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# Super-elements and flexibility methods exploiting hybrid equilibrium formulations.

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## ABSTRACT

At ADMOS2019 [1] we proposed an extension of the macro-element concept to formulate a solid hybrid equilibrium hexahedral element. In this paper we detail a similar concept as a super-element for a 2D quadrilateral element in plane elasticity.

A quadrilateral is considered as an assembly of 8 triangular primitives with a common internal vertex. The quadrilateral has straight sides split into pairs of collinear sides by its mesh of primitives. Each pair of corresponding primitives are formulated as a single triangular quasi-primitive, which are then assembled in a similar way to the usual macro-element [2].

As a macro-element, it has 8 sides with $8\times2(d + 1) = 64$ degrees of freedom when degree $d = 3$. As a super-element we define 4 discrete “covers” to align with the 4 pairs of collinear sides. The covers serve as interfaces between adjacent elements, and with the boundaries. They receive simplified continuous tractions of degree $d_c \leq d$ as external loads and transmit them to adjacent elements. The idea here is to project such tractions from the covers onto the collinear sides without exciting spurious kinematic modes [2]. Dual to the simplified continuous tractions, the displacements of the covers are mapped back from the displacements of the collinear sides.

We will illustrate the super-element concepts with a quadrilateral when $d = 3$ and $d_c = 1$ and the cover degrees of freedom represent just its 3 rigid body modes, with linear tractions representing the corresponding 3 resultant actions: normal and tangential forces and a moment. The constitutive relations then lead directly to a form of flexibility matrix for a super-element, which prompts the idea of analysing a mesh using a flexibility method [3]. This will be illustrated by modelling a portal frame with deep structural members and 6 super-elements that require only 3 compatibility equations to be solved for the system, before recovering all the displacement and traction variables and the equilibrating internal stress fields.

## REFERENCES

[1] E.A.W. Maunder and J.P. Moitinho de Almeida, ”Towards a hexahedral hybrid equilibrium macro-element”, ADMOS 2019, 27-29 May, 2019, El Campello.

[2] J.P.Moitinho de Almeida and E.A.W.Maunder, *Equilibrium Finite Element Formulations*, Wiley,2017

[3] Maunder EAW, Savage GJ, A graph-theoretic model for finite elements with side variables, *Civil Eng. Sys*. 11, 1994:111-141.