MODELING AN UNSTEADY ELASTIC DIFFUSION PROCESSES IN AN ISOTROPIC RECTANGULAR TIMOSHENKO PLATE

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Abstract. We investigated unsteady elastic diffusion vibrations of a rectangular isotropic Timoshenko plate. For the mathematical problem formulation, a model of coupled elastic diffusion processes in a multicomponent continuum is used. Using the d'Alembert variational principle, the equations of transverse vibrations of a rectangular isotropic Timoshenko plate taking into account diffusion are obtained from this model. An initial-boundary value problem of a simply supported plate bending is formulated.

INTRODUCTION

We considered an unsteady elastic diffusion vibrations problem of a Timoshenko plate. This model is a refinement of the classical Kirchhoff - Love plate model by considering the influence of inertial forces upon rotation of the normal line to the middle surface and shear deformation. It is assumed that the material fiber, which was straight and normal to the middle surface before deformation, remains straight but ceases to be normal to the middle surface. Thus, this theory takes into account shear deformations and shear stresses. Considering shear stresses is very important for the design of composites and wood parts since their destruction can occur due to the destruction of the binder during shear.

It is known that the classical Kirchhoff - Love plate model is quite simple and provides sufficient accuracy in solving many engineering problems, and therefore it is used most often. However, taking shear deformations into account may turn out to be essential, for example, for rods made of anisotropic material, in which the shear modulus is much lower than Young's modulus. Some fibrous and composite materials have these properties (in particular, human bones). It is also important to consider shear deformations in the problems of stability of three-layer rods and plates, where two bearing layers are thin and made of high-strength rigid material, and between them, there is a light and less durable filler. On the other hand, when the stiffness characteristics of the filler layer are significantly lower than the stiffness characteristics of the bearing layers, a simplified calculation using the classical Kirchhoff - Love model can lead to significant errors in the calculation of critical loads. This, in turn, leads to a decrease in the economic efficiency of the design or an underestimation of the potential resources of biological systems.

In addition, the presented model takes into account the mutual influence of the mechanical and diffusion fields on each other. As early as the beginning of the 20th century, different scientific groups experimentally proved that due to beam and plates bending, the deformation gradient initiates the process of ascending diffusion [1-3]. This leads to the formation of a concentration gradient, and as a consequence, to a redistribution of the solute atoms. As a result, there is a transfer of matter from areas of compression in the area of tension. This phenomenon is called the Gorsky effect. The result of scientific research was published in 1936 [1].

A review of publications in this scientific sphere shows that analyzing the interaction of mechanical and diffusion fields in thin-walled structural elements is also relevant today. Among the few publications on this topic are articles [4, 5]. They investigate the influence of diffusion processes on the bearing capacity of a smooth transversely isotropic shell. Contact interaction of a rod with an elastic half-space is considered in [6, 7]. Publications [8–10] are devoted to the study of elastic diffusion processes in plates. The calculation of elastic diffusion spherical shells is considered in [11].

It should be noted that all these problems are solved in a stationary formulation. Problem formulations on unsteady elastic diffusion vibrations of beams and plates and methods for their solution are absent in the publications known to date.

PROBLEM FORMULATION

We considered the unsteady vibrations problem of a rectangular orthotropic multicomponent Kirchhoff-Love plate under the action of mechanical and diffusion perturbations. Fig. 1 shows the orientation of Cartesian axes and how the forces and the moments are applied.

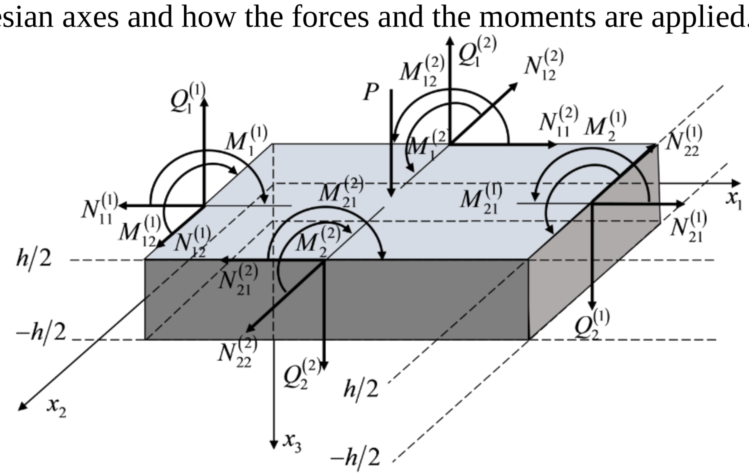

Fig. 1. Illustration for the problem formulation.

