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Bayesian Model updating of Linear dynamic systems using complex modal data

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ABSTRACT:

In the lifetime of structures, they may be exposed to damage that deviates their parameters to a new uncertain state. To predict these parameters, they must be updated using experimental data through various model updating methods. Among model updating methods, the Bayesian approach enables the exploration of all the probable models. Many approaches are available, like those that require the solution of the eigenvalue problem and those that do not require the resolution of the eigenvalue problem. Most existing studies have assumed proportional damping, which gives real modal data, which is not the case always. In this paper, a new Bayesian model updating methodology is proposed based on introducing system mode shapes, damping ratios and natural frequencies as additional uncertain parameters. A dynamic condensation technique is used to restrain the model updating problem to work on the observed degrees of freedom (DOF) field only. To decrease the number of uncertain parameters, system mode shapes are integrated out, and Transitional Monte Carlo Markov Chain (TMCMC) is used to sample from the posterior probability density function. The proposed approach has been applied to the 3-storey shear building model. Results show that the proposed methodology can predict the updated structural parameters in many cases, like the cases where the number of observed modes is greater than the number of observed DOF and if the observed modes are not the lowest-frequency modes.

1. INTRODUCTION

Finite element (FE) is widely used in civil engineering applications, specially, FE model updating are found to be useful in many applications such as Structural Health Monitoring (SHM), estimating the structural responses, and structural control applications. In most of the works in the literature [1–3] two assumptions are made: first, the system is assumed to be linear, and second, the system is assumed to be undamped or classically damped, giving identical results. However, the second assumption does not always reflect the actual systems for many reasons, such as the availability of different damping sources in the structural model that will dis-preserve the modes orthogonality [4] or if the system is equipped with supplemental viscous dampers.

The deterministic [5] or probabilistic [1, 6, 7] approach can be followed to update the FE model. The probabilistic approach has the advantage of exploring the whole family of probable models and quantifying for uncertainties [8]. Many probabilistic model updating approaches are available; the Bayesian approach is the most used one [9–11] Recently, the approaches that do not require the eigenvalue problem's solution have gained importance since they can be applied with a considerable reduction in computational efforts [12, 13].

Model updating requires measured data to update the FE model. Various form of dynamic tests data has been used. Modal data, i.e., (natural frequencies, damping ratios, mode shapes) acquired using modal identification techniques [14–16] are universally used in model updating. In this paper, a stochastic approach for updating linear dynamic systems using modal data acquired from non-classically damped mechanical system. To demonstrate the effectiveness of the approach, a 3-DOF simulation example has been applied and the results shows that the approach is working properly when the number of the measured DOFs are less than the number of the full DOFs of the model.

1. BAYESIAN APPROACH FOR MODEL UPDATING

The Bayesian approach for model updating is a probabilistic approach based on the Bayes theorem. As a probabilistic model updating approach, probability represents the degree of belief of an uncertain event conditional on some given information [17].

In Bayes' theorem, conditional probability is used to measure the plausibility of some parameter given other fixed parameters in the model. Using the standard notations in the literature:

$$p(\boldsymbol{\theta}|D) = p(D|\boldsymbol{\theta}).\frac{p(\theta)}{p(D)} \tag{1}$$

In Eq. (1), $\boldsymbol{\theta} \in \Theta \in \mathrm{R}^{n_\theta}$ (where Θ denotes the bounded space of the uncertain parameters) is the uncertain model parameters, D is data, $\mathrm{p}(\boldsymbol{\theta})$ is the prior probability density function (PDF) which is constructed in the absence of the observed data, $\mathrm{p}(D|\boldsymbol{\theta})$ is the likelihood function which represents the likeliness of observing the measured data given some model parameters. The posterior PDF $\mathrm{p}(\boldsymbol{\theta}|D)$ gives the probability model for the updated parameters and the denominator $p(D)$ is the normalization constant.

