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A MULTI-FIDELITY FRAMEWORK WITH ADAPTIVE TRANSFER LEARNING FOR RISK ASSESSMENT OF STRUCTURAL SYSTEMS SUBJECT TO NATURAL HAZARDS

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Abstract. Recently, low-fidelity models (e.g., data-driven emulators) have gained extensive acceptance for accelerating the evaluation of nonlinear dynamic systems, thereby enhancing the computational efficiency in estimating small failure probabilities against extreme events, such as natural hazards. However, directly employing low-fidelity models for uncertainty propagation can potentially lead to biased predictions. To address this challenge, this work develops a multi-fidelity scheme with adaptive transfer learning for efficiently estimating small failure probabilities. In this approach, the probability space is partitioned into several strata, within which deep learning-based emulators are developed by adaptive transfer learning. These emulators serve as strata-wise low-fidelity models. By effectively combining the strata-wise high- and low-fidelity evaluations, this method allows for efficient estimation of small probabilities of failure associated with rare events. Through an illustration on an full-scale steel building subject to seismic excitation, the proposed scheme is demonstrated to enable accurate estimation of small failure probabilities, while enabling remarkable speedup compared to stratified sampling schemes relying entirely on high-fidelity model evaluations.

1 INTRODUCTION

Performance-based design (PBD) has been widely recognized for its capability to investigate nonlinear structural responses and associated probabilities of failure under a full range of hazard intensities [1, 2, 3, 4]. To achieve this objective, frameworks that integrate variance reduction techniques (e.g., subset simulation (SuS), importance sampling (IS) and stratified sampling (SS)) and high-fidelity modeling environments have been extensively explored [5, 6]. However, these approaches in general still require several thousand model evaluations to ensure accuracy for estimating small failure probabilities, which potentially lead to computationally intensive problems. An alternative solution involves utilizing low-fidelity models, such as data-driven emulators, to rapidly approximate nonlinear system behaviors and accelerate uncertainty propagation [7, 8, 9]. Despite remarkable computational efficiency, low-fidelity models may produce biased estimates compared to those derived from high-fidelity modeling environments. To balance approximation accuracy and computational efficiency, multi-fidelity schemes that combine high- and low-fidelity model outputs have emerged [10]. One such method is multi-fidelity Monte Carlo (MFMC), which provides unbiased estimates with substantial variance reduction [11, 12, 13]. To ensure the efficiency of MFMC, computationally efficient low-fidelity models with adequate correlation with the high-fidelity model are essential. Consequently, several MFMC applications employ data-driven emulators as low-fidelity models to provide rapid approximation of the high-fidelity outputs. However, developing an emulator capable of capturing complex nonlinear features over a full range of hazard intensities requires substantial training data. To tackle this, this work proposes an transfer learning-enabled multi-fidelity stratified sampling scheme for efficient estimation of small failure probabilities associated with rare events.

2 MULTI-FIDELITY STRATIFIED SAMPLING

Consider a structural system subject to random input $\boldsymbol{\theta} \in \mathcal{R}^{n_\theta}$, where n_θ is the input dimension. The problem of interest is to estimate the small probability of the structural response, $y(\boldsymbol{\theta})$, exceeding a threshold, z_i, under rare events, expressed as $P_f = P(y > z_i)$. Accurately estimating P_f within high-fidelity modeling environments involves substantial computational demand, even when employing variance reduction techniques (e.g., SS) [5]. To address this challenge, a multi-fidelity stratified sampling (MFSS) scheme is developed. In this context, the high-fidelity model, which captures the essential inelastic system behavior, relies on computationally demanding numerical simulations (e.g., finite element analysis). The deep learning-based emulator serves as the low-fidelity model, rapidly approximating the high-fidelity model outputs. For clarity, in the following the notions HF and LF will indicate the high- and low-fidelity models.