For the problem formulation, we use the coupled elastic diffusion continuum model in a rectangular Cartesian coordinate system, which in the case of a homogeneous continuum has the next form [12-17]:

$$\ddot{u}_i = \frac{\partial \sigma_{ij}}{\partial x_j} + F_i, \quad \dot{\eta}^{(q)} = -\frac{\partial J_i^{(q)}}{\partial x_i} + Y^{(q)}, \quad \eta^{(N+1)} = -\sum_{q=1}^{N} \eta^{(q)} \quad \left(q = \overline{1, N}\right). \tag{1.1}$$

where σ_{ij} and $J_i^{(q)}$ are components of the stress tensor and the diffusion flux vector, which are defined as follows

$$\sigma_{ij}=C_{ijkl}\frac{\partial u_k}{\partial x_l}-\sum_{q=1}^{N}\alpha_{ij}^{(q)}\eta^{(q)},\quad J_i^{(q)}+\tau_q\dot{J}_i^{(q)}=-\sum_{t=1}^{N}D_{ij}^{(qt)}\frac{\partial\eta^{(t)}}{\partial x_j}+\Lambda_{ijkl}^{(q)}\frac{\partial^2 u_k}{\partial x_j\partial x_l}\quad\left(q=\overline{1,N}\right). \tag{1.2}$$

Here the dots denote the time derivative. All quantities in (1.1) and (1.2) are dimensionless. We accepted the following notation:

$$\begin{gathered}x_i=\frac{x^*_i}{l},u_i=\frac{u^*_i}{l},\tau=\frac{ct}{l},C_{ijkl}=\frac{C^*_{ijkl}}{C^*_{1111}},C^2=\frac{C^*_{1111}}{\rho},\alpha_{ij}^{(q)}=\frac{\alpha_{ij}^{*(q)}}{C^*_{1111}},l_m=\frac{l^*_m}{l},D_{ij}^{(qt)}=D_{ij}^{(t)}g^{(qt)},\\ D_{ij}^{(q)}=\frac{D_{ij}^{*(q)}}{Cl},\Lambda_{ijkl}^{(q)}=\frac{m^{(q)}D_{ij}^{*(q)}\alpha_{kl}^{*(q)}n_0^{(q)}}{\rho RT_0Cl},F_i=\frac{F_i^*}{C^*_{1111}},Y^{(q)}=\frac{LY^{*(q)}}{C},h=\frac{h^*}{l},\tau_q=\frac{c\tau^{(q)}}{l},\end{gathered} \tag{1.3}$$

where t is time; x_i^* are rectangular Cartesian coordinates; u_i^* are displacement vector components; l is the characteristic linear dimension in the problem; l_1^*, l_2^* are length and width of the plate; h^* is plate thickness; $\eta^{(q)}=n^{(q)}-n_0^{(q)}$ is the concentration increment of q-th component in the multicomponent continuum; $n^{(q)}$ and $n_0^{(q)}$ are the actual and initial concentrations (mass fractions) of q-th component; C_{ijkl}^* are components of the elastic constant tensor; ρ is density of the medium; $\alpha_{ij}^{*(q)}$ are coefficients characterizing the volumetric changes of the medium due to diffusion; $D_{ij}^{*(q)}$ are the self-diffusion coefficients; R is the universal gas constant; T_0 is initial temperature; $m^{(q)}$ is the molar mass q-th component; $\tau^{(q)}$ is relaxation time of diffusion perturbations; $g^{(qt)}$ is the Darken's thermodynamic coefficients.

