompassing the initial displacement of the structural system.

3. Create Radial Basis Function (RBF) connections between fluid faces and structural nodes, if structural mesh is not the same as fluid boundary mesh [7]

4. At every time interval Δt:

   (a) Compute the forces exerted on the structural mesh due to fluid pressure.

   (b) Solve equation of motion equ (6) using modal vector

   (c) Determine the updated deformation vector using equ (3) and apply the resulting deformation to the mesh.

   (d) Solve the fluid equations for time $t + \Delta t$, taking into account of structural deformation.

The modal coupling solver is constructed by solving the equation of motion dynamically with external loads, leveraging the mode superposition technique aided by pertinent mode shapes and natural frequencies determined in the pre-calculation stage (step 1). This modal coupling is integrated into the Lagrangian Differencing Dynamics (LDD) method, which adopts finite differences within a Lagrangian framework, offering a robust and implicit formulation of the Navier-Stokes equations to simulate incompressible free-surface fluid dynamics. The displacement resulting from fluid forces (step 4a) is calculated within the modal coupling algorithm through direct numerical integration (step 4b). This deformation is then applied in the fluid flow solver to account for geometric changes (step 4c), consequently generating new flow pressure and velocity fields (step 4d).

4 NUMERICAL VALIDATION

4.1 Static cantilever gate

In this section, an experiment originally conducted by Antoci et al. [6] is replicated. The experiment resembles the classic dam-breaking scenario; however, the gate in this instance isn't rigid or movable, but rather elastic and deformable. This rubber gate is affixed along its upper edge to a rigid wall and undergoes deformation when exposed to fluid forces acting behind it.

The experimental setup features a fluid column within a tank with dimensions: length $(A) = 100mm$ and height $(H) = 140mm$. The rubber gate, supported by a rigid obstruction, extends downward to touch the floor. The gate's height is $L = 79mm$. For modeling, an elastic isotropic material with a density of $\rho_{gate} = 1100kg/m$ and Young's modulus $E = 12MPa$ is employed. It's noteworthy that due to inherent uncertainty in estimating the Young's modulus for rubber, future endeavors will incorporate accurate rubber hyper-elastic properties. The simulation considers only the first mode shape and its associated natural frequency. Validation is undertaken with the tank filled with water, density $\rho = 1000kg/m^3$ and dynamic viscosity $\mu = 10^{-3}Pa.s$. The depicted elastic deformation of the cantilever due to fluid loading is illustrated in Figure 2.

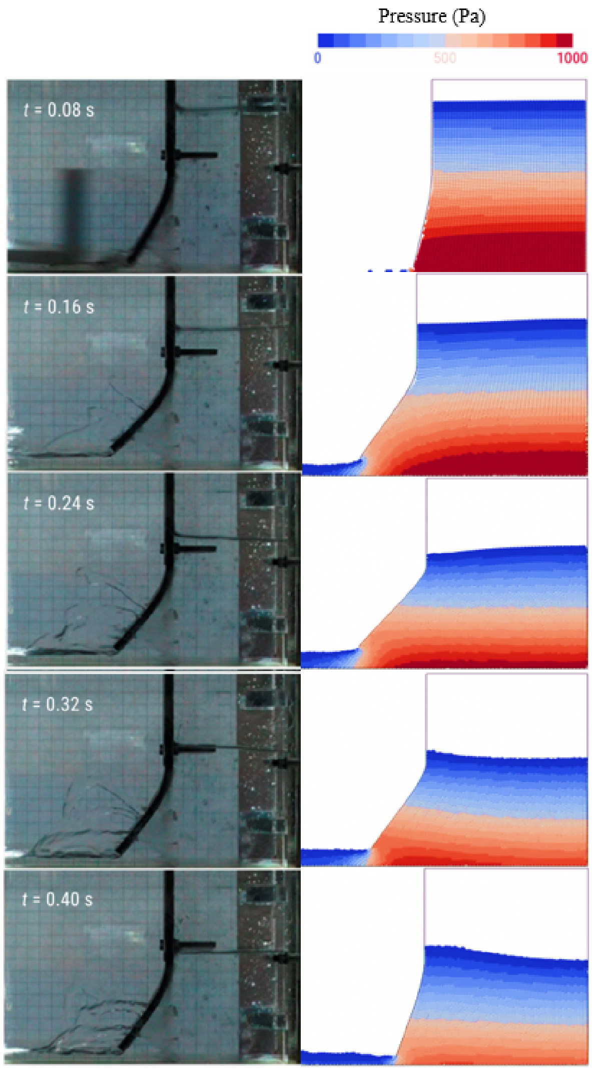

Figure 2: Cantilever– Pressure contour at different time step

This specific case is not optimal for the ultimate aim of mode superposition, and the solution for this instance is static. Thus, the first bending mode suffices for relevance. An assessment is performed to verify whether the attained deflection aligns with the anticipated order of magnitude, thereby assessing the accuracy of the dynamic solver's equation.