To implement the MFSS scheme, the probability space of a dominate random variable, e.g., hazard intensity measure (IM), is partitioned into N_s mutually exclusive and collectively exhaustive strata, denoted as E_k. Within each stratum, an MFMC estimator, $\hat{H}^k_{MF}$, is established for the conditional probability of failure, $P(y > z_i|E_k)$. Subsequently, the estimation of P_f, indicated as $\hat{H}_{MF}$, can be achieved using the total probability theorem:

$$P_f \approx \hat{H}_{MF} = \sum_{k=1}^{N_s} \hat{H}^k_{MF} \cdot P(E_k) \tag{1}$$

$$\mathbb{V}\left[\hat{H}_{MF}\right] = \sum_{k=1}^{N_s} \mathbb{V}\left[\hat{H}^k_{MF}\right] \cdot [P(E_k)]^2 \tag{2}$$

where $P(E_k)$ denotes the probability of E_k; and $\mathbb{V}[\cdot]$ is the variance operator.

By integrating the strata-wise HF and LF evaluations, $\hat{H}^k_{MF}$ in each stratum, E_k, can be formulated as follows:

$$\hat{H}^{(k)}_{MF} = \frac{1}{N^k_{HF}} \sum_{j=1}^{N^k_{HF}} h^k_{HF,j} + a^k \left(\frac{1}{N^k_{LF}} \sum_{j=1}^{N^k_{LF}} h^k_{LF,j} - \frac{1}{N^k_{HF}} \sum_{j=1}^{N^k_{HF}} h^k_{LF,j} \right) \tag{3}$$

where $h^{(k)}_{HF,j}$ and $h^{(k)}_{LF,j}$ represent consequence measures (e.g., indictor functions, kernel estimators) based on the j^{th} HF and LF model evaluation in the k^{th} stratum; N^k_{HF} and N^k_{LF} are the number of

HF and LF model evaluations in the k^{th} stratum; a^k denotes the control variate coefficient associated with the k^{th} stratum. Computational efficiency can be further improved by optimally determining a^k as well as the sample allocation of HF and LF models [11]. To facilitate the efficiency of MFSS, a LF model, characterized by high computational efficiency and high correlation with the HF model is desired [13].

3 STRATA-WISE LF MODEL DEVELOPMENT

3.1 Adaptive training strategy

In the MFSS setting, data-driven emulators capable of rapidly approximating HF outputs can be taken as a desirable LF model. To develop an emulator mapping from the stochastic excitation to system output, Recurrent Neural Networks (e.g., Gated Recurrent Unit (GRU)) can be adopted. To tackle the challenges associated with high-dimensionality (i.e., in the order of hundreds or higher in practical engineering problems), a proper orthogonal decomposition (POD)-based model order reduction technique is employed [7, 8]. Once reduced, a weighted correlation coefficient in the reduced space can be defined to quantify the correlation between the HF and LF models.

To derive a trade-off between the approximation quality of the emulator and computational demand, an adaptive scheme is proposed to seek the minimum training data that ensures an adequate correlation with the HF model, as determined through K-fold cross-validation. In this approach, the correlation coefficient, $\bar{\rho}_v$, and its coefficient of variation (COV), δ_v, are determined by averaging ρ_v and computing the COV across all K rounds. The core idea of the adaptive training strategy involves initially training the model with a small dataset. In each subsequent iteration, a fixed number of training samples are incrementally added, until reaching a target correlation, ρ^*, and target COV, δ^*.

3.2 Transfer learning-enabled LF model development

Notwithstanding the efficiency of the adaptive training strategy, developing a single emulator with desired correlation still requires substantial training data to capture the variation in system response under a full range of intensities. To further enhance efficiency, strata-wise LF models are developed by transfer learning. The process begins with creating an emulator for the critical stratum (e.g., the last stratum, E_{N_s}), characterized by the highest nonlinearity. An adaptive training scheme is employed to develop an emulator with adequate correlation, ρ^{N_S}, while using minimal training data. The developed emulator serves as the LF model for E_{N_s}, denoted as LF_{N_s}. Subsequently, LF_{N_s} is propagated to the preceding stratum E_{N_s-1}. The model LF_{N_s} is evaluated using a set of test samples generated from E_{N_s-1}. If the correlation satisfies the target ρ^*, the model LF_{N_s} can be transferred directly, i.e., $LF_{N_s-1} = LF_{N_s}$. Otherwise, the adaptive training scheme, with model parameters initialized from the source model, is employed to develop a new emulator LF_{N_s-1} with desired correlation ρ^{N_S-1}. Once LF_{N_s-1} is established, it can be propagated to the next preceding stratum. This iterative process continues until reaching the first stratum. By combining adaptive training with transfer learning, strata-wise LF models with desired correlation can be developed by using a minimal computational budget, as illustrated in Figure 1.