Experimental modal data can contain an N_s set of data identified from the real structure. A two-stage Bayesian approach is presented by [18] states that for model updating to be performed, a two-stage is followed in which firstly the modal data is identified and then they are used to update the FE model. A major assumption in this approach is to assume a uniform prior to the first stage. Furthermore, since the modal data are globally identifiable from the vibration test, then it will be reasonable to model the likelihood function in the first stage as a normal distribution with mean and covariance matrix identified from the posterior statistics of the first stage. Thus, the posterior PDF of the first stage is given by:

$$p_0(\omega_r \mid D) = (2\pi\hat{C}_{\omega,r})^{-\frac{1}{2}} \exp\left(-\frac{(\omega_r - \hat{\omega}_r)^2}{2\hat{C}_{\omega,r}} \right) \tag{2}$$

where, $\hat{\omega}_r \in \mathbb{R}^+$ denote the most probable values (MPVs) of the modal frequency; $\hat{C}_{\omega,r} \in \mathbb{R}$ denote the posterior variance of ω_r. Similarly, the initial posterior PDF is formulated for the damping ratios $\hat{\zeta}_r \in \mathbb{R}^+$ and the mode shapes $\hat{\phi}_r \in \mathbb{C}^{n_m}$ represents the mode n_m observed DOF ($\| \hat{\phi}_r \| = (\hat{\phi}_r^* \hat{\phi}_r)^{1/2} = 1$, where $\| \cdot \|$ denotes the Euclidean norm and ϕ_r^* denotes the Hermitian transpose of complex vector ϕ_r). The posterior PDF given in Eq. (2) is subscripted with '0' to denote that this posterior is related to the stage one assuming uniform prior.

2. NON-CLASSICALLY DAMPED LINEAR DYNAMIC SYSTEMS

In the systems that incurred non-classical damping, the conventional solution to the eigenequation Eq. (3) will not be feasible. Some or all the N-differential equations are coupled through the mode shapes term and cannot be reduced to an N second-order uncoupled equation.

$$\mathbf{M}\ddot{x}(t) + \mathbf{C}\dot{x}(t) + \mathbf{K}x(\mathrm{t}) = 0 \tag{3}$$

In such cases, the equation of motion can be expressed in state space form by changing of variables and defining the velocity $\nu(t) = \dot{x}(t)$, so that $\ddot{x}(t) = \dot{\nu}(t)$ and solving Eq. (3) for $\ddot{x}(t)$ which gives :$\frac{d}{dt}\nu(t) \equiv \ddot{x}(t) = -\mathbf{M}^{-1}\mathbf{K}x(t) - \mathbf{M}^{-1}\mathbf{C}\dot{x}(t)$ and re-write these two sets of first order differential equations in matrix form:

$$\frac{d}{dt}\begin{Bmatrix} x(t) \\ \nu(t) \end{Bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \begin{Bmatrix} x(t) \\ \nu(t) \end{Bmatrix} \tag{4}$$

where the matrix $\mathbf{A} = \begin{bmatrix} 0_{N\times N} & \mathbf{I_{NXN}} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$ is the system matrix. The solution to the eigenvalue problem specified by the system matrix $\mathbf{A}$ yields a complex eigenvalue $\bar{\lambda}_r$ and complex eigenvectors $\bar{\psi}_r$ given by:

$$\bar{\boldsymbol{\psi}}_r = \begin{bmatrix} \bar{\varphi}_r \\ \bar{\lambda}_r \bar{\varphi}_r \end{bmatrix}, \qquad \bar{\lambda}_r = -\bar{\zeta}_r \bar{\omega}_r + i\bar{\omega}_r \sqrt{1 - \bar{\zeta}_r^2} \tag{5}$$

where $(\bar{\omega}_r, \bar{\zeta}_r, \bar{\varphi}_r)$ represents the r_{th} mode modal frequency, damping ratio and mode shapes, respectively.

In this paper, the modal pair $[\bar{\lambda}_r, \bar{\varphi}_r]$ is replaced with additional uncertain parameter pair $[\lambda_r, \phi_r]$ which need to be updated and hence avoid the solution of the eigenvalue problem.

3. MODEL REDUCTION

Dynamic tests usually are limited to the available sensors. Thus, the full system is reduced to a system that is corresponding to the measured DOFs. This can be accomplished by the using of dynamic condensation method.

