A posteriori error estimation and adaptivity for second-order optimally convergent G/XFEM and FEM

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ABSTRACT

The Generalized/eXtended Finite Element Method (G/XFEM) is known to efficiently and accurately solve problems that are challenging for standard methodologies. The method can deliver optimal convergence rates in the energy norm and global matrices with a scaled condition number that has the same order as in the Finite Element Method (FEM). This is achieved even for problems of Linear Elastic Fracture Mechanics (LEFM), which have solutions containing singularities and discontinuities. Despite delivering optimal convergence rates, it has been shown [1], however, that first-order G/XFEM are not competitive with second-order FEM that uses quarter-point elements, especially for three-dimensional (3-D) problems. Because of this, optimally convergent second-order G/XFEM, customized to solve LEFM problems, have been recently proposed [1, 2, 3]. The formulations presented in these works augment both standard lagrangian FEM approximation spaces [3] and pFEM approximation spaces [1, 2] in order to insert into the G/XFEM numerical approximation the discontinuous and singular behaviors of fractures. It is important to note that, in addition to using enrichment functions, G/XFEM still needs local mesh refinement around crack fronts in order to achieve optimal convergence. This must be considered especially for 3-D problems that violate the assumptions of the adopted singular enrichments. While this local mesh refinement can be easily performed for simple cases, the level of refinement that must be used with more complicated problems can be difficult to be defined. A posteriori error estimators and adaptive refinement algorithms can address this issue and this is investigated herein. First, a recently proposed [4] Zienkiewicz and Zhu block-diagonal (ZZ-BD) error estimator is expanded in order to also estimate errors of these second-order G/XFEM formulations [1, 2, 3] for LEFM problems. The associated recovery procedure involves locally weighted L^2 projections of raw stresses on approximation spaces including high-order discontinuous and singular stress fields. The basis functions for these stress approximations are defined using a low-order partition of unity together with polynomial, discontinuous, and singular enrichment functions. The results show that the proposed ZZ-BD error estimator can estimate well discretization errors in the energy norm, with the estimated discretization error converging at the same rate as the exact discretization error. Also, the computed effectivity index of the error estimator is shown to be close to one. Furthermore, the results also demonstrate that the error in the recovered stress field converges at least at the same rate as the exact discretization error in the energy norm. All these results provide upper and lower bounds for the effectivity index of the error estimator [5]. Finally, the coupling between these numerical tools is a natural step to be performed in order to derive adaptive algorithms. The procedures proposed herein allow for robust and reliable simulations of 3-D LEFM problems, with minimal user intervention. The definition of discretization spaces that provide on the fly solutions meeting a pre-specified tolerance to the discretization error and ways of robustly refining the finite element mesh around crack fronts are addressed in this work.

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