4.2 Dam break with cantilever beam

After successfully validating the dynamic equations through the analysis of a static cantilever gate in Section 4.1, we proceed to extend our exploration by conducting a dynamic simulation of a cantilever beam. The geometric dimensions remain consistent with the static case, maintaining a thickness of 5 mm, while the fluid properties remain unchanged. The simulation involves a domain measuring 0.5 x 0.2 m, where an initial

Table 1: Modes and natural frequencies

Mode	1	2	3	4	5
Natural Frequency(Hz)	15.09	93.433	257.66	374.8	495.57

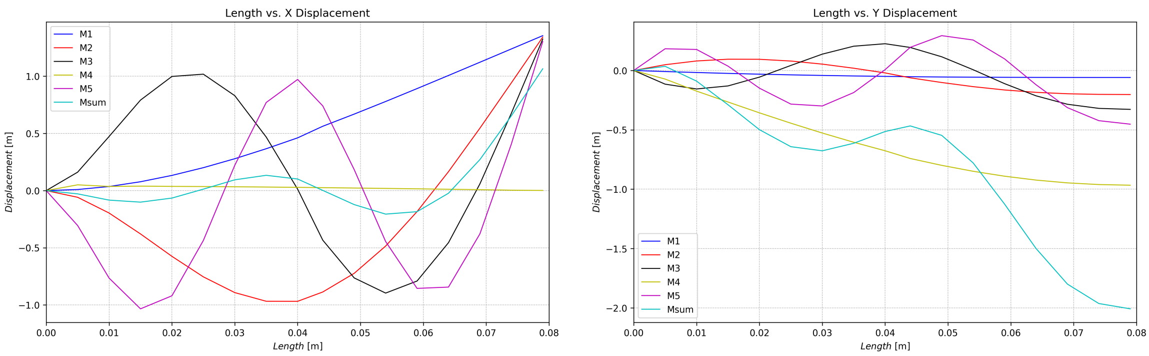

Figure 3: Mode shape of the beam along the length, Left: X-Displacement and Right: Y-Displacement

fluid column of dimensions 0.1 x 0.14 m is positioned at the left end of the domain. A beam is centrally placed within this domain, with its fixed end touching the bottom. The simulation replicates a scenario akin to a dam break, spanning a total time of 10 seconds with intervals of 1×10^{-3} seconds.

In this simulation, we utilize the first five natural frequencies (as detailed in Table 1) alongside their corresponding mode shapes (illustrated in Figure 3). The subsequent dynamic behavior of the cantilever beam is vividly portrayed in Figure 5, further substantiating the accuracy and effectiveness of our approach.

Furthermore, we examine the displacement in both x and y directions over time for both the tip and mid-section of the beam, as showcased in Figure 4. This comprehensive analysis offers valuable insights into the dynamic response characteristics of the structure within a fluid-structure interaction context. Notably, Figure 6 unveils minor oscillations overlaying more pronounced oscillations. This observation underscores the successful

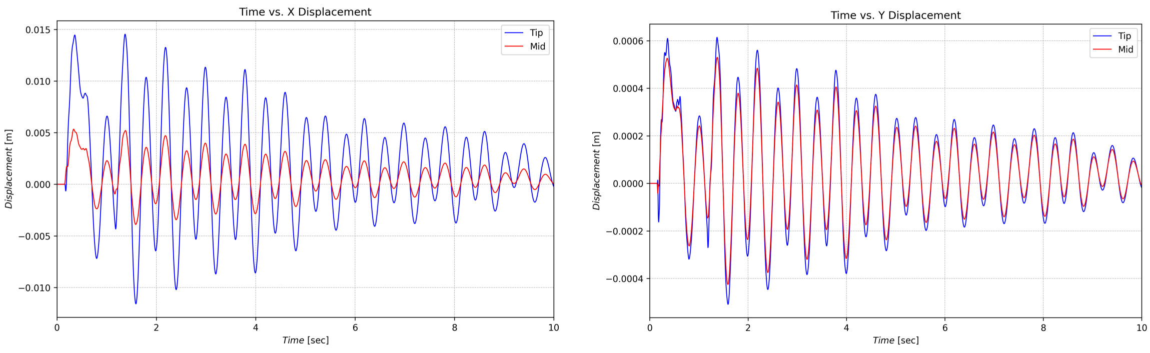

Figure 4: Dynamic response over a time at the mid and tip section, Left: X-Displacement and Right: Y-Displacement