To reduce the full system model into a smaller model, mass $\mathbf{M}(\boldsymbol{\theta}_s) \in \mathbb{R}^{n_d \times n_d}$, damping $\mathbf{C}(\boldsymbol{\theta}_s) \in \mathbb{R}^{n_d \times n_d}$ and stiffness $\mathbf{K}(\boldsymbol{\theta}_s) \in \mathbb{R}^{n_d \times n_d}$ matrices (with n_d being the number of DOF of the structural model) parameterized by uncertain structural model parameter set $\boldsymbol{\theta}_s \in \mathbb{R}^{n_{\theta_s}}$, are split into master n_m and slave n_s DOF, $(n_d = n_m + n_s)$ where master is corresponding to observed and slave to unobserved DOFs as follows:

$$\left(\begin{bmatrix} \mathbf{K}^{\mathbf{mm}}(\boldsymbol{\theta}_\mathbf{s}) & \mathbf{K}^{\mathbf{ms}}(\boldsymbol{\theta}_\mathbf{s}) \\ \mathbf{K}^{\mathbf{sm}}(\boldsymbol{\theta}_\mathbf{s}) & \mathbf{K}^{\mathbf{ss}}(\boldsymbol{\theta}_\mathbf{s}) \end{bmatrix} + \bar{\lambda}_r \begin{bmatrix} \mathbf{C}^{\mathbf{mm}}(\boldsymbol{\theta}_\mathbf{s}) & \mathbf{C}^{\mathbf{ms}}(\boldsymbol{\theta}_\mathbf{s}) \\ \mathbf{C}^{\mathbf{sm}}(\boldsymbol{\theta}_\mathbf{s}) & \mathbf{C}^{\mathbf{ss}}(\boldsymbol{\theta}_\mathbf{s}) \end{bmatrix} + \bar{\lambda}_r{}^2 \begin{bmatrix} \mathbf{M}^{\mathbf{mm}}(\boldsymbol{\theta}_\mathbf{s}) & \mathbf{M}^{\mathbf{mm}}(\boldsymbol{\theta}_\mathbf{s}) \\ \mathbf{M}^{\mathbf{mm}}(\boldsymbol{\theta}_\mathbf{s}) & \mathbf{M}^{\mathbf{mm}}(\boldsymbol{\theta}_\mathbf{s}) \end{bmatrix}\right) \begin{bmatrix} \varphi^m \\ \varphi_s \end{bmatrix} = \mathbf{0}. \tag{6}$$

By eliminating the slave DOF field, Eq. (6) yields:

$$\begin{aligned} \mathbf{M}^{\mathrm{R}}(\bar{\lambda}_r, \boldsymbol{\theta}_\mathrm{s}) &= \mathbf{T}(\bar{\lambda}_r, \boldsymbol{\theta}_\mathrm{s})^T \mathbf{M}(\boldsymbol{\theta}_\mathbf{s}) \mathbf{T}(\bar{\lambda}_r, \boldsymbol{\theta}_\mathrm{s}) \\ \mathbf{C}^{\mathrm{R}}(\bar{\lambda}_r, \boldsymbol{\theta}_\mathrm{s}) &= \mathbf{T}(\bar{\lambda}_r, \boldsymbol{\theta}_s)^T \mathbf{C}(\boldsymbol{\theta}_\mathbf{s}) \mathbf{T}(\bar{\lambda}_r, \boldsymbol{\theta}_\mathrm{s}) \\ \mathbf{K}^{\mathrm{R}}(\bar{\lambda}_r, \boldsymbol{\theta}_\mathrm{s}) &= \mathbf{T}(\bar{\lambda}_r, \boldsymbol{\theta}_\mathrm{s})^T \mathbf{K}(\boldsymbol{\theta}_\mathbf{s}) \mathbf{T}(\bar{\lambda}_r, \boldsymbol{\theta}_\mathrm{s}) \end{aligned} \tag{7}$$

where, $\mathbf{M}^{\mathrm{R}} \in \mathbb{C}^{n_m \times n_m}$, Damping $\mathbf{C}^{\mathrm{R}} \in \mathbb{C}^{\mathrm{n_m} \times \mathrm{n_m}}$ and Stiffness $\mathbf{K}^{\mathrm{R}} \in \mathbb{C}^{\mathrm{n_m} \times \mathrm{n_m}}$, are the reduced mass, damping, stiffness matrices corresponding to the rth mode.

