

## Deep Neural Network-driven $hp$ -adaptive Finite Element Method in three dimensions

Maciej Paszyński<sup>1</sup>, Rafał Grzeszczuk<sup>2</sup>, Witold Dzwiniel<sup>3</sup>, and David Pardo<sup>4</sup>

<sup>1</sup> AGH University of Science and Technology, Al. Mickiewicza 30, 30-059, Kraków, Poland  
maciej.paszynski@agh.edu.pl, home.agh.edu.pl/paszynsk

<sup>2</sup> AGH University of Science and Technology, Al. Mickiewicza 30, 30-059, Kraków, Poland  
grzeszczuk@agh.edu.pl

<sup>3</sup> AGH University of Science and Technology, Al. Mickiewicza 30, 30-059, Kraków, Poland  
dzwiniel@agh.edu.pl

<sup>4</sup> Basque Center for Applied Mathematics, Mazarredo Zumarkalea, 14, 48009 Bilbo, Bizkaia, Spain  
dzubiaur@gmail.com

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The deterministic  $hp$  adaptive finite element method delivers exponential convergence of the numerical accuracy with respect to the mesh size. The method employs a non-regular hierarchically adapted computational grid constructed from hexahedral finite elements. Our grids fulfill the 1-irregularity rule, which ensures that each finite element is broken only once without breaking neighboring elements. Additionally, the method employs polynomials of different order on finite element edges, polynomials of different orders in both directions on finite element faces, and polynomials of different orders in  $x$ ,  $y$ , and  $z$  directions in finite element interiors. Therefore, the procedure of selecting optimal  $hp$  refinement has to take into account the hierarchical structure of elements and relations between selected polynomial orders of approximation on finite element edges, faces, and interiors.

This talk presents the Deep-Neural-Networks train to select optimal refinements on finite element edges, faces, and interiors. We show that replacing the deterministic kernel of the self-adaptive  $hp$  finite element method code selecting the optimal refinements by the Deep Neural Network preserves the exponential convergence rate. We test our code on the three-dimensional Fichera model problem [1], with different boundary conditions. This work extends the two-dimensional DNN-driven  $hp$ -FEM described in [2].

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### REFERENCES

- [1] L. Demkowicz, J. Kurtz, D. Pardo, M. Paszyński, W. Rachowicz, A. Zdunek, Computing with  $hp$ -Adaptive Finite Elements, vol. 2. Chapman & Hall/CRC Applied Mathematics & Non-linear Science (2007)
- [2] M. Paszyński, R. Grzeszczuk, W. Dzwiniel, D. Pardo, Deep Neural Network Driven Self-adaptive HP Finite Element Method, Lecture Notes in Computer Science, **12742** (2021) 114-121