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MATERIAL COST MINIMIZATION PROBLEM FOR ALUMINUM ALLOY BEAM USING BEAM STRING STRUCTURE

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Abstract. *Aluminum alloy is a light-weight material with excellent corrosion resistance but low rigidity. When the aluminum alloy is used to a girder bridge, it takes high costs owing to the increment of its stiffness. Therefore in order to reduce a material cost, the cost minimization problem was performed on beam string structure (BSS) made of the aluminum alloy material based on the results of the topology optimization. We focused on the layout of the BSS and diameter of the cable. The conducted simulation made clear the effectivity of the BSS to the aluminum alloy material for a reduction of material cost and increment of the beam span.*

1 INTRODUCTION

Aluminum alloy materials are lighter than steel materials at approximately one-third the density and have superior corrosion resistance, hence their life-cycle costs are considered lower than those of the steel bridges. On the other hand, the aluminum alloy materials are more expensive than the steel and are prone to deflection because their rigidity is approximately one-third that of the steel. In the specification for highway bridges in Japan, the aluminum alloy materials are defined same deflection limits as the steel materials, which requires a larger cross section, resulting in higher material costs.

Therefore, this study proposes a beam string structure (BSS) made of the aluminum alloy, which is expected to improve stiffness and reduce costs. The BSS is a structure in which the cable members are installed through the strut at the bottom of the beam. The cable members bear the tensile force, and the beam bears the compressive force. The lifting of the strut generates a negative bending moment in the beam, which suppresses deflection.

In this study, two-stage optimization problem, that is a topology optimization and material cost minimization problem, is considered, as shown in Figure 1. An optimal form of BSS is created based on the groundstructure method. Then, the size optimization of the cross section

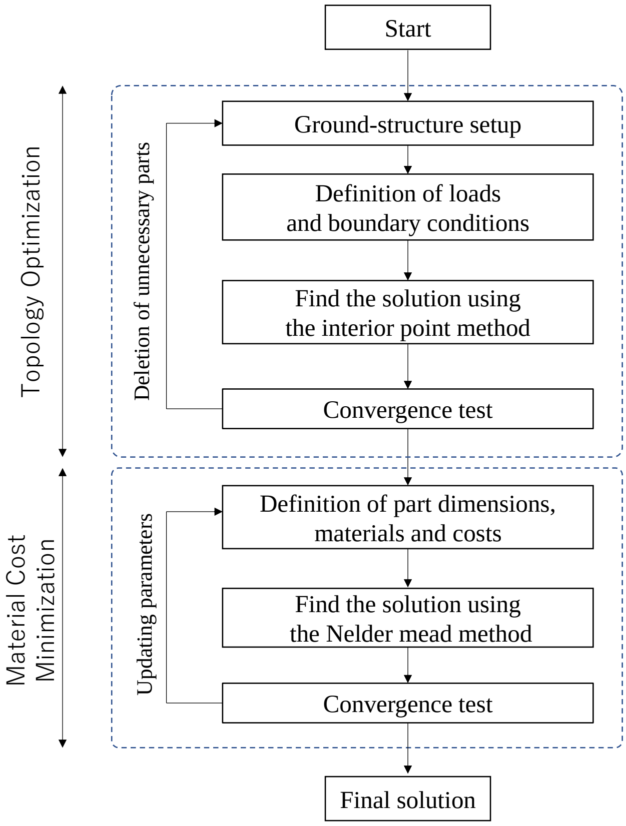


Figure 1: Flowchart of optimization procedure

of each member is performed with considering material cost of the BSS in order to clear the one advantage of the aluminum alloy beam. Mathematica version 12.3 was used to solve the optimization problem.

2 DESING OF BEAM STRING STRUCTURE BY TOPOLOGY OPTIMIZATION

Topology optimization is performed for the design area with aspect ratios of 2:5 and 1:5 to in order to propose optimal structures.