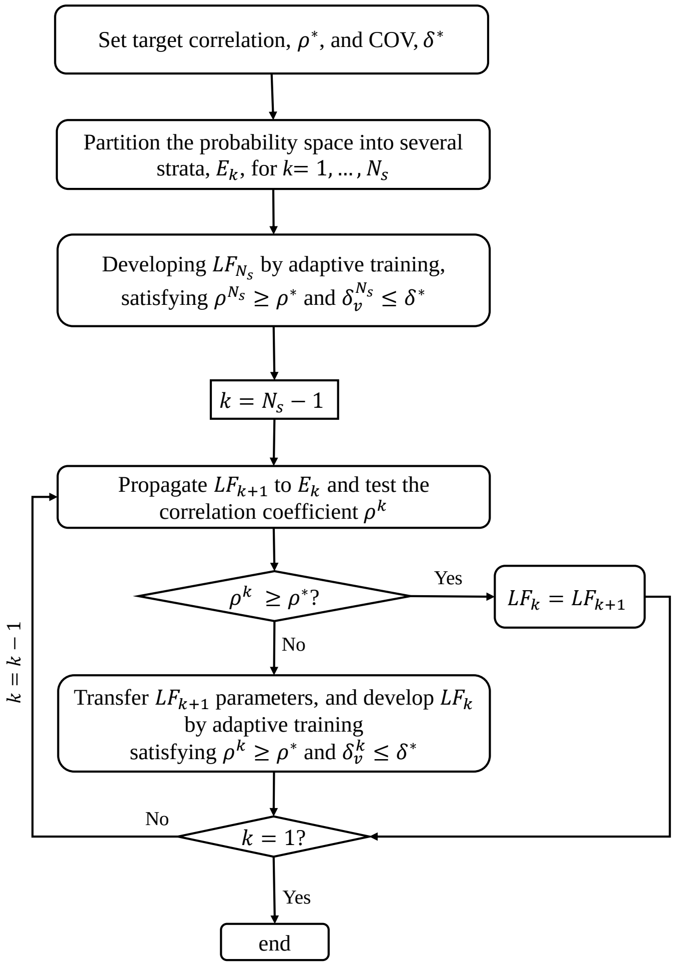

Figure 1: Workflow for developing strata-wise LF models by adaptive transfer learning.

4 ILLUSTRATIVE EXAMPLE

4.1 Building description and uncertainties

To illustrate the efficiency of the proposed framework, a case study consisting in the two-dimensional (2D) 37-story steel moment-resisting frame located in San Francisco subject to stochastic seismic excitation is considered [7]. A fiber-discretized nonlinear model of the 2D frame is established in $OpenSees$ [14], serving as the HF model in this study. A Giuffre-Menegotto-Pinto model is adopted for each fiber of the discretization. Damage resulting from low-cycle fatigue is modeled by warping the $OpenSees$ fatigue material around each fiber. Large displacement effects are accounted for by utilizing a corotational transformation. A point-source stochastic model as outlined in [5, 15, 16] is adopted for ground motion modeling. In this model, the record-to-record variability (stochasticity) serves as input uncertainties.

