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DEVELOPMENT AND ANALYSIS OF PRE-STRESSED CABLE ROOF WITH STIFFENING GIRDER AND POLYMER MEMBRANE CLADDING

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Summary. *The roof construction considered in the research consists of a flexible framework with a stiffening girder and architectural membrane. The framework is a two-chord truss-like structure. The chords are made of pre-stressed steel cables linked together by vertical struts. The girder mitigates deformations of the roof. It takes horizontal reactions of the chords allowing lightweight supporting structures. Design clearances prevent overstressing the girder. Computational technique for static analysis of the roof is developed. The technique allows satisfying the limit state conditions. Uniform and half-span loads are taken into account. The membrane cladding is included into the structural model. The results of the work are to be used for the preliminary design stage of pre-stressed cable roofs. The work contributes to the development of hybrid building constructions.*

1 INTRODUCTION

Pre-stressed cable and membrane structures become an integral part of modern public buildings such as railway stations, airports, business centres and stadiums[1,2]. They are an efficient solution for sheltering large-span areas[3]. Along with civil engineering sector, cable and membrane roof structures have great potential in the field of industrial construction as well[4].

Cable structures, however, exhibit complex behavior under load. Numerical methods for nonlinear structural analysis are based on iterative solving the equations of equilibrium[5]. Numerical methods, however, require the main structural parameters to be given. Thus, analytical technique is needed to determine stiffness properties of the cables and the magnitude of pre-stressing of the structural elements. Simplified approach is also required for understanding the structural behavior and for assessing the numerical results[6]. Linear static solution for cable roofs is proposed[7]. More sophisticated approach for nonlinear analysis of cable structures includes closed-form solutions for biconvex and biconcave trusses[8].

Being highly deformable, cable systems are combined with stiffening girders. The girders effectively mitigate structural deflections under load, including, so-called, kinematic displacements. They also take horizontal reactions brought about by the cables[9,10,11]. It allows applying lightweight supporting structures instead of complicated solutions for equilibrating the thrust.

The roof construction, considered in the present research, consists of a framework and flexible polymer membrane (figure 1). The framework is composed of two-chord truss-like cable structures with stiffening girders. The girders are arranged between the opposite supports of the trusses. The chords of the framework are made of pre-stressed steel cables linked together by vertical struts. The membrane is attached to the top chord of the roof.

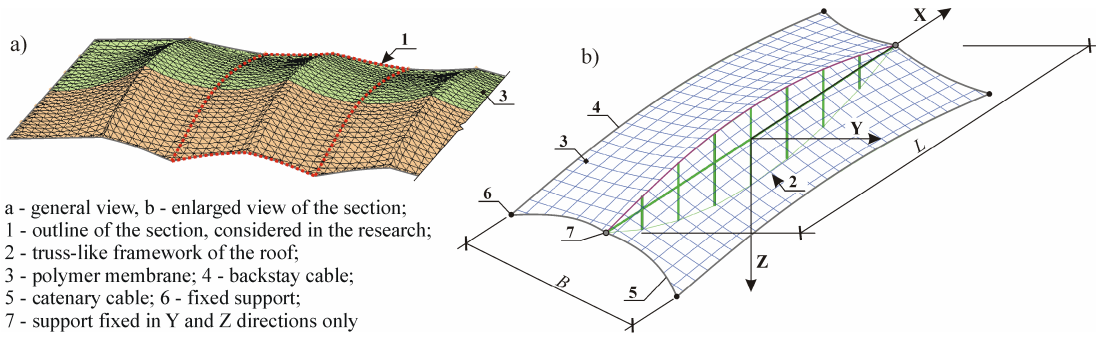

Figure 1: The cable roof structure

Due to high relative stiffness, the girder, however, bears the major portion of external transverse impacts while the flexible chords remain underused. In order to enhance the efficiency of high-strength steel cables, the girder and the struts of the framework are linked together by means of design clearances[12,13] (figure 2). The clearance, being a gap between the girder and a retainer of the strut, allows the framework to move freely a distance Δ in the vertical direction. It prevents overstressing of the girder under uniformly distributed loads. Having been exhausted by non-uniform impacts, the clearances do not preclude leveling the loads and reducing deformations of the roof.