$\mathbf{T}(\bar{\lambda}_r, \boldsymbol{\theta}_\mathrm{s}) = \begin{pmatrix} \mathbf{I} \\ \mathbf{t}(\bar{\lambda}_r, \boldsymbol{\theta}_\mathrm{s}) \end{pmatrix}$ is the $r t$h transformation matrix and $\mathbf{I}$: identity matrix of size n_m.

$$\mathbf{t}(\bar{\lambda}_r, \boldsymbol{\theta}_\mathrm{s}) \varphi_r^m = -\left(\mathbf{K}^{\mathrm{ss}}(\boldsymbol{\theta}_\mathrm{s}) + \bar{\lambda}_r \mathbf{C}^{\mathrm{ss}}(\boldsymbol{\theta}_\mathrm{s}) + \bar{\lambda}_r^2 \mathbf{M}^{\mathrm{ss}}(\boldsymbol{\theta}_\mathrm{s})\right)^{-1} \left(\mathbf{K}^{\mathrm{sm}}(\boldsymbol{\theta}_\mathrm{s}) + \bar{\lambda}_r \mathbf{C}^{\mathrm{sm}}(\boldsymbol{\theta}_\mathrm{s}) + \bar{\lambda}_r^2 \mathbf{M}^{\mathrm{sm}}(\boldsymbol{\theta}_\mathrm{s})\right) \varphi_r^m = \varphi_r^s \tag{8}$$

where $\mathbf{t}(\bar{\lambda}_r, \boldsymbol{\theta}_\mathrm{s})$ is the dynamic reduction matrix for the r^{th} mode.

4. BAYESIAN FORMULATION

Bayesian theorem is applied to the current problem having the uncertain parameters set $(\boldsymbol{\theta}, \boldsymbol{\omega}, \boldsymbol{\zeta}, \mathbf{x}_\phi)$ and the measured data $\mathbf{D} = \{\widehat{\omega}_r, \hat{\zeta}_r, \widehat{\boldsymbol{\phi}}_r, \hat{C}_{\omega,r}, \hat{C}_{\zeta,r}, \hat{\mathbf{C}}_{\boldsymbol{\phi},r}, r = 1, \ldots, N_M\}$, applying this to Eq. (1) gives:

$$p(\boldsymbol{\theta}, \boldsymbol{\omega}, \boldsymbol{\zeta}, \mathbf{x}_\phi | D) = \frac{p(D|\boldsymbol{\theta}, \boldsymbol{\omega}, \boldsymbol{\zeta}, \mathbf{x}_\phi) p(\boldsymbol{\theta}, \boldsymbol{\omega}, \boldsymbol{\zeta}, \mathbf{x}_\phi)}{p(D)} \tag{9}$$

According to the fact that the PDF of modal data do not require the information of the uncertain parameter $\boldsymbol{\theta}$ i.e. $p(D|\boldsymbol{\theta}, \boldsymbol{\omega}, \boldsymbol{\zeta}, \mathbf{x}_{\boldsymbol{\phi}}) = p(D|\boldsymbol{\omega}, \boldsymbol{\zeta}, \mathbf{x}_{\boldsymbol{\phi}})$ and that $p(\boldsymbol{\theta}, \boldsymbol{\omega}, \boldsymbol{\zeta}, \mathbf{x}_\phi) = p(\boldsymbol{\omega}, \boldsymbol{\zeta}, \mathbf{x}_\phi | \boldsymbol{\theta}) p(\boldsymbol{\theta})$.Moreover, from total probability theorem gives:

$$p(D|\boldsymbol{\omega}, \boldsymbol{\zeta} \boldsymbol{x}_\phi) = c p_0(\boldsymbol{\omega}|D) p_0(\boldsymbol{\zeta}|D) p_0(\mathbf{x}_{\boldsymbol{\phi}}|D) p(D) \quad \textit{(10)}$$

This will change the Eq. (9) to:

$$p(\boldsymbol{\theta}, \boldsymbol{\omega}, \boldsymbol{\zeta}, \mathbf{x}_\phi | D) = \frac{c p(\boldsymbol{\theta}) p_0(\boldsymbol{\omega}|D) p_0(\mathbf{x}_\phi|D) p_0(\boldsymbol{\zeta}|D) p(\boldsymbol{\omega}, \boldsymbol{\zeta}, \mathbf{x}_\phi|\theta) p(\boldsymbol{\omega}, \boldsymbol{\zeta}, \mathbf{x}_\phi|\boldsymbol{\theta}) p(\boldsymbol{\theta})}{p(D)} \tag{10}$$

