Model Order Reduction for Non Linear Mechanics.

**PhD student:**
Rubén Ibáñez Pinillo

**Supervisors:**
F. Chinesta, E. Abisset-Chavane, A. Huerta

**Industrial Correspondants:**
E. Abenius, J.L. Duval

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Motivation of the project

Context:
Automotive industry is moving towards a new generation of cars.

Main idea:
Cars are furnished with radars, cameras, sensors, etc... providing useful information about the environment surrounding the car.

Goals:
Provide an efficient model for the radar input/output.
Reducing computational costs by means of big data techniques.
Introduction

Ways to reduce the computational cost:

1. Reduce the complexity of the model.

Maxwell equations need a very fine mesh due to high frequency constraints.

Far-Field approaches tend to be less accurate in near field.

2. Use manifold learning techniques to unveil relevant information.

<table>
<thead>
<tr>
<th>Models</th>
<th>Maxwell Equations</th>
<th>Far Field Approaches</th>
<th>Geometrical Optics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Time</td>
<td>Red</td>
<td>Green</td>
<td>Green</td>
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<tr>
<td>Accuracy</td>
<td>Green</td>
<td>Red</td>
<td>??</td>
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The Concept of Manifold

**Manifold:** A subspace of dimension $N$ belonging to a space of dimension $D$, where physics is organized.

$$\mathcal{M} = \{ x \in \mathbb{R}^D | f(x) = 0 \}$$

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The Concept of Manifold

How can a manifold help in model order reduction?

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Motivation of the project

Context:

Nonlinear dimensionality reduction becomes a powerful tool for extracting the manifolds that can be then used for making safely interpolations, for extracting the uncorrelated parameters that models involve and for defining general parametric solutions.

Gather the information in the manifold as an off-line stage, then making simulations faster in the on-line stage.

Goals:

Extract relevant information is extracted when the data is organized in some specific pattern.

Reduce the computational cost associated to the electromagnetic simulation of the autonomous car.
Outline

- Geometrical Optics
- Fundaments
- Scenario
- Convergence
- BlackBoxing the Scenario
- Scenario Manifold
- Future Work
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Geometrical Optics: Fundamentals

Optical reflections are produced inside the domain.

Each time a reflection is produced, some amount of energy is retained in the surface controlled by a parameter called absorption coefficient.

\[ E_{\text{ext}}(x_0) = E_{\text{int}}(x), x \in \partial \Omega \]

In our case the external energy will be the energy sent out by the radar and we will capture only the energy coming back to the receptor.
Geometrical Optics: Fundamentals

Single Ray Equation emisor and receptor located at x₀

Repeating the procedure for any possible angle of departure:

\[ e_{absorbed} = M e_{external} \]

The (i,j) component of the matrix is the energy coming back to the source with a discretized arrival angle \( \alpha_a = i \Delta \alpha \) which has been thrown from an angle of departure \( \alpha_d = j \Delta \alpha \).
Geometrical Optics: Fundamentals

Possible Quantities of Interest

- **Dep./ Arr. Energy**
  \[ E_{DA} = E(\alpha_d, \alpha_a) \]

- **Departure Energy**
  \[ E_D = \int_{-\pi/2}^{\pi/2} E(\alpha_d, \alpha_a) d\alpha_a \]

- **Arrival Energy**
  \[ E_A = \int_{-\pi/2}^{\pi/2} E(\alpha_d, \alpha_a) d\alpha_d \]

- **Total Energy**
  \[ E_T = \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} E(\alpha_d, \alpha_a) d\alpha_d d\alpha_a \]
Geometrical Optics: Simple Case

Both source and receptor are located at the middle point of the south wall. (Red point)
Geometrical Optics: Simple Case

\[ E_A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} E(\alpha_d, \alpha_a) d\alpha_d \]
Geometrical Optics: Convergence

Convergence analysis of total energy coming back to the radar.

\[
\varepsilon = \frac{\left| E^H_T - E^L_T \right|}{E^H_T}
\]

It converges!
BLACK-BOXING
THE SCENARIO
Black Boxing the Scenario

Replace objects inside the scenario by a “black-box” and a transfer function.