2.1 Formulation of topology optimization problem

A groundstructure is created by connecting the nodes in the design area with the truss elements as shown in Figure 2. Then the axial forces are determined by a mathematical optimization method. In the groundstructure method, the remaining shape after removing the truss elements with small axial forces is the optimal structure.

In this study, the minimum volume solution (Pareto-optimal solution) under stress constraints is obtained by linear programming for the topology optimization problem concerning the total volume and compliance of members [1]. The Pareto-optimal solution of the truss structure under constant external force in this multi-objective optimization problem can be computed by simple equilibrium of forces.

In a multi-objective optimization problem targeting the minimization of the total volume V

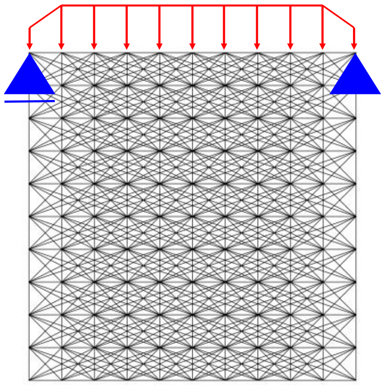


Figure 2: Example of ground-structure

and compliance C of the truss members, the problem is formulated as follows:

$$\{V, C\} \longrightarrow \text{min.} \tag{1}$$

$$\begin{cases} V = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{l} \\ C = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{\delta} \end{cases} \tag{2}$$

where $\boldsymbol{A}$ $(= \{A_1, \ldots, A_{\mathrm{m}}\}^{\mathrm{T}})$ is the cross-section of the members vector, $\boldsymbol{l}$ $(= \{l_1, \ldots, l_{\mathrm{m}}\}^{\mathrm{T}})$ is the length of the members vector, $\boldsymbol{N}$ $(\in \mathcal{R}^{\mathrm{m}})$ is the axial force vector of the member, and $\boldsymbol{\delta}$ $(\in \mathcal{R}^{\mathrm{m}})$ is the axial deformation vector of the member.

The relationship between the total volume of the truss members V and compliance C in the Pareto-optimal solution is

$$C = \frac{\sigma^2 V}{E} \tag{3}$$

$$VC = \frac{(|\boldsymbol{N}|^{\mathrm{T}}\boldsymbol{l})^2}{E} \tag{4}$$

where σ is the absolute value of stress in all members and E is the elastic modulus of all members.

Considering equations (4), the multi-objective optimization problem in equations (1) can be formulated as a linear programming problem as follows. This linear programming problem can be solved by the interior point method.

$$\begin{aligned} f &= \boldsymbol{l}^{\mathrm{T}}\boldsymbol{N}_{+} + \boldsymbol{l}^{\mathrm{T}}\boldsymbol{N}_{-} \\ \boldsymbol{P} &= \boldsymbol{B}\boldsymbol{N}_{+} - \boldsymbol{B}\boldsymbol{N}_{-} \end{aligned} \tag{5}$$

2.2 Structure simplification by ESO method

The optimal structure obtained from initial calculations is often a complex geometry with a mixture of members with low axial forces. In this study, the Evolutionary Structural Optimization (ESO) method [2] is used in combination with the groundstructure method to make the axial forces acting on the members more uniform and to derive a simpler form. In this study, a lower limit of axial force $\alpha N_{\max}$ is defined based on the maximum value of axial force $N_{\max}$

obtained from the calculation, then, a new group of nodes is defined as the grandstructure for recalculation, excluding the group of nodes where unnecessary members are gathered, and the calculation is repeated.

$$\alpha {N_{\max}}^k < {\boldsymbol{N}_i}^k \leq {N_{\max}}^k \tag{6}$$

where $i(= 1,2,3 \cdots i_{\max})$ is the number of members and $k(= 1,2,3 \cdots k_{\max})$ is the number of calculations. The initial value of α is 0, and it is increased by 0.01 for each calculation.