TIMOSHENKO ELASTIC DIFFUSION PLATE

To construct the equations for the plate bending, we use the variational formulation of the problem (1.1) – (1.3). According to the d'Alembert variational principle, relations (1.1) - (1.3) can be written in the form [18, 19]:

$$\begin{gathered}\int_G\left(\ddot{u}_i-\frac{\partial\sigma_{ij}}{\partial x_j}-F_i\right)\delta u_i dG+\sum_{q=1}^{N}\int_G\left(1+\tau_q\frac{\partial}{\partial\tau}\right)\left(\dot{\eta}^{(q)}+\frac{\partial J_i^{(q)}}{\partial x_i}-Y^{(q)}\right)\delta\eta^{(q)}dG+\\ +\iint_{\Pi_\sigma}\left(\sigma_{ij}n_j-P_i\right)\delta u_i dS+\sum_{q=1}^{N}\iint_{\Pi_J}\left(J_i^{(q)}+\tau_q\ddot{J}_i^{(q)}-I_i^{(q)}\right)n_i\delta\eta^{(q)}dS=0.\end{gathered} \tag{2.1}$$

Here U_i and $N^{(q)}$ are surface kinematic perturbations; P_i, $I_i^{(q)}$ are dynamic kinematic perturbations; δu_i, $\delta\eta^{(q)}$ are virtual displacement and concentrations increments; n_i are components of the outer normal unit vector to the surface of the plate.

For further transformations, it is necessary to formulate the following assumptions:

1) The domain G is a rectangular parallelepiped $G=D\times\left[-h/2,h/2\right]$, where $D=\left[0,l_1\right]\times\left[0,l_2\right]$ is the rectangular region of the plate middle surface $x_3=0$; $\Gamma=\partial D$ is the middle surface boundary (Fig. 1)

2) The plate surface has the form $\Pi=\Pi_-\cup\Pi_+\cup\Pi_b$, where Π_- are bottom surface corresponding $x_3=-h/2$; Π_+ are top surface corresponding $x_3=h/2$; $\Pi_b=\Pi_{11}\cup\Pi_{21}\cup\Pi_{12}\cup\Pi_{22}$

. The surfaces Π_{1k} corresponding $x_k = 0$; the surfaces Π_{2k} corresponding $x_k = l_k$, $k = 1,2$. We assume that the plate side surface is free from mechanical loads and mass transfer

$$\sigma_{ij} n_j \big|_{\Pi_-} = \sigma_{ij} n_j \big|_{\Pi_+} = 0. \tag{2.2}$$

$$J_i^{(q)} \big|_{\Pi_-} = J_i^{(q)} \big|_{\Pi_+} = 0. \tag{2.3}$$

3) Plate material is orthotropic perfect solid solution:

$$\begin{gathered} C_{\alpha\beta} = C_{\alpha\alpha\beta\beta}, \quad C_{44} = C_{2323}, \quad C_{55} = C_{1313}, \quad C_{66} = C_{1212}, \\ D_\alpha^{(q)} = D_{\alpha\alpha}^{(q)}, \quad D_\alpha^{(qr)} = D_\alpha^{(r)} g^{(qr)}, \quad \alpha_\alpha^{(q)} = \alpha_{\alpha\alpha}^{(q)}, \quad \Lambda_{\alpha\beta}^{(q)} = \Lambda_{\alpha\alpha\beta\beta}^{(q)}. \end{gathered} \tag{2.4}$$

4) Transverse plate deflections are considered small. Then the linearization of the unknown quantities with respect to the variable x_3 will has the form (here the approximate equality is replaced by the exact one)

$$\begin{gathered} u_1(x_1,x_2,x_3,\tau) = u(x_1,x_2,\tau) - x_3\chi_1(x_1,x_2,\tau), \; u_2(x_1,x_2,\tau) = v(x_1,x_2,\tau) - x_3\chi_2(x_1,x_2,\tau), \\ u_3(x_1,x_2,\tau) = w(x_1,x_2,\tau) + x_3\psi(x_1,x_2,\tau), \quad \eta^{(q)} = N_q(x_1,x_2,\tau) + x_3 H_q(x_1,x_2,\tau). \end{gathered} \tag{2.5}$$

5) There is considered that straight lines normal to the mid-surface remain straight after deformation and straight lines normal to the mid-surface remain normal to the mid-surface after deformation (straight normal hypothesis). Given (2.2) we will assume that there are no deformations along the axis Ox_3. Then [18-21]

$$\varepsilon_{33} = \frac{\partial u_3}{\partial x_3} = \psi = 0 \Rightarrow \psi = 0, \tag{2.6}$$