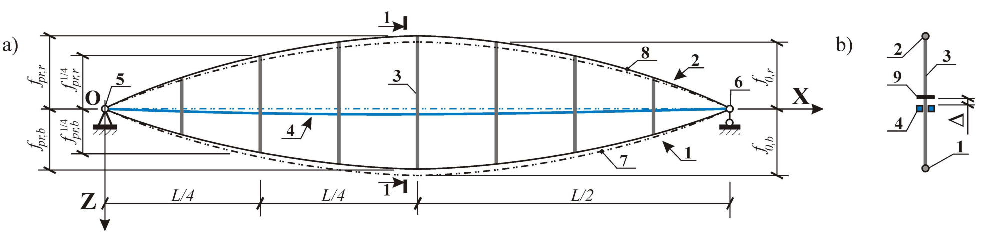

Figure 2: Structural model of the roof. a – general view, b – view along the line 1-1; 1 – bearer chord; 2 – restraining chord; 3 – vertical strut; 4 – stiffening girder; 5 – fixed support; 6 – support fixed in the Z direction only; 7, 8 – previous and current positions of the framework, respectively; 9 – retainer; Δ - design clearance

A section of the roof, highlighted in figure 1a, is analyzed in the present paper. Computational technique for static analysis is developed. The polymer membrane cladding is included into the structural model. The technique allows satisfying the limit state conditions. It provides the allowable deformations of the roof at the pre-stressing and operational stages. Stiffness properties of the cables and the girder, required pre-tensioning of the bottom chord and the size of the design clearance are given.

2 THE STRUCTURAL MODEL

Two-chord cable truss with the girder is adopted as the structural model of the roof's section (figure 2). The model is situated in XOZ -plane. Nodal displacements are only allowed in the plane. The struts shown schematically are taken incompressible, while the span of the roof L is considered constant.

The bearer chord of the truss consists of steel cables with the stiffness EA_b while the restraining chord is composed of a cable and polymer membrane. The overall stiffness of the restraining chord EA_r splits into the cable's stiffness $EA_{r,cab}$ and the effective stiffness of the membrane $\overline{EA}_{r,m}$:

$$EA_r = EA_{r,cab} + \overline{EA}_{r,m} . \tag{1}$$

At the initial (undeformed) state the chords of the truss take on parabola-like shape with the rises $f_{0,b}$ and $f_{0,r}$. Hereinafter, the initial configuration is marked by the subscript '0'. The indexes $'b'$ and $'r'$ refer to the bearer and restraining chords, respectively (figure 2).

The pre-stressing of the roof is implemented by means of the tensioning the bearer chord. Having been pre-stressed, the truss transforms into the operational state with rises $f_{pr,b}$, $f_{pr,r}$ in the center of the span and $f^{1/4}{}_{pr,b}$, $f^{1/4}{}_{pr,r}$ in the quarter of the span:

$$f_{pr,b} = f_{0,b} - \Delta f_{pr} , \tag{2a}$$

$$f_{pr,r} = f_{0,r} + \Delta f_{pr} , \tag{2b}$$

where Δf_{pr} is the cambering of the roof at the stage of the pre-stressing (figure 3).

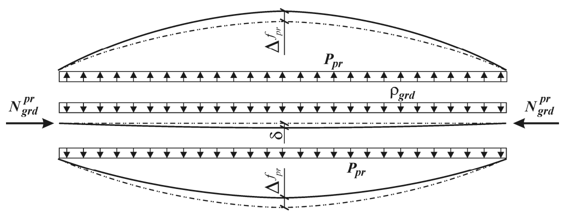

Figure 3: Structural model at the pre-stressing stage

Considering uniform pre-stressing, the chords of the roof keep parabola-like shape. Thus, the ordinate at the quarter of the span $f^{1/4}{}_{pr}$ obeys the following condition: $f^{1/4}{}_{pr} = 0.75 \cdot f_{pr}$.