Since different modes are statistically independent, then:

$$p(\boldsymbol{\theta}, \boldsymbol{\omega}, \boldsymbol{\zeta}, \mathbf{x}_\phi | D) = c p(\boldsymbol{\theta}) \left[\prod_r p_0(\omega_r|D) p_0(\zeta_r|D) p_0(\mathbf{x}_{\phi,r}|D) p(\omega_r, \boldsymbol{\zeta}_r \mathbf{x}_{\phi,r}|\boldsymbol{\theta})\right] \tag{11}$$

5. PROBABILITY MODEL OF PREDICTION ERROR

The accounting for uncertainties in Bayesian model updating is done by introducing the prediction error probabilistic model. The prediction error is defined as the discrepancy between the measured responses of the system and the response of a system model parameterized by the uncertain parameter set $\boldsymbol{\theta}_s$. In the proposed methodology the prediction error, $\boldsymbol{\varepsilon}_r \in \mathbb{C}^{n_m}$, is given by the following equation:

$$\left[\mathbf{K}^R(\lambda_r,\boldsymbol{\theta}_s)+\lambda_r\mathbf{C}^R(\lambda_r,\boldsymbol{\theta}_s)+\lambda_r^2\mathbf{M}^R(\lambda_r,\boldsymbol{\theta}_s)\right]\phi_r=\boldsymbol{\varepsilon}_r \tag{12}$$

Motivated by [6,19], the probability model for the prediction error $\mathbf{e}_r$ is designed to produce the maximum uncertainty based on the Principle of Maximum Entropy [20]. Thus, $\mathbf{e}_r$ is modelled as a discrete Gaussian process with mean assumed to be equal to zero and covariance matrix, $\mathbf{C}_{e,r} \in \mathbb{R}^{2n_m\times 2n_m}$ modelled as, $\mathbf{C}_{e,r}=\begin{bmatrix}\sigma^2_{\mathrm{Re},r}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \sigma^2_{\mathrm{Im},r}\mathbf{I}\end{bmatrix}$, where $\mathbf{I}\in\mathbb{R}^{n_m\times n_m}$ is an identity matrix, and $\sigma^2_{\mathrm{Re},r}$, $\sigma^2_{\mathrm{Im},r}$ are prediction error variance.

6. FORMULATION OF THE POSTERIOR PDF

The posterior PDF given by the Eq. (11) has large number of uncertain parameters making its estimation using any available any Markov Chain Monte Carlo (MCMC) algorithms a nontrivial task. In this approach, the sampling is intended to be done from the marginal PDF $p(\boldsymbol{\theta},\boldsymbol{\zeta},\boldsymbol{\omega}\,|\,D)$ instead of sampling from the full Posterior PDF $p(\boldsymbol{\theta},\boldsymbol{\omega},\boldsymbol{\zeta},\mathbf{x}_\phi\,|\,D)$. This demands the integration of the variable $\mathbf{x}_\phi$ from Eq. (11) as follows:

$$p(\boldsymbol{\theta},\boldsymbol{\zeta},\boldsymbol{\omega}\,|\,D)=cp(\boldsymbol{\theta})p_0(\boldsymbol{\omega}\,|\,D)p_0(\boldsymbol{\zeta}\,|\,D)\int p_0(\mathbf{x}_\phi\,|\,D)p(\boldsymbol{\omega},\boldsymbol{\zeta},\mathbf{x}_\phi\,|\,\boldsymbol{\theta})d\mathbf{x}_\phi \tag{13}$$

Sampling from the marginal PDF above decreases the number of uncertain parameters from $(n_{\theta_s}+2N_M)+N_M+N_M+2N_M n_m$ to $n_{\theta_s}+4N_M$ which can be handled using suitable MCMC algorithms.

7. SAMPLING USING MARKOV CHAIN MONTE CARLO

To deal with relatively large number of uncertain parameters, a special Monte Markov Chain algorithm is used which is Transitional Monte Carlo Markov Chain algorithm (TMCMC). TMCMC is an efficient simulation algorithm to sample from difficult PDFs. The distinctive feature behind the TMCMC is to sample from a series of intermediate PDFs that converge to the target PDF.