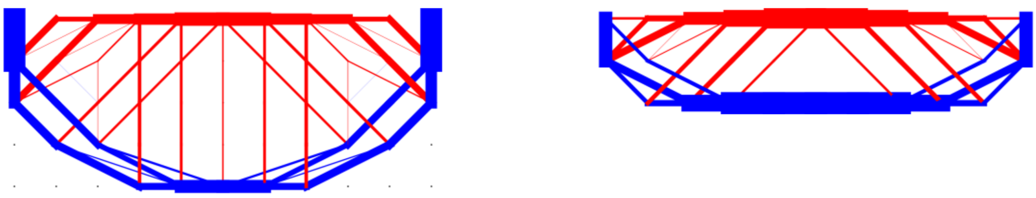
2.3 Design model

As shown in Figure 2, hinge and roller supports were placed at both ends of the top of the groundstructure, and the load was applied to the uppermost part of the structure. The design area was set so that the aspect ratio of the groundstructure was 2:5 and 1:5.

2.4 Topology optimization results

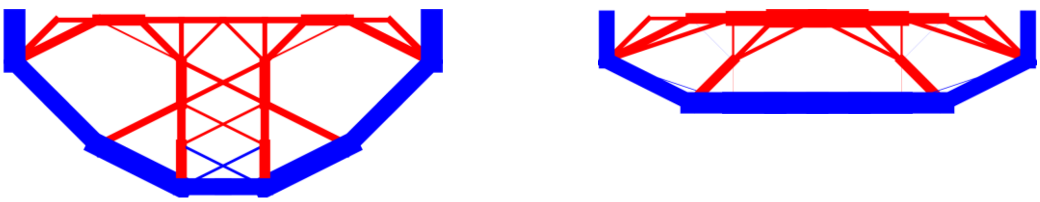
Figures 3 and 4 show the initial and final structures for the topology optimization, respectively. The members in red are subjected to compressive forces, while the members in blue are subjected to tensile forces. The thickness of the member indicates the magnitude of the axial force, and the thicker the member, the greater the axial force acting on it.

In the process of the topology optimization, the number of members was reduced by approximately 26% and 51% for aspect ratios of 2:5 and 1:5, respectively. In all the resultant figures, a compression member appeared at the top where the load acts, and an arched tension member appeared at the bottom of the design area. Focusing on the final step, in Figure 4(a), a single vertical member joined by diagonal members was observed in the center of the span. In Figure 4(b), where the aspect ratio is larger, two diagonals are observed in the center of the span, indicating that the distance between the diagonals is wider.



(a) Aspect ratio 2:5 (b) Aspect ratio 1:5

Figure 3: Topology optimization results (initial layout)



(a) Aspect ratio 2:5 (b) Aspect ratio 1:5

Figure 4: Topology optimization results (final layout)

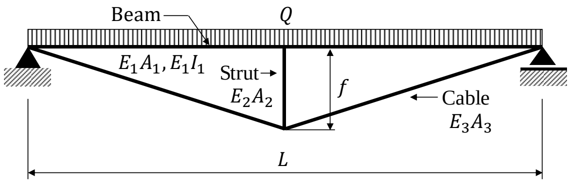


Figure 5: Beam string structure with a strut

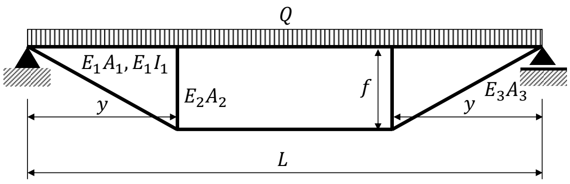


Figure 6: Beam string structure with two struts

3 MATERIAL COST MINIMIZATION PROBLEM FOR ALUMINUM ALLOY BEAM USING BEAM STRING STRUCTURE

We proposed the aluminum alloy beams using BSS with one or two struts and cables based on the topology optimization in previous section as shown in Figures 5 and 6. Then, material cost minimization was conducted under the simple supoprted condition. In addition, the steel and the aluminum alloy simple supported beam was considered as comparative models.