Advantages:
- Faster on-line simulations since only a box has to be meshed.
- Parameterization of the scenario becomes easier.
The transfer function can be seen as a manifold establishing geometrical and energetic relationships between input/output ray.

Indeed, input and output position and orientation of the ray just like the ratio of energy between incoming and outcoming ray will appear in the transfer function.
Black Boxing the Scenario

**Example:** Circle

**X axis:** Arc length of the square (starting from south-west corner, counterclockwise)

**Y axis:** Input ray orientation. **Absortion coefficient:** 0.5
Black Boxing the Scenario

Example: Circle

\[
\text{Angular Deviation}
\]

\[
\begin{array}{c}
\chi_1 \\
0 & 1 & 2 & 3 & 4 \\
0 & 100 & 200 & 300 & 400 & 500
\end{array}
\]

AdMoRe
Black Boxing the Scenario

Example: Circle

Output Length

\[
\begin{array}{c}
\chi_1 \\
\alpha_{\text{in}}
\end{array}
\]

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Black Boxing the Scenario

Example: Circle

Energy Ratio

\[ \chi_1 \]

\[ \alpha_{in} \]

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Black Boxing the Scenario

Example: Star

X axis: Arc length of the square (starting from south-west corner, counterclockwise)

Y axis: Input ray orientation. Absortion coefficient: 0.5
Black Boxing the Scenario

Example: Star

Angular Deviation

\[ \chi \]

\[ x \]

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Black Boxing the Scenario

Example: Star

Output Length

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Black Boxing the Scenario

Example: Star

Energy Ratio

$\chi_1$, $\alpha_{in}$

0, 100, 200, 300, 400, 500

0, 2, 4

$X$
SCENARIO MANIFOLD
Scenario Manifold

Goals: Scenario identification knowing the electromagnetic response.

Determine where and how many receptors are needed to determine unequivocally a scenario.
Let’s assume that $M$ scenarios are precomputed off-line.

The electromagnetic response of a single scenario is written as:

$$f(\alpha; p)$$

Where the blue terms are angular coordinates and the red ones are the parameterization of the manifold.

$$\min(||f(\alpha; p_n) - f(\alpha; p_i)||_{L_2}) \forall p_i \in \mathcal{M}_S$$

Those scenarios minimizing the functional will potential candidates for $p_n$. 
Scenario Manifold

**Step 1:** Find among all the data set, those scenarios with “similar” electromagnetic response.

**Result:** Many scenarios satisfied the same electromagnetic response, but they were geometrically different.
**Scenario Manifold**

**AdMoRe**

**Step 2:** A small subset of scenarios sharing the electromagnetic response is obtained. Therefore, there will be some obstacles causing an impact in the electromagnetic response and some other which does not.

That is what we will call, **active** and **non active** scenario, respectively.

Making a covariance analysis:

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<th>Colour</th>
<th>Covar. (%)</th>
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<td></td>
<td>92</td>
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<td>60</td>
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<td>61</td>
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Scenario Manifold

Using three sensors..

Sensor locations

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New subset of scenarios gives a covariance.

3 Sensors covariance

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1 Sensor covariance

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How to distinguish redundant data in the parameterization of the manifold?
Computing the difference of the electromagnetic response between two scenarios (red line) can be seen as a directional derivative in the manifold.

\[ \nabla_p f \cdot \frac{d}{||d||} \simeq \frac{f_{S2} - f_{S1}}{||d||} \]

Making all pairwise combinations in the local neighbourhood allows to estimate the gradient in a point.

All \( p \) parameters whose derivative is negligible can be inferred as redundant data.
Future prospects

**Geometrical Optics**
- Comparison with Maxwell Equations.

**Black-Boxing the Scenario**
- Reduce the cost of computing a transfer function. Smart selection of query points to do interpolation in the rest of the input domain.

**Scenario Manifold**
- Add more sensors to better distinguish different scenarios.
- Differentiate active obstacles from redundant obstacles based on the directional derivative or covariogram based.

**Domain Decomposition**
- Having a physical domain partitioned in such a way that in some areas geometrical optics is solved and in another areas Maxwell equations.
THANK YOU FOR THE ATTENTION.

QUESTIONS?