8. ILLUSTRATVE EXAMPLE

The example given in this section is a 3-storey shear building system ($n_d=3$), taken from [21] as shown in Fig. 1 The model has the system parameters shown in Table 1.
For simulation of acceleration response, 100 independent acceleration response sets have been recorded. The acceleration response is corresponding to the first and third floor ($n_m=2$) with an impact load applied to the second floor. Data has been recorded for 10 seconds at a sampling frequency of 1000 Hz. The measured acceleration is found by perturbing the acceleration of the nominal system with noises simulated from a Gaussian distribution with mean zero and a

standard deviation equal to $0.05 \times max(abs(\ddot{x}_t(t)))$, with $\ddot{x}_t(t)$ being the jth floor acceleration response.

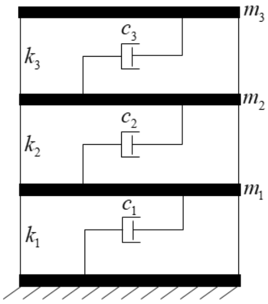

Fig. 1. 3-story shear building

Modal data: Modal identification is executed firstly using an approach based on Hilbert-Hung analysis [21]. The statistical properties (MPVs and covariances) of the modal parameters have been acquired from 100 independent sets of simulated noisy data. Table 2 shows the nominal system modal frequencies, damping ratios, and mode shapes and the statistics of the modal parameters identified from measured data.

Identification model: For identification of uncertain model parameters, the same 3-story shear model is considered with stiffnesses parameterized as $k_j = k_{0j}\theta_j$, and damping coefficients parameterized as $c_j = c_{0j}\theta_{n_d+j}$, for $j = 1,2,3$, where $\boldsymbol{\theta}_s = [\theta_1 \cdots \theta_6]$ are uncertain scaling parameters to be identified, that is $\boldsymbol{\theta}_s = [\theta_1 \cdots \theta_6]$. Masses are assumed to be known.

Prior PDFs: Scaling parameters θ_1, θ_2 and θ_3 are assumed to be independently uniformly distributed in the range [0.5, 1.5], θ_4, θ_5 and θ_6 are independently uniformly distributed in the range [0, 2], and prediction error variance, $\sigma^2_{\mathrm{Re},r}$ and $\sigma^2_{\mathrm{Im},r}$, are independently uniformly distributed over [0, 2000] $(\mathrm{Nm}^{-1})^2$. The updating has been done using three observed modes ($N_M = 3$). Correspondingly, in such a case: $\boldsymbol{\theta} = [\boldsymbol{\theta}_s, \sigma^2_{\mathrm{Re},1}, \sigma^2_{\mathrm{Im},1}, \sigma^2_{\mathrm{Re},2}, \sigma^2_{\mathrm{Im},2}, \sigma^2_{\mathrm{Re},3}, \sigma^2_{\mathrm{Im},3}]$.

Results from the proposed approach**:** Results from a model updating using TMCMC algorithm (characterized by number of samples N = 5,000/stage and scaling parameter $\beta = 0.2$), are presented in Fig. 2. These results represent the samples simulated in the last single run transitional PDF. The results show that the large uncertainty of the stiffness and damping parameters $\boldsymbol{\theta}_s$ has been significantly reduced in the posterior samples which are distributed in a small region around the true mean values (true mean values=1).

Table 1. System Parameters for 3-DOF shear model

Stiffness	$k_1 = k_2 = k_3$980 kN/m
Lumped Mass	$m_1 = m_2 = m_3$980 kN/m
damping coefficients	$c_1 = 7.035 KNs/m, c_2 = 2.814 KNs/m, c_3 = 0.704$ kNs/m

Table 2. Modal properties of 3-story shear building system

Mode	Theoretical values			Identified MPVs and COV*		
	Freq. (Hz)	Damping ratio (%)	Mode shape	Freq. (Hz)	Damping ratio (%)	Mode shape
1	2.22	3.47	1.0000 + 0.0000i 1.8020 + 0.0543i 2.2455 + 0.1006i	2.22 (0.0016)	3.54 (0.0354)	1.0000 + 0.0000i - 2.2190 + 0.0982i
2	6.24	6.26	1.0000 + 0.0000i 0.4399 + 0.1489i -0.7957 - 0.0495i	6.28 (0.0116)	5.69 (0.0797)	1.0000 + 0.0000i - -0.7733 - 0.0136i
3	8.94	7.30	1.0000 + 0.0000i -1.1909 + 0.2892i 0.5077 - 0.2095i	8.89 (0.0026)	7.44 (0.0361)	1.0000 + 0.0000i -1.2043 + 0.2137i 0.5225 - 0.1513i

*The COV values are provided in the parenthesis.