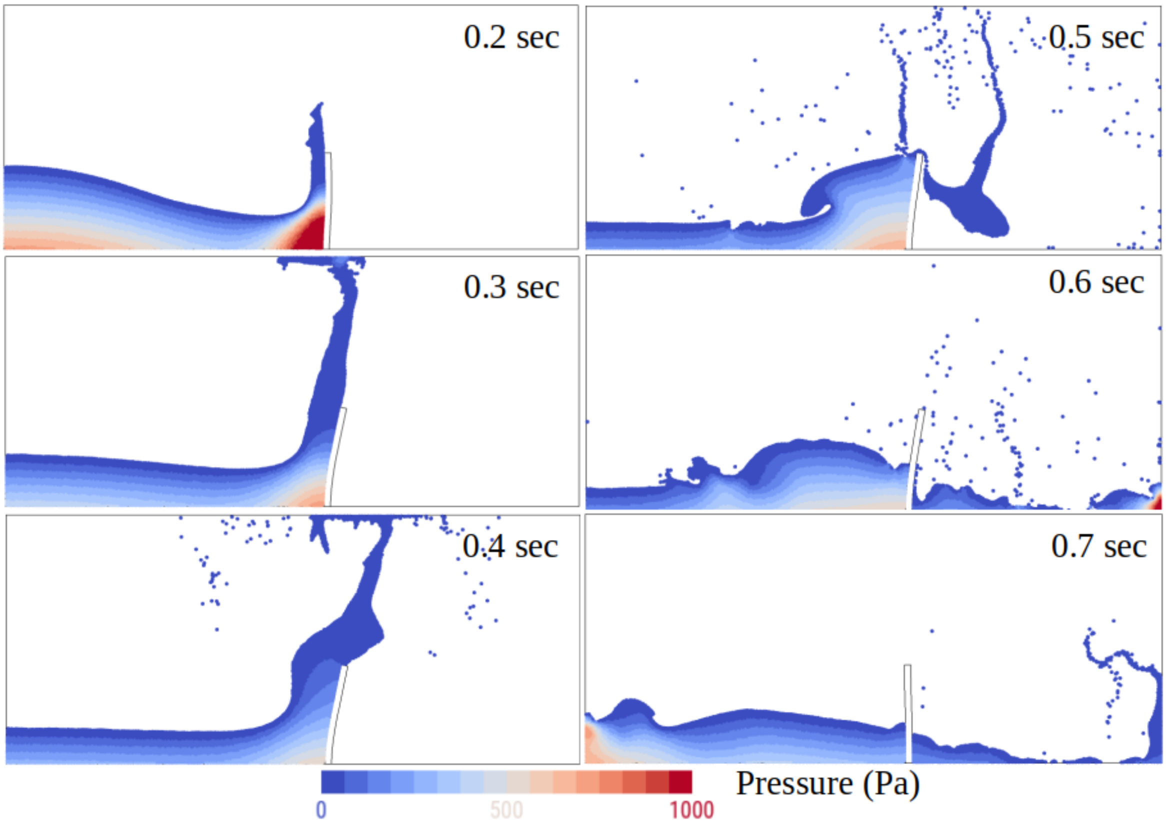

Figure 5: Dynamic response of cantilever beam at different time step

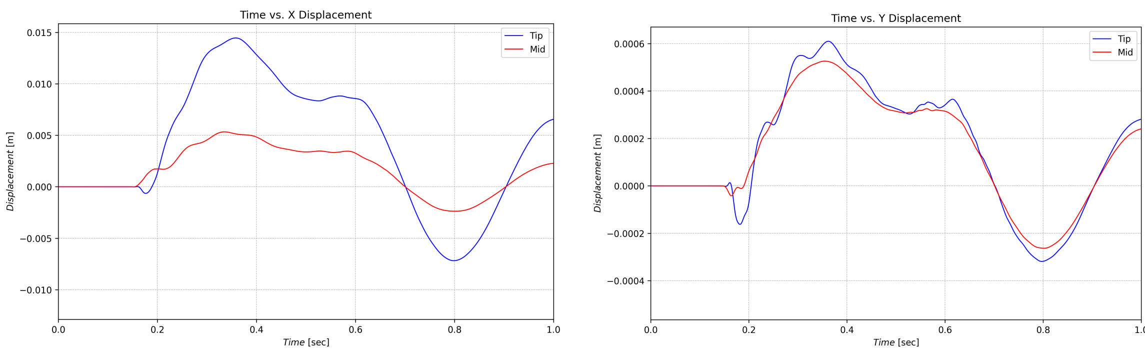

Figure 6: Displacement at the mid and tip section till 1 sec, Left: X-Displacement and Right: Y-Displacement

operation of the mode superposition technique, adeptly capturing the interplay of various modes in the system's response.

5 CONCLUSION

The Modal Coupling solver has been seamlessly integrated into the Lagrangian Differencing Dynamics (LDD) solver, effectively enabling two-way fluid-structure interaction coupling. The Modal Coupling solver's success is attributed to its foundation in the mode superposition method. The validation of the bidirectional fluid-structure coupling is effectively demonstrated through the simulation of a dam-break scenario involving an elastic gate.

The implementation process of the coupling scheme substantiates the tool's capability to facilitate the effortless coupling of diverse solvers, all without necessitating changes to solver algorithms or input files. Looking ahead, the research trajectory involves more intricate simulations encompassing three-dimensional structures, and these will be rigorously validated against experimental data. Furthermore, the incorporation of the energy equation into the framework is planned. This expansion aims to explore the influence of energy on fluid-structure interaction, thereby broadening the scope and insights of the analysis.

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