External loads are applied to the membrane cladding at the operational stage. They result in the deflection of the roof Δf and the following alteration of the rises of the chords:

$$f_{ld,b} = f_{pr,b} + \Delta f\ , \tag{3a}$$

$$f_{ld,r} = f_{pr,r} - \Delta f\ . \tag{3b}$$

The limit state approach is applied for the analysis of the truss. The serviceability limit state implies the following conditions:

$$\Delta f \le \Omega_{\lim}\ , \tag{4}$$

$$\Theta_{\lim,1} \le \Theta_c\ , \tag{5}$$

where $\Omega_{\lim}$ is the allowable displacement; $\Theta_{\lim,1} = 0.01$ is the minimal relative tensioning which prevents slackening of the flexible chords; Θ_c is the following ratio:

$$\Theta_c = \varepsilon / \zeta_c\ , \tag{6}$$

where ε is the relative deformation of the chord; ζ_c is the allowable relative deformation:

$$\zeta_c = R_c / E_c\ , \tag{7}$$

$$\varepsilon = \frac{L_c}{L_{c0}} - 1\ , \tag{8}$$

where R_c and E_c are the strength and the modulus of elasticity of the material; L_{c0} is the initial chord's length; L_c is the current length of parabola-shaped chord:

$$L_c = \Psi_4 \cdot f^4 + \Psi_2 \cdot f^2 + L\ , \tag{9}$$

where f is the rise of the chord at the center of the span; $\Psi_2 = 8/(3 \cdot L)$ and $\Psi_4 = -32/(5 \cdot L^3)$ are the coefficients.

The initial length of the cable L_{c0} is the difference between its geometric length L_g and the tensioning ΔL_p at the stage of the pre-stressing:

$$L_{c0} = L_g - \Delta L_p\ . \tag{10}$$

The geometric length of the cable L_g is calculated from (9) given the initial sag f_0.

The ultimate limit state condition is written as follows:

$$\Theta \le \Theta_{\lim,2}\ , \tag{11}$$

where $\Theta = \Theta_c$ for the chords of the truss (6), while $\Theta = \Theta_{grd}$ is the ratio for the girder; $\Theta_{\lim,2} = 1.0$ is the upper bound for the Θ-ratio.

The girder's ratio Θ_{grd} is obtained as follows[14]:

$$\Theta_{grd} = \frac{N_{grd}}{\varphi_e \cdot A_{grd} \cdot R_{grd}}\text{, if } m_{ef} < 20\text{;}\quad \Theta_{grd} = \frac{M_{grd}}{W_{grd} \cdot R_{grd}}\ \text{otherwise,} \tag{12}$$

where N_{grd} and M_{grd} are the axial force and the bending moment in the girder, respectively; A_{grd} and W_{grd} are the area and the elastic section modulus of the girder's cross section; R_{grd} is the material strength of the girder; φ_e is the buckling coefficient of the girder subjected to compression with bending.

The buckling coefficient φ_e is obtained from the design code[14] given the effective slenderness $\bar{\lambda}$ and the adjusted relative eccentricity m_{ef}:

$$\bar{\lambda} = \frac{L}{\sqrt{I_{grd} / A_{grd}}} \cdot \sqrt{\frac{R_{grd}}{E_{grd}}}\,, \qquad (13a)$$

$$m_{ef} = \eta \cdot \frac{M_{grd}}{N_{grd}} \cdot \frac{A_{grd}}{W_{grd}}\,, \qquad (13b)$$

where I_{grd} is the moment of inertia of the girder's cross section; E_{grd} is the Young's modulus of the girder's material; L is the length of the girder; η is the shape factor[14].