3.1 Theoretical Model of Beam String Structure

Okada derived a theoretical equation for the stress deformation behavior of a BSS with one strut based on the unit load method [3]. In this section, the theoretical equation is derived using the unit load method, including the case with two struts.

Figure 5 shows a mechanical model of the BSS with one strut under a distributed load Q. where Q is the span of the BSS, E_1A_1 is the elongation stiffness of the beam, E_1I_1 is the bending stiffness of the beam, f is the height of the strut, E_2A_2 is the elongation stiffness of the strut, E_3A_3 is the elongation stiffness of the cable material. The BSS with one strut is statically indeterminate structure. If the axial force on the strut of the BSS is a statically indeterminate force X, the displacements of the 0th and first systems can be expressed by the following equation:

$$\delta_{10} = 2\int_0^{\frac{L}{2}} \frac{M_0 M_1}{E_1 I_1} dx \tag{6}$$

$$\delta_{11} = 2\int_0^{\frac{L}{2}} \frac{{M_1}^2}{E_1 I_1} dx + \frac{{N_1}^2}{E_1 A_1} L + \frac{f}{E_2 A_2} + 2\frac{{T_1}^2}{E_3 A_3} \sqrt{\left(\frac{L}{2}\right)^2 + f^2} \tag{7}$$

where $M_i$and N_i are the bending moment and axial force of the beam in the i-th system. The tension T_1 in the cable material of the first system under unit load can be expressed by the following equation:

$$T_1 = \frac{\sqrt{\left(\frac{L}{2}\right)^2 + f^2}}{2f} \tag{8}$$

Therefore, from the displacement compatibility, the statically indeterminate force $X = (-\delta_{10}/\delta_{11})$ on the strut can be obtained as follows:

$$X = -\frac{\frac{5QL^4}{384E_1I_1}}{\frac{L^3}{48E_1I_1} + \frac{f}{2E_1A_1}\left(2\alpha + \beta(1 + \frac{3}{2}\gamma^3) + \gamma^3\right)} \tag{9}$$

where $\alpha = E_1I_1/\, E_2A_2$, $\beta = E_1A_1/\, E_3A_3$,$\gamma = L/2f$. Using the derived statically indeterminate force X, it is possible to obtain the axial force N_B, bending moment M_B, deflection δ_B , and axial force of the cable.

Similarly, the statically indeterminate force X on the struts of the BSS with two struts under the distributed load Q shown in Figure 4 is given by the following equation.

$$X = -\frac{\frac{Q(L^2 + Ly - y^2)(L - y)y}{12E_1I_1}}{\frac{(3L + 2y)y^2}{3E_1I_1} + \frac{2f}{E_1A_1}\left(1 + \alpha + (1 + \beta)\gamma\delta^2 + (\frac{3}{2} - \beta)\delta^3\right)} \tag{10}$$

where $\alpha = E_1I_1/\, E_2A_2$, $\beta = E_1A_1/\, E_3A_3$, $\gamma = L/2f$ and $\delta = y/f$. where y is the distance from the support to the strut.

3.2 Load conditions and cost index

For the load acting on the BSS, a crowd load of Q_l =350N/mm^2 was used, assuming a pedestrian bridge. The aluminum alloy A6061-T6 and the steel SS400 were assumed for the beam and struts. Three types of structural stainless steel wire ropes (JIS G 3550) with different cable diameters were used as the cable materials. The material properties of each material and cable material are shown in Tables 1 and 2, respectively. To simplify the material cost in this study, we used the relative cost per unit mass [4]. In other words, the cost indices listed in Tables 1 and 2 indicate that the price of the aluminum alloy material is 6.5 times and the stainless steel material is 6.0 times the price of the steel material when the price of the steel material is 1.0.