Hence

$$\begin{gathered} u_1(x_1,x_2,x_3,\tau) = u(x_1,x_2,\tau) - x_3\chi_1(x_1,x_2,\tau), \; u_2(x_1,x_2,\tau) = v(x_1,x_2,\tau) - x_3\chi_2(x_1,x_2,\tau), \\ u_3(x_1,x_2,\tau) = w(x_1,x_2,\tau), \quad \eta^{(q)} = N_q(x_1,x_2,\tau) + x_3 H_q(x_1,x_2,\tau). \end{gathered} \tag{2.7}$$

It follows from (2.2) that

$$\sigma_{33} n_3 \big|_{\Pi_-} = \sigma_{33} \big|_{x_3 = -h/2} = 0, \quad \sigma_{33} n_3 \big|_{\Pi_+} = \sigma_{33} \big|_{x_3 = h/2} = 0 \;\Rightarrow \int\limits_{-h/2}^{h/2} \frac{\partial \sigma_{33}}{\partial x_3} dx_3 = \sigma_{33} \big|_{-h/2}^{h/2} = 0. \tag{2.8}$$

$$\int\limits_G \frac{\partial \sigma_{33}}{\partial x_3} dG = \iint\limits_D dx_2 dx_3 \int\limits_{-h/2}^{h/2} \frac{\partial \sigma_{33}}{\partial x_3} dx_3 = \iint\limits_D \sigma_{33} \big|_{-h/2}^{h/2} dx_2 dx_3 = \iint\limits_D 0 dx_2 dx_3 = 0. \tag{2.9}$$

The components of the stress tensor and the diffusion flux vector are written as

$$\begin{gathered} \sigma_{11} = \frac{\partial u}{\partial x_1} - x_3 \frac{\partial \chi_1}{\partial x_1} + C_{12}\left(\frac{\partial v}{\partial x_2} - x_3 \frac{\partial \chi_2}{\partial x_2} \right) - \sum_{q=1}^{N} \alpha_1^{(q)} \left(N_q + x_3 H_q \right), \\ \sigma_{22} = C_{12}\left(\frac{\partial u}{\partial x_1} - x_3 \frac{\partial \chi_1}{\partial x_1} \right) + C_{22}\left(\frac{\partial v}{\partial x_2} - x_3 \frac{\partial \chi_2}{\partial x_2} \right) - \sum_{q=1}^{N} \alpha_2^{(q)} \left(N_q + x_3 H_q \right), \\ \sigma_{33} = C_{13}\left(\frac{\partial u}{\partial x_1} - x_3 \frac{\partial \chi_1}{\partial x_1} \right) + C_{23}\left(\frac{\partial v}{\partial x_2} - x_3 \frac{\partial \chi_2}{\partial x_2} \right) - \sum_{q=1}^{N} \alpha_3^{(q)} H_q, \\ \sigma_{12} = C_{66}\left(\frac{\partial u}{\partial x_2} + \frac{\partial v}{\partial x_1} - x_3 \frac{\partial \chi_1}{\partial x_2} - x_3 \frac{\partial \chi_2}{\partial x_1} \right), \quad \sigma_{13} = C_{55}\left(-\chi_1 + \frac{\partial w}{\partial x_1} \right), \quad \sigma_{23} = C_{44}\left(-\chi_2 + \frac{\partial w}{\partial x_2} \right), \end{gathered} \tag{2.10}$$