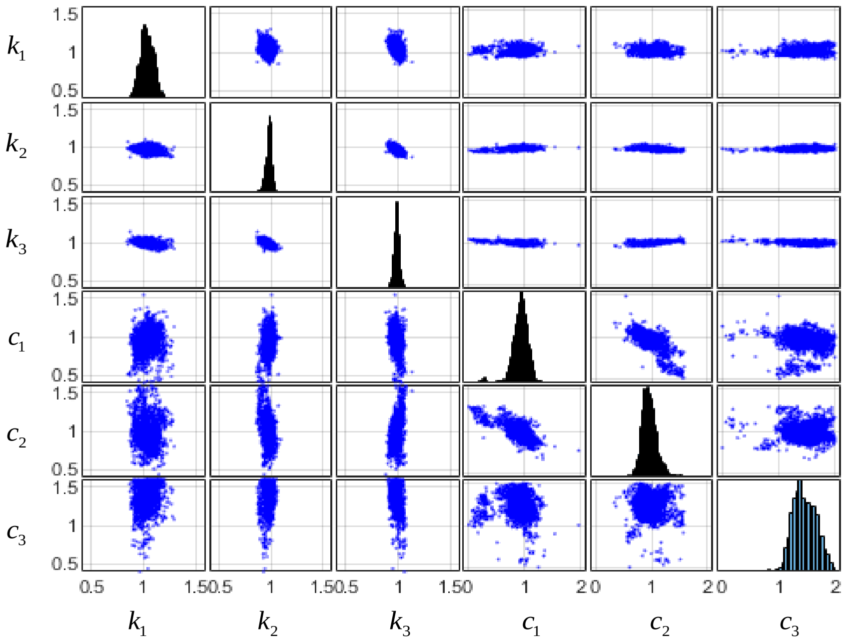

Fig. 2. Posterior histograms and scatter plot of stiffness and damping contribution parameters.

Table 3. Statistics of frequency and damping ratio parameters of 3-story shear building system

Case	Mode	Statistics of ω and ζ samples generated at the last stage of TMCMC		Statistics of ω and ζ obtained by solving eigen-equation for each θ_s sample of the last stage of TMCMC	
		Freq (Hz)	Damping Ratio (%)	Freq (Hz)	Damping Ratio (%)
1	1	2.22 (0.0014)	3.59 (0.0352)	2.22 (0.0543)	3.22 (0.4177)
	2	-	-	6.32 (0.0551)	6.02 (0.3103)
	3	8.90 (0.0025)	7.33 (0.0310)	8.87 (0.0064)	7.31 (0.0966)
2	1	2.22 (0.0007)	3.57 (0.0180)	2.23 (0.0150)	3.16 (0.1176)
	2	6.23 (0.0071)	5.91 (0.0249)	6.24 (0.0103)	6.10 (0.1161)
	3	8.90 (0.0011)	7.68 (0.0258)	8.90 (0.0058)	7.40 (0.0690)

*The COV values are provided in the parenthesis.

The statistics of the posterior frequencies and damping ratios samples from a single run are summarized in Table 3. It is observed that the posterior system frequencies and damping ratios are Gaussian with their sample mean and sample COV close to the MPV and COV of the observed modal frequencies. Table 2 also presents the statistics of $\boldsymbol{\omega}$ and $\boldsymbol{\zeta}$ samples obtained by solving eigen system equation for each $\boldsymbol{\theta}_s$ sample generated at the last stage of TMCMC. It is observed that the natural frequencies and the damping ratios of the updated system are close to those for the nominal system.

9. CONCLUSION

A model updating approach for reduced linear non-classically damped dynamical systems is presented. Results from the example with simulated modal data show that when updating the model's structural parameters, the updated parameters are distributed in a smaller region than the prior region, indicating the efficiency of the updating approach. Moreover, the additional uncertain parameters, i.e., modal characteristics, have been successfully identified with matching posterior statistics. Finally, integrating the system mode shapes and sampling from marginal PDF proved helpful in reducing the dimensionality of the problem if an efficient MCMC algorithm was used.

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