Bending moment in the girder M_{grd} is obtained considering transverse and longitudinal impacts:

$$M_{grd} = M_Q + N_{grd} \cdot \delta\,, \qquad (14)$$

where M_Q is the bending moment brought about by the transverse load; δ is the deflection of the girder under load[15]:

$$\delta = \frac{\delta_Q}{1 - N_{grd} / N_{el}}\,, \qquad (15)$$

where δ_Q is the deflection by transverse load only; N_{el} is the Euler load, $N_{grd} < N_{el}$:

$$N_{el} = \pi^2 \cdot E_{grd} \cdot I_{grd} / L^2\,. \qquad (16)$$

The axial force in the girder is induced by the chords of the truss:

$$N_{grd} \approx N_{b,cab} + N_{r,cab} + \frac{N_{r,m}}{2}\,. \qquad (17)$$

where $N_{b,cab}$ and $N_{r,cab}$ are the axial force in the bearer chord and the force in the cable of the restraining chord, respectively; $N_{r,m}$ is the membrane effective force.

The membrane effective force is distributed by the catenary cables into the girder and the neighboring fixed supports (figure 1), so the factor ½ is applied in (17).

The axial forces N are obtained by the Hook's law given the stiffness EA and the relative deformation ε:

$$N = EA \cdot \varepsilon\,. \qquad (18)$$

Considering the conditions (5) and (11), the relative deformations of the cable and the membrane elements, ε_{cab} and ε_m, are bounded as follows:

$$\varepsilon_{cab} \in [\varepsilon_{cab,1} \quad ... \quad \varepsilon_{cab,2}], \tag{19a}$$

$$\varepsilon_{m} \in [\varepsilon_{m,1} \quad ... \quad \varepsilon_{m,2}], \tag{19b}$$

where $\varepsilon_{cab,i} = \Theta_{\lim,i} \cdot \zeta_{cab}$ and $\varepsilon_{m,i} = \Theta_{\lim,i} \cdot \zeta_{m}$ are the boundary values for the cables and the membrane, respectively; $i = \{1, \quad 2\}$.

Considering the restraining chord in a whole, the relative deformations must be in the following range:

$$\varepsilon_{r} \in [\varepsilon_{r,1} \quad ... \quad \varepsilon_{r,2}], \tag{19c}$$

where $\varepsilon_{r,1}$ and $\varepsilon_{r,2}$ are the following boundary values:

$$\varepsilon_{r,1} = \max(\varepsilon_{cab,1}, \quad \varepsilon_{m,1}), \tag{20a}$$

$$\varepsilon_{r,2} = \min(\varepsilon_{cab,2}, \quad \varepsilon_{m,2}). \tag{20b}$$

3 STATIC ANALYSIS OF THE ROOF STRUCTURE

The stage of the pre-stressing of the roof and the operational stage are analyzed separately. The following load cases are taken into account:

1. The full uniform load Q_{tot}.
2. Partial uniform load Q_{prt} and the load Q_{hlf} acting on a half of the span of the roof.

The overall sum of the partial and the half-span loads is considered not greater than the full load: $Q_{prt} + Q_{hlf} \le Q_{tot}$. The girder is subjected to its own weight ρ_{grd} which sums up with residual load from the truss. The loads are considered to exert the roof in the vertical direction from top to bottom.

3.1 Pre-stressing of the truss

Cambering of the roof at the stage of the pre-stressing is confined by the range $\Delta f_{pr} \in (0...\Delta f_{pr,\lim}]$, where the allowable cambering $\Delta f_{pr,\lim}$ is obtained as follows:

$$\Delta f_{pr,\lim} = f_{\lim_2,r} - f_{0,r}, \tag{21}$$

where $f_{\lim_2,r}$ is the upper bound for the rise of the restraining chord, obtained from (22).

The allowable rise of a parabola-shaped chord:

$$f_{\lim} = \frac{L}{2} \cdot \sqrt{\frac{5}{6}} \cdot \sqrt{1 - \sqrt{1 - 3.6 \cdot \left(\frac{L^{\lim}{}_{c}}{L} - 1 \right)}}, \tag{22}$$

where $L^{\lim}{}_{c}$ is the chord's allowable length.