Table 1: Materials properties of steel and aluminum alloy

materials	Aluminum alloy (A6061-T6)	Steel (SS400)
Modulus of elasticity [GPa]	70.0	200.0
Density [ton/mm^3]	2.7×10^{-9}	7.9×10^{-9}
Yield stress [N/mm^2]	235.0	255.0
Poisson's ratio	0.34	0.30
Cost index [1/ kg]	6.5	1.0

Table 2: Material properties of cable

Diameter [mm]	14	20	28
Cross-sectional area [mm^2]	93.5	191.0	374.0
Modulus of elasticity [GPa]	88.0		
Unit mass [kg/mm]	0.796×10^{-3}	1.63×10^{-3}	3.18×10^{-3}
Rupture load [N]	121	234	432
Cost index [1/ kg]	6.0 (Stainless steel)		

3.3 Formulation of the material cost minimization problem

The material cost of the BSS can be expressed by the following equation using the cost index α_i and the mass W_i of any member.

$$C = \alpha_i W_i = \alpha_{\mathrm{B}} W_{\mathrm{B}} + \alpha_{\mathrm{T}} W_{\mathrm{T}} + \alpha_{\mathrm{C}} W_{\mathrm{C}} \tag{11}$$

where $\alpha_{\mathrm{B}} W_{\mathrm{B}}$ denotes the material cost of the beam, $\alpha_{\mathrm{T}} W_{\mathrm{T}}$ denotes the material cost of the strut, and $\alpha_{\mathrm{C}} W_{\mathrm{C}}$ denotes the material cost of the cable material, respectively. The member mass W_i of the BSS can be expressed by the following equation:

$$W_i = \rho_i A_i l_i \tag{12}$$

where ρ_i is the material density, A_i is the cross-sectional area, and l_i is the member length. To solve the cost minimization problem, the beam and struts are assumed to have rectangular cross sections and the cable members have circular cross sections. The design variable X is defined as follows:

$$\boldsymbol{x} = \{\boldsymbol{x}^{\mathbf{B}}, \boldsymbol{x}^{\mathbf{T}}, \boldsymbol{x}^{\mathbf{C}}\} \tag{13}$$

where the design variables $\boldsymbol{x}^{\mathbf{B}}$ for beams, $\boldsymbol{x}^{\mathbf{T}}$ for struts, and $\boldsymbol{x}^{\mathbf{C}}$ for cables can each be expressed by the following equation:

$$\boldsymbol{x}^{\mathbf{B}} = \{b_{\mathrm{B}}, L\} \tag{14}$$

$$\boldsymbol{x}^{\mathbf{T}} = \{b_{\mathrm{T}}, f, y\} \tag{15}$$

$$\boldsymbol{x}^{\mathbf{C}} = \{D\} \tag{16}$$

where L is the span of the beam, b_{B} is the height of the beam, b_{T} is the thickness of the strut, f is the height of the strut, y is the distance from the support to the strut, and D is the diameter of the cable material. The beam and strut widths h_{B}, h_{T} are uniformly 1500 mm, and the diameters of the cable materials D are given as 14, 20 and 28 mm, as shown in Table 2.

In the cost minimization optimization, the span L was evaluated every 5 m, and the material cost was considered up to the maximum span L_{max}, which no longer satisfies the constraint condition. The optimization problem is formulated as follows for the objective function $f(\boldsymbol{x})$ led in equation (11), with constraints on the beams, struts, and cable.

$$\text{Min.} \quad f(\boldsymbol{x})$$

$$\text{s.t.} \quad \begin{cases} |\sigma_{\mathrm{B}}| \leq \mathrm{Min.}\left(\sigma_{\mathrm{cr}}, \sigma_y\right) \\ |\sigma_{\mathrm{T}}| \leq \mathrm{Min.}\left(\sigma_{\mathrm{crT}}, \sigma_y\right) \\ \sigma_{\mathrm{C}} \leq \sigma_{\mathrm{y}} \\ \delta_{\mathrm{max}} \leq \delta_{\mathrm{y}} \end{cases} \tag{17}$$

where $\boldsymbol{x}$ denotes the design variable satisfying with the following equation.