$$J_1^{(q)}+\tau_q\dot{J}_1^{(q)}=-\sum_{r=1}^{N}D_1^{(qr)}\left(\frac{\partial N_r}{\partial x_1}+x_3\frac{\partial H_r}{\partial x_1}\right)+\Lambda_{11}^{(q)}\left(\frac{\partial^2 u}{\partial x_1^2}-x_3\frac{\partial^2\chi_1}{\partial x_1^2}\right)+\Lambda_{12}^{(q)}\left(\frac{\partial^2 v}{\partial x_1\partial x_2}-x_3\frac{\partial^2\chi_2}{\partial x_1\partial x_2}\right),$$
$$J_2^{(q)}+\tau_q\dot{J}_2^{(q)}=-\sum_{r=1}^{N}D_2^{(qr)}\left(\frac{\partial N_r}{\partial x_2}+x_3\frac{\partial H_r}{\partial x_2}\right)+\Lambda_{21}^{(q)}\left(\frac{\partial^2 u}{\partial x_1\partial x_2}-x_3\frac{\partial^2\chi_1}{\partial x_1\partial x_2}\right)+\Lambda_{22}^{(q)}\left(\frac{\partial^2 v}{\partial x_2^2}-x_3\frac{\partial^3\chi_2}{\partial x_2^2}\right),$$
$$J_3^{(q)}+\tau_q\dot{J}_3^{(q)}=-\sum_{r=1}^{N}D_3^{(qr)}H_r-\Lambda_{31}^{(q)}\frac{\partial\chi_1}{\partial x_1}-\Lambda_{32}^{(q)}\frac{\partial\chi_2}{\partial x_2},\qquad \frac{\partial J_3^{(q)}}{\partial x_3}=0\quad\left(q=\overline{1,N}\right),$$

Substituting equalities (2.7), (2.10) into (2.1), we obtain the equations of transverse elastic diffusion vibrations of the Timoshenko plate

$$\ddot{\chi}_1=\frac{\partial^2\chi_1}{\partial x_1^2}+C_{66}\frac{\partial^2\chi_1}{\partial x_2^2}+C_{55}k^2\left(\frac{\partial w}{\partial x_1}-\chi_1\right)+\left(C_{12}+C_{66}\right)\frac{\partial^2\chi_2}{\partial x_1\partial x_2}+\sum_{q=1}^{N}\alpha_1^{(q)}\frac{\partial H_q}{\partial x_1}+\frac{12}{h^3}m_1,$$
$$\ddot{\chi}_2=C_{66}\frac{\partial^2\chi_2}{\partial x_1^2}+C_{22}\frac{\partial^2\chi_2}{\partial x_2^2}+C_{44}k^2\left(\frac{\partial w}{\partial x_2}-\chi_2\right)+\left(C_{12}+C_{66}\right)\frac{\partial^2\chi_1}{\partial x_1\partial x_2}+\sum_{q=1}^{N}\alpha_2^{(q)}\frac{\partial H_q}{\partial x_2}+\frac{12}{h^3}m_2,$$
$$\ddot{w}=C_{55}k^2\left(\frac{\partial^2 w}{\partial x_1^2}-\frac{\partial\chi_1}{\partial x_1}\right)+C_{44}k^2\left(\frac{\partial^2 w}{\partial x_2^2}-\frac{\partial\chi_2}{\partial x_2}\right)+\frac{q}{h},$$
$$\dot{H}_q+\tau_q\ddot{H}_q=\sum_{r=1}^{N}\left(D_1^{(qr)}\frac{\partial^2 H_r}{\partial x_1^2}+D_2^{(qr)}\frac{\partial^2 H_r}{\partial x_2^2}\right)+$$
$$+\Lambda_{11}^{(q)}\frac{\partial^3\chi_1}{\partial x_1^3}+\Lambda_{12}^{(q)}\frac{\partial^3\chi_2}{\partial x_1^2\partial x_2}+\Lambda_{21}^{(q)}\frac{\partial^3\chi_1}{\partial x_1\partial x_2^2}+\Lambda_{22}^{(q)}\frac{\partial^3\chi_2}{\partial x_2^3}+\frac{12}{h^3}z_q; \qquad (2.11)$$

where k are coefficient taking into account the uneven distribution of shear stresses over the plate thickness.