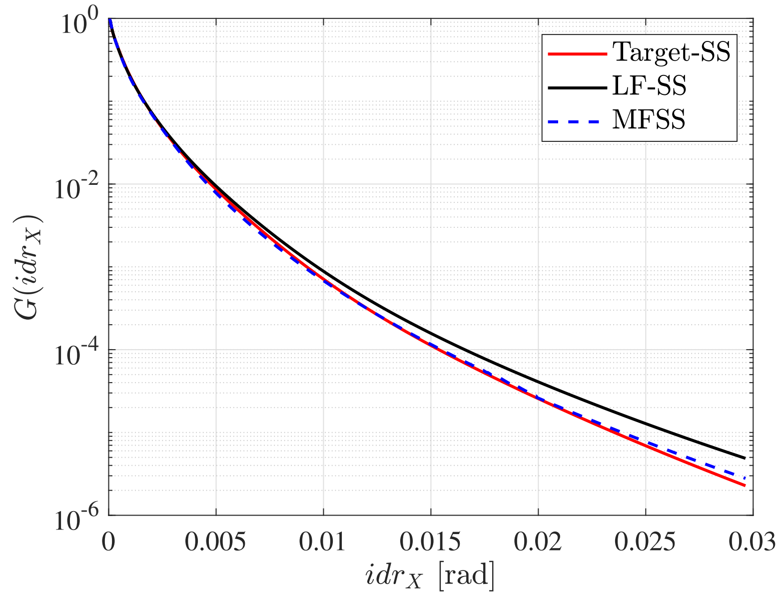

Figure 2: Comparison of the exceeding probability curves of the horizontal peak inter-story drift ratio at the top floor.

4.2 Transfer learning-enabled strata-wise LF models

The probability space of spectral acceleration at the fundamental period of the building, $S_a(T)$, was partitioned into 5 strata, by fixing the lower bound of the last stratum at 10^{-3}. To ensure the approximation quality of the LF model, the target correlation and COV were set to $\rho^* = 0.95$ and $\delta^* = 0.03$. To balance dimensionality and accuracy, the truncation criterion was specified as $\eta = 99.999\%$ for the POD reduction. Subsequently, strata-wise LF models were developed using the proposed adaptive transfer learning strategy. For the critical stratum E_5, a GRU-based deep neural network was adopted to develop LF_5 using the adaptive training. Following this, LF_5 was propagated to the previous stratum. In this stratum, 20 samples were generated to evaluate the coefficient correlation between LF_5 and the HF model, determining whether LF_5 could be directly transferred or if a new emulator required to be trained. This process continued until the first stratum, E_1, was reached.

4.3 Calibration of the proposed scheme and results

To demonstrate the efficiency of the proposed scheme, a HF-based SS scheme with an optimal allocation [6] was utilized as a reference. The computational cost ratio, c_{HF}/c_{LF}, was estimated to be 10,000 showcasing the computational efficiency of neural network over the HF model. Figure 2 compares the exceedance probability curves of the peak horizontal inter-story drift ratio at the top floor, idr_X, obtained by the proposed scheme, LF-based SS, and the target. Evidently, the proposed scheme shows remarkable accuracy in reproducing the target exceedance probability curve. The advantage over the LF model is also clear from how the proposed scheme removes the discrepancy observed in LF-based SS. Given a specific limit state of interest, $idr_X \geq 1/50$, the proposed scheme can accurately estimate the small failure probability (e.g., to 10^{-5}) with a similar COV, while only around 7% of the computational budget of the target, as shown in Table 1.

Table 1: Comparison of probability of failure, COV, and computational cost given the limit state: $idr_X \geq 1/50$.

Methods	HF (MFSS)	LF(MFSS)	Target
LF-SS	4.1×10^{-5}	0.01	6%
MFSS	2.6×10^{-5}	0.02	7%
Target	2.6×10^{-5}	0.04	100%

5 CONCLUSION

This paper proposes a multi-fidelity stratified sampling scheme with adaptive transfer learning for efficient estimation of small failure probabilities related to rare events (e.g., natural hazards). This method is centered on developing strata-wise low-fidelity models (e.g., data-driven emulators), which are well correlated with the HF model, by an adaptive training strategy and transfer learning. Subsequently, the probabilities of failure can be efficiently estimated by integrating strata-wise high- and low-fidelity outputs. The proposed scheme is shown to be capable of accurately predicting small probabilities of failure of nonlinear structural systems, while providing significant computational speedup (i.e., using approximately 6% of the computational demand of the HF-based stratified sampling scheme).

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