Considering the ultimate limit state condition (11), the allowable length of the restraining chord is the following:

$$L^{\lim_2}{}_{c,r} = L_{c0,r} \cdot (\varepsilon_{r,2} + 1), \tag{23}$$

where $\varepsilon_{r,2}$ is the right-hand side bound for the relative elongation of the restraining chord (20b); $L_{c0,r}$ is the initial length of the restraining chord, obtained from (10) given $\Delta L_p = 0$.

The tensioning of the bearer chord ΔL_p is obtained from (10) given the initial length $L_{c0,b}$ which is calculated as follows:

$$L_{c0,b} = L^{\omega}{}_{c,b} \cdot \frac{1}{1+\varepsilon_{cab,2}}, \tag{24a}$$

where $\varepsilon_{cab,2}$ is the right-hand side bound for the cable elongation (19a); $L^{\omega}{}_{c,b}$ is the length (9) of the bearer cable having the rise (3a) given the allowable deformation $\omega_{\lim}$:

$$f_{ld,b} = f_{pr,b} + \omega_{\lim}. \tag{24b}$$

The allowable deformation $\omega_{\lim}$ at the center of the span, obtained from (4) and (5), may be written as follows:

$$\omega_{\lim} = \min(\Omega_{\lim}, \quad \omega_{\lim,r}, \quad \omega_{\lim,b}), \tag{25}$$

where

$$\omega_{\lim,r} = f_{pr,r} - f_{\lim_1,r}, \tag{26a}$$

$$\omega_{\lim,b} = f_{\lim_2,b} - f_{pr,b}, \tag{26b}$$

where $f_{pr,r}$ and $f_{pr,b}$ are the chord rises (2); $f_{\lim_1,r}$ and $f_{\lim_2,b}$ are the allowable rises (22) given the allowable chord lengths $L^{\lim_1}{}_{c,r}$ and $L^{\lim_2}{}_{c,b}$, respectively:

$$L^{\lim_1}{}_{c,r} = L_{c0,r} \cdot (\varepsilon_{r,1} + 1), \tag{27a}$$

$$L^{\lim_2}{}_{c,b} = L^{pr}{}_{c,b} \cdot \frac{\varepsilon_{cab,2} + 1}{\varepsilon_{cab,1} + 1}, \tag{27b}$$

where $L^{pr}{}_{c,b}$ is the length of the bearer cable (9) at the stage of the pre-stressing, having the rise at the center of the span $f_{pr,b}$ (2a); $\varepsilon_{r,1}$, $\varepsilon_{cab,1}$ and $\varepsilon_{cab,2}$ are the limiting relative deformations (19).

At the stage of the pre-stressing the girder takes the axial force $N^{pr}{}_{grd}$ and the transverse load. The transverse load is the girder's own weight ρ_{grd} (figure 3). Considering (17) and (18), the axial force is obtained as follows:

$$N^{pr}{}_{grd} = EA_b \cdot \varepsilon_{pr,b} + (EA_{r,cab} + \bar{E}A_{r,m}/2) \cdot \varepsilon_{r,2}, \tag{28}$$

where $\varepsilon_{r,2}$ is given by (20b); $\varepsilon_{pr,b}$ is the relative deformation (8) of the bearer chord having the rise $f_{pr,b}$ (2a).

The stiffness of the cable of the bearer chord is the following:

$$EA_b = P_{pr} \cdot \frac{L^2}{8 \cdot f_{pr,b} \cdot \varepsilon_{pr,b}}, \tag{29}$$

where P_{pr} is the link load between the chords (figure 3) at the stage of the pre-stressing.