If the shear stresses are distributed according to the Zhuravsky formula, then for a plate of thickness h we have [20, 21]

$$k^2=\frac{5}{6}.$$

The remaining quantities in (2.11) are defined as follows:

1) $\int_{-h/2}^{h/2}F_1dx_3=n_1$ and $\int_{-h/2}^{h/2}F_2dx_3=n_2$ are the surface distributed axial loads,

2) $\int_{-h/2}^{h/2}x_3F_1dx_3=m_1+\int_{-h/2}^{h/2}\sigma_{13}dx_3$ and $\int_{-h/2}^{h/2}x_3F_2dx_3=m_2+\int_{-h/2}^{h/2}\sigma_{23}dx_3$, where m_1 и m_2 are the surface distributed moments,

3) $\int_{-h/2}^{h/2}F_3dx_3=q$ is the surface distributed transverse load,

4) $\int_{-h/2}^{h/2}Y^{(q)}dx_3=y_q$ and $\int_{-h/2}^{h/2}Y^{(q)}x_3dx_3=z_q$ is the surface density of mass transfer sources.

Equations (2.11) are supplemented with boundary conditions, which are also obtained from the variational equation (2.1). In the case of simply support plate they have the form

$$\left.\left(\frac{\partial\chi_1}{\partial x_1}+C_{12}\frac{\partial\chi_2}{\partial x_2}+\sum_{q=1}^{N}\alpha_1^{(q)}H_q\right)\right|_{x_1=0}=-\frac{12}{h^3}M_1^{(1)},\ \left.\left(\frac{\partial\chi_1}{\partial x_1}+C_{12}\frac{\partial\chi_2}{\partial x_2}+\sum_{q=1}^{N}\alpha_1^{(q)}H_q\right)\right|_{x_1=l_1}=-\frac{12}{h^3}M_2^{(1)};\quad(2.12)$$

$$\begin{gathered}\left.\left(C_{12}\frac{\partial^2\chi_1}{\partial x_1}+C_{22}\frac{\partial^2\chi_2}{\partial x_2}+\sum_{q=1}^{N}\alpha_2^{(q)}H_q\right)\right|_{x_2=0}=-\frac{12}{h^3}M_1^{(2)},\\ \left.\left(C_{12}\frac{\partial^2\chi_1}{\partial x_1}+C_{22}\frac{\partial^2\chi_2}{\partial x_2}+\sum_{q=1}^{N}\alpha_2^{(q)}H_q\right)\right|_{x_2=l_2}=-\frac{12}{h^3}M_2^{(2)};\end{gathered}\quad(2.13)$$

$$w|_{x_1=0}=W_1^{(1)}(x_2,\tau),\ \ w|_{x_1=l_1}=W_2^{(1)}(x_2,\tau),\ \ w|_{x_2=0}=W_1^{(2)}(x_1,\tau),\ \ w|_{x_2=l_2}=W_2^{(2)}(x_1,\tau);\quad(2.14)$$

$$H_q\big|_{x_1=0}=H_{q1}^{(1)}(x_2,\tau),\ \ H_q\big|_{x_1=l_1}=H_{q2}^{(1)}(x_2,\tau),\ \ H_q\big|_{x_2=0}=H_{q1}^{(2)}(x_1,\tau),\ \ H_q\big|_{x_2=l_2}=H_{q2}^{(2)}(x_1,\tau).\quad(2.15)$$

Here $M_k^{(l)}$ are bending moments shown in the figure 1.

The initial conditions are assumed to be zero.

TRANSITION TO THE KIRCHHOFF-LOVE PLATE

The Timoshenko model is a refinement of the Kirchhoff-Love plate model, by taking into account shear deformations and the influence of inertial forces when the normal is rotated to the middle surface. To check the transition to the Kirchhoff-Love model, we set [18-21]

$$\chi_k(x_1,x_2,\tau)=\frac{\partial w(x_1,x_2,\tau)}{\partial x_k}.\quad(3.1)$$

In this case

$$\varepsilon_{13}=\frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right)=-\chi_1+\frac{\partial w}{\partial x_1}=0,\ \ \varepsilon_{23}=\frac{1}{2}\left(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}\right)=-\chi_2+\frac{\partial w}{\partial x_2}=0.$$