$$\begin{cases} b_{\mathrm{B}} \leq 200 \\ b_{\mathrm{T}} \leq 200 \\ 0.05L \leq f \leq 0.5L \\ y < 0.5L \end{cases} \tag{18}$$

Here, the strut height f is limited to half of the span. In the case of two struts, the distance y from the support to the strut is assumed to vary so that the struts do not overlap each other. The $\sigma_{\mathrm{B}}, \sigma_{\mathrm{T}}$ and σ_{c} on the left-hand side of the constraints are the maximum or minimum stresses in beams, struts, and cables, and $\delta_{\max}$ is the maximum deflection of the BSS. On the right-hand side of the constraints, σ_{cr} is the buckling stress of the beam calculated from the Bernulli-Euler beam, σ_{crT} is the buckling stress of the strut calculated from the Timoshenko beam, σ_y is the yield stress of the material and δ_{y} is the deflection limit. The deflection limit was set to $L/600$. Local buckling of flanges and lateral buckling of beams were not considered to simplify the calculations.

3.4 Results of material cost minimization problem

In this section, the optimal solutions for the aluminum alloy using the BSS with one and two struts are described in comparison with simple supported beams.

3.4.1 Results for simple supported beams

Figure 7 shows the results of the cost minimization problem for the steel and aluminum simple supported beams as a comparison for the aluminum alloy BSS. Figure 7 shows the relationship between material cost and span, respectively. In addition to the 2.5 m span, numerical optimization was performed in 5 m increments starting from 5 m span. When the constraint condition was no longer satisfied, the maximum span was defined as the largest span that satisfied the constraint condition, calculated in 0.5 m increments from the immediately preceding span. The black line in the figure indicates the optimal solution for the steel simple supported beams (SS-ST) and the red line indicates the optimal solution for the aluminum alloy simple supported beams (SS-AL). The X-marc in the figure indicates the result for the maximum span.

Figure 7 shows that the material cost of the aluminum alloy simple supported beams was always higher than that of the steel simple supported beams. For example, the material cost of the aluminum alloy simple supported beam at L=10 m was 44057, which was 3.2 times that of the steel one, 13976. Comparing the maximum spans of both simple supported beams, the maximum span of the aluminum-alloy simple supported beam was $L_{\max} = 11.5\text{m}$, which is approximately 30% shorter than that of the steel simple supported beam. This is because the

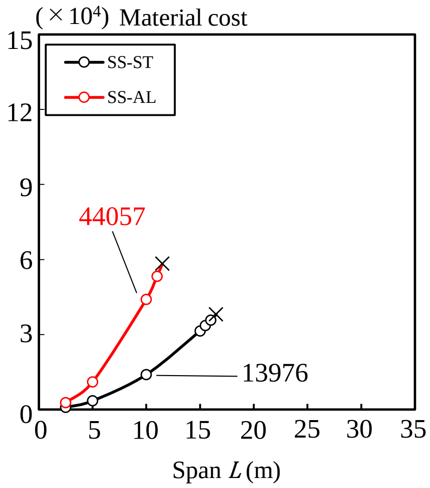


Figure 7: Material cost of simple supported beam versus span

aluminum alloy material is less rigid than the steel material and no longer satisfies the deflection limit at the early stage, resulting in the shorter maximum span.

3.4.2 Results for BSSs with one strut

The results of the cost minimization problem for the aluminum alloy BSSs with one strut are shown in Figure 8. The vertical and horizontal axes in the figure are the same as in Figure 7. The blue dotted, dashed, and solid lines in the figure indicate the optimal solutions for cable diameters of $D = 14$, 20 and 28mm, respectively.

