Implicit shock tracking for unsteady flows by the method of lines

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ABSTRACT

We present the time-dependent extension of our recently developed high-order implicit shock tracking (HOIST) framework for resolving discontinuous solutions of inviscid conservation laws. Central to the framework is an optimization problem which simultaneously computes a discontinuity-aligned mesh and the corresponding high-order approximation to the flow, which provides nonlinear stabilization and a high-order approximation to the solution. The time discretization is based on method of lines and diagonally implicit Runge-Kutta (DIRK) methods, and we formulate and solve an optimization problem that produces a feature-aligned mesh and solution at each Runge-Kutta stage of each timestep. A Rankine-Hugoniot based prediction of the shock location together with a high-order untangling mesh smoothing procedure provides a high-quality initial guess for the optimization problem at each time, which results in Newton-like convergence of the sequential quadratic programming (SQP) optimization solver. This method is shown to deliver highly accurate solutions on coarse, high-order discretizations without nonlinear stabilization and recovers the design accuracy of the Runge-Kutta scheme. We demonstrate our framework using several inviscid unsteady conservation laws, and we verify that our method is able to recover the design order of accuracy of our time integrator in the presence of strong discontinuities.