The stiffness of the cable in the restraining chord $EA_{r,cab}$ is obtained from (1) given the membrane effective stiffness $\overline{EA}_{r,m}$ and the overall stiffness of the chord EA_r:

$$EA_r = P_{pr} \cdot \frac{L^2}{8 \cdot f_{pr,r} \cdot \varepsilon_{r,2}}. \tag{30}$$

Bending moment in the girder and the girder's deflection brought about by its own weight are the following:

$$M_Q = \rho_{grd} \cdot L^2 / 8, \tag{31}$$

$$\delta_Q = \frac{5}{384} \cdot \rho_{grd} \cdot \frac{L^4}{E_{grd} \cdot I_{grd}}. \tag{32}$$

3.2 Impact of the full uniform operational load Q_{tot}

The full uniform load Q_{tot} is considered to be completely taken by the cable truss. Deformation of the truss is assumed equal to the allowable deflection $\Delta f_{ld} = \omega_{\lim}$ (25). Under this condition, the link load between the chords P_{pr} at the stage of the pre-stressing is determined by the following expression:

$$P_{pr} = \frac{Q_{tot}}{\dfrac{f_{ld,b} \cdot \varepsilon_{cab,2}}{f_{pr,b} \cdot \varepsilon_{pr,b}} - \dfrac{f_{ld,r} \cdot \varepsilon_{ld,r}}{f_{pr,r} \cdot \varepsilon_{r,2}}}, \tag{33}$$

where $\varepsilon_{cab,2}$ and $\varepsilon_{r,2}$ are the limiting relative deformations (19); $\varepsilon_{pr,b}$ and $\varepsilon_{ld,r}$ are the relative deformations (8) of the bearer and the restraining chords given the rises $f_{pr,b}$ (2a) and $f_{ld,r} = f_{pr,r} - \omega_{\lim}$, respectively; $f_{pr,r}$ and $f_{ld,b}$ are the chord rises (2b) and (24b).

The girder takes its own weight ρ_{grd} in the transverse direction, while the following axial load influences the girder in the longitudinal direction:

$$N^{ld}{}_{grd} = EA_b \cdot \varepsilon_{cab,2} + (EA_{r,cab} + \bar{E}A_{r,m}/2) \cdot \varepsilon_{ld,r}. \tag{34}$$

Bending moment in the girder M_Q and the girder deflection δ_Q brought about by the transverse load stay the same as for the pre-stressing stage (31, 32).

Considering uniformity of the load Q_{tot}, the design clearance values throughout the span are taken to be determined by the parabola-shaped curve:

$$\Delta(x) = 4 \cdot \Delta_{1/2} \cdot x \cdot (L - x) / L^2, \tag{35}$$

where $x \in [0...L]$ is the X-coordinate along the span; $\Delta_{1/2}$ is the design clearance at the center of the span:

$$\Delta_{1/2} = \omega_{\lim} . \tag{36}$$

3.3 Impact of the partial uniform load Q_{prt} and the load Q_{hlf} acting on a half of the span of the roof

Partial uniform load Q_{prt} is completely taken by the cable truss. Half-span load Q_{hlf} is divided into $Q_{hlf,t}$ and $Q_{hlf,grd}$. The first part influences the truss, while the remaining one impacts the girder (figure 4).

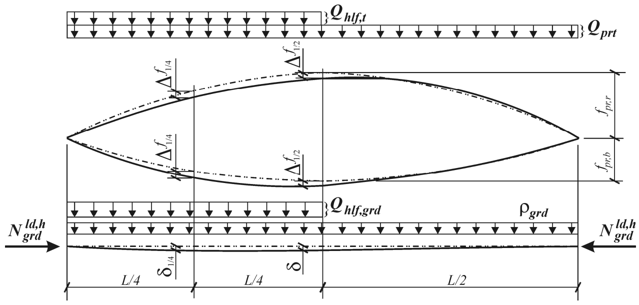

Figure 4: Partial uniform load and the load acting on a half of the span

Determination of loads $Q_{hlf,t}$ and $Q_{hlf,grd}$ is performed using the following conditions:

$$Q_{hlf,t} + Q_{hlf,grd} = Q_{hlf} , \tag{37a}$$

$$Q_{hlf,t} \le Q_{hlf} , \tag{37b}$$

$$\Delta f_{1/4} = \Delta_{1/4} + \delta_{hlf,1/4} , \tag{37c}$$

where $\Delta f_{1/4}$ is the deflection of the truss in the quarter of the span, brought about by the loads Q_{prt} and $Q_{hlf,t}$; $\delta_{hlf,1/4}$ is the girder's deflection (15) caused by the transverse load $Q_{hlf,grd}$ and the axial force induced by the truss $N^{ld,h}{}_{grd}$; $\Delta_{1/4}$ is the size of the design clearance in the quarter of the span:

$$\Delta_{1/4} = 0.75 \cdot \Delta_{1/2} . \tag{38}$$

The axial force in the girder $N^{ld,h}{}_{grd}$ is obtained as follows:

$$N^{ld,h}{}_{grd} = N^{ld,h}{}_{b} + (EA_{r,cab} + \bar{E}A_{r,m}/2) \cdot \frac{N^{ld,h}{}_{r}}{EA_r} , \tag{39}$$

where $N^{ld,h}{}_{b}$ and $N^{ld,h}{}_{r}$ are the axial forces in the chords of the cable truss subjected to non-uniform loads $Q_{hlf,t}$ and Q_{prt}.

If the condition (37b) is not met, the girder does not take transverse load: $Q_{hlf,grd} = 0$,

$Q_{hlf,t} = Q_{hlf}$ and the condition (37c) is skipped.

Bending moment in the girder and the girder deflection consist of the parts brought about by the load $Q_{hlf,grd}$ and the girder's own weight ρ_{grd} :

$$M_Q = M_{Q,hlf} + M_\rho , \tag{40a}$$

$$\delta_Q = \delta_{Q,hlf} + \delta_\rho . \tag{40b}$$

The resultant moment M_{grd} for verifying the ultimate limit state condition (11) is obtained by (14).

4 NUMERICAL EXAMPLE

A section of the roof (figure 1b) is considered as an example. Its span L is 12 m, while the width B is 6 m. Initial rises of the chords are $f_{0,b} = 1.5$ m and $f_{0,r} = 1.0$ m. The cables, forming the chords, are adopted of high-strength steel, while the membrane cladding is made of architectural fabrics. The allowable relative deformations (7) are $\zeta_{cab} = 5.385 \cdot 10^{-3}$ and $\zeta_m = 2.231 \cdot 10^{-2}$ for the cables and the membrane, respectively. The girder is made of two steel channel bars with $R_{grd} = 2.1 \cdot 10^5 kN/m^2$. Geometrical parameters of the overall cross section are the following: $A_{grd} = 7.04 \cdot 10^{-3} m^2$, $W_{grd} = 6.16 \cdot 10^{-4} m^3$ and $I_{grd} = 8.32 \cdot 10^{-5} m^4$. The full uniform operational load is taken $Q_{tot} = 18.0$ kN/m, while partial uniform load Q_{prt} and half-span load Q_{hlf} are assumed equal to 9.0 kN/m.

The cambering of the roof during the pre-stressing stage is adopted equal to the allowable one (21): $\Delta f_{pr} = 0.1436$ m. The allowable displacement (25) of the roof under external loads is taken $\omega_{\lim} = 0.1$ m, so the design clearances are the following: $\Delta_{1/2} = 0.1$ m and $\Delta_{1/4} = 0.075$ m. The tensioning of the bearer chord during the pre-stressing $\Delta L_p = 0.0933$ m is obtained from (10), considering (24a).

The link load at the stage of the pre-stressing is $P_{pr} = 2.16$ kN/m (33). Substituting it into (29) and (30) yields in the stiffness properties of the chords: $EA_b = 42719$ kN and $EA_r = 6314$ kN.