Figure 8 shows that the cost of the aluminum alloy BSS with one strut is reduced for all cable diameters and the maximum span is increased compared to the aluminum alloy simple supported beam. For example, in the case of $L = 10$m, the cost of the aluminum alloy simple supported beam is $C = 44057$, while it decreases as the cable diameter D changes from 14,20, to 28mm, $C = 33248$ (25% reduction), 16508 (63% reduction), 12987 (71% reduction). The cost of a tension beam with a cable diameter of $D = 28$mm was almost equal to that of the steel simple supported beam. The maximum span of the BSS with $D = 28$mm was $L_{\max} = 25.5$m, 1.5 times longer than that of the steel simple supported beam.

3.4.3 Results for BSSs with two struts

The results of the cost minimization problem for the aluminum alloy BSS with two struts are shown in Figure 9. The vertical and horizontal axes as well as the black and red lines in the figure follow Figure 7. The green dotted, dashed, and solid lines in the figure indicate the optimal solutions when the cable diameter is $D = 14$, 20 and 28mm, respectively.

Figure 9 shows that the BSS with two struts has a lower cost and a higher maximum span than the aluminum alloy simple supported beam, as well as the BSS with one strut. For example, for $L = 10$m, changing the cable diameter to $D = 14, 20$ or 28mm resulted in $C = 31182$ (29% reduction), 10253 (77% reduction) and 8836 (80% reduction), indicating that the cost of

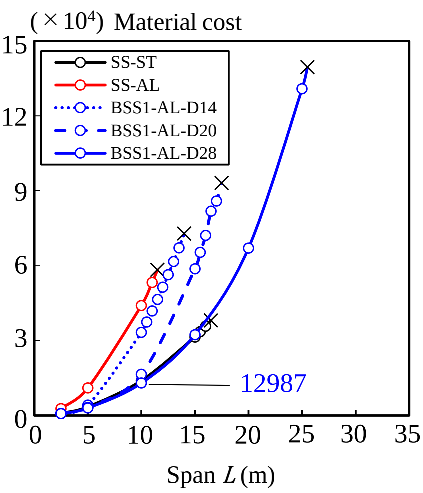


Figure 8: Material cost of BSSs with a strut versus span

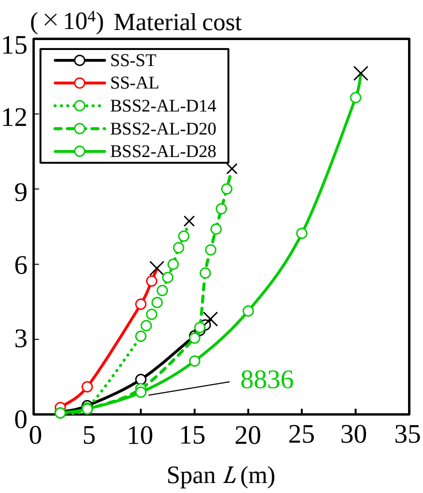


Figure 9: Material cost of BSSs with two struts versus span

the BSSs with $D = 28$mm cable diameter was lower than that of the steel simple supported beams. The maximum span of the BSSs with $D = 28$mm was $L_{\max} = 30.5$m, 1.8 times longer than that of the steel simple supported beam.

4 CONCLUSION

In this study, we proposed the BSS based on the topology optimization for the aluminum alloy materials, which are still rarely applied as the materials in civil engineering. The superiority of the material cost over steel materials was evaluated from the viewpoint of the material cost minimization problem. The results of this study are summarized as follows:

(1) The cost of the aluminum alloy simple supported beam was approximately 3 times higher than that of the steel one. However, the use of the BSSs allows a longer span and lower cost than the steel simple supported beam, indicating the usefulness of the BSSs.
(2) The maximum span was longer when the number of struts in the BSSs was greater. And, the cost was found to decrease as the cable diameter increased regardless of the number of struts.

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