The roof is numerically simulated by means of the software package of non-linear structural analysis EASY.2020[16]. Considering the membrane effective stiffness $\overline{EA}_{r,m} = 3600$ kN and the modulus of elasticity of the cables $E_{cab} = 1.3 \cdot 10^5$ MPa, the bearer and restraining chords are adopted of 24.1 mm and 6.1 mm steel cables, respectively. Thus, the stiffness properties of the chords, used for the numerical simulation, are $EA_{b,E} = 43940$ kN and $EA_{r,E} = 6460$ kN. Hereinafter, the subscript '*E*' refers to the values substituted into or obtained by the EASY.2020 software.

The cambering of the roof at the pre-stressing stage is $\Delta f_{pr,E} = 0.1389$ m. It deviates from the adopted value Δf_{pr} by 3.3%. The deflection $\Delta f_{ld,E}$, brought about by the uniform load

Q_{tot}, is 0.099 m versus the allowable value $\omega_{\lim} = 0.1$ m.

The Θ_c-ratios obtained for the bearer and the restraining chords given the forces provided by the EASY software are the following: $\Theta_{c,pr,b} = 0.11$, $\Theta_{c,pr,r} = 0.99$, $\Theta_{c,ld,b} = 0.96$, $\Theta_{c,ld,r} = 0.26$. All the ratios meet the conditions (5) and (11).

Under the load Q_{tot} the axial force in the girder is obtained $N^{ld}{}_{grd} = 243.9$ kN (34). The discrepancy from the result by the EASY software ($N^{ld}{}_{grd,E} = 235.0$ kN) is 3.8%. The girder's ratio (12), $\Theta_{grd} = 0.41$, meets the conditions (11).

The load $Q_{hlf} = 9.0$ kN/m acting on a half of the span splits into $Q_{hlf,t} = 4.8$ kN/m and $Q_{hlf,grd} = 4.2$ kN/m. The ratios (6, 12) for the girder and for the chords of the truss, being equal to $\Theta_{grd} = 0.58$, $\Theta_{c,r} = 0.69$ and $\Theta_{c,b} = 0.74$, satisfy the limit state conditions (5, 11). The deflection of the cable truss in the quarter of the span[17] and the axial force in the girder are the following: $\Delta f_{1/4} = 0.102$ m and $N^{ld,h}{}_{grd} = 165.4$ kN. The corresponding results by the EASY software are $\Delta f_{1/4,E} = 0.096$ m and $N^{ld,h}{}_{grd,E} = 174.3$ kN. The deviations are less than 6.5%.

In order to highlight the effect of the girder and the design clearances, the following ordinary embodiments of the roof are considered. If no design clearances are provided, the stresses in the girder become substantially higher and the girder's ratio $\Theta_{grd} = 1.14$ obtained for the load Q_{tot} exceeds the allowable value $\Theta_{\lim,2} = 1.0$. On the other hand, having no girder's support, the framework deforms by $\Delta f_{1/4} = 0.145$ m under the half-span load Q_{hlf}. It fails the serviceability limit state condition (4) and requires increasing of cross section areas and additional pres-stressing of structural elements.

CONCLUSIONS

- Computational technique for static analysis of the cable roof with stiffening girder and membrane cladding is developed. The results provided by the technique are very close to the ones obtained by the specialized software package EASY.2020.
- Allowable cambering of the roof, required tensioning of the bearer chord and the link load at the pre-stressing stage are proposed. Stiffness properties of the chords and the design clearances between the struts and the girder are given.
- The developed technique allows achieving chord ratios ($\Theta_{c,pr,r} = 0.99$ and $\Theta_{c,ld,b} = 0.96$) close to the upper bound $\Theta_{\lim,2} = 1.0$. It prevents inefficient consumption of high-strength cables under the limit state conditions satisfied.
- The favorable effect of the girder and the design clearances is highlighted. The deformation of non-uniformly loaded roof without the girder is 1.45 times as large as the deformation of the roof with the girder installed. Skipping the design clearances leads to overstressing the girder by the uniform load.
- The results of the work are to be used for the preliminary design stage of pre-stressed cable roofs. The work contributes to the development of hybrid building constructions.

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