Further, we substitute relations (3.1) into equations (2.11) and boundary conditions (2.12) - (2.15). In this case, the first equation in (2.11) we differentiate with respect to the variable x_1, and the second equation we differentiate with respect to the variable x_2, we obtain [18, 19]:

$$\begin{aligned}&\frac{\partial^2\ddot{w}}{\partial x_1^2}+\frac{\partial^2\ddot{w}}{\partial x_2^2}-\frac{12}{h^2}\ddot{w}=\frac{\partial^4 w}{\partial x_1^4}+2(C_{12}+2C_{66})\frac{\partial^4 w}{\partial x_1^2\partial x_2^2}+C_{22}\frac{\partial^4 w}{\partial x_2^4}+\\&+\sum_{q=1}^{N}\left(\alpha_1^{(q)}\frac{\partial^2 H_q}{\partial x_1^2}+\alpha_2^{(q)}\frac{\partial^2 H_q}{\partial x_2^2}\right)-\frac{12}{h^3}\left(\frac{\partial m_2}{\partial x_2}+\frac{\partial m_1}{\partial x_1}+q\right),\\&\dot{H}_q+\tau_q\ddot{H}_q=\sum_{r=1}^{N}\left(D_1^{(qr)}\frac{\partial^2 H_r}{\partial x_1^2}+D_2^{(qr)}\frac{\partial^2 H_r}{\partial x_2^2}\right)+\\&+\Lambda_{11}^{(q)}\frac{\partial^4 w}{\partial x_1^4}+\left(\Lambda_{12}^{(q)}+\Lambda_{21}^{(q)}\right)\frac{\partial^4 w}{\partial x_1^2\partial x_2^2}+\Lambda_{22}^{(q)}\frac{\partial^4 w}{\partial x_2^4}+\frac{12}{h^3}z_q\end{aligned}\quad(3.2)$$

Neglecting the inertial forces associated with the rotation of the normal, we pass to the classical model of the Kirchhoff-Love plate

$$
\begin{aligned}
&-\frac{12}{h^2}\ddot{w}=\frac{\partial^4 w}{\partial x_1^4}+2\left(C_{12}+2C_{66}\right)\frac{\partial^4 w}{\partial x_1^2\partial x_2^2}+C_{22}\frac{\partial^4 w}{\partial x_2^4}+\\
&+\sum_{q=1}^{N}\left(\alpha_1^{(q)}\frac{\partial^2 H_q}{\partial x_1^2}+\alpha_2^{(q)}\frac{\partial^2 H_q}{\partial x_2^2}\right)-\frac{12}{h^3}\left(\frac{\partial m_2}{\partial x_2}+\frac{\partial m_1}{\partial x_1}+q\right),\\
&\dot{H}_q+\tau_q\ddot{H}_q=\sum_{r=1}^{N}\left(D_1^{(qr)}\frac{\partial^2 H_r}{\partial x_1^2}+D_2^{(qr)}\frac{\partial^2 H_r}{\partial x_2^2}\right)+\\
&+\Lambda_{11}^{(q)}\frac{\partial^4 w}{\partial x_1^4}+\left(\Lambda_{12}^{(q)}+\Lambda_{21}^{(q)}\right)\frac{\partial^4 w}{\partial x_1^2\partial x_2^2}+\Lambda_{22}^{(q)}\frac{\partial^4 w}{\partial x_2^4}+\frac{12}{h^3}z_q
\end{aligned}
\qquad (3.3)
$$

An analogue of boundary conditions (2.12) - (2.15) to equations (3.3) can be written as

$$
\begin{aligned}
&\left.\left(\frac{\partial^2 w}{\partial x_1^2}+C_{12}\frac{\partial^2 w}{\partial x_2^2}+\sum_{q=1}^{N}\alpha_1^{(q)}H_q\right)\right|_{x_1=0}=-\frac{12}{h^3}M_1^{(1)},\\
&\left.\left(\frac{\partial^2 w}{\partial x_1^2}+C_{12}\frac{\partial^2 w}{\partial x_2^2}+\sum_{q=1}^{N}\alpha_1^{(q)}H_q\right)\right|_{x_1=l_1}=-\frac{12}{h^3}M_2^{(1)},\\
&\left.\left(C_{12}\frac{\partial^2 w}{\partial x_1^2}+C_{22}\frac{\partial^2 w}{\partial x_2^2}+\sum_{q=1}^{N}\alpha_2^{(q)}H_q\right)\right|_{x_2=0}=-\frac{12}{h^3}M_1^{(2)},\\
&\left.\left(C_{12}\frac{\partial^2 w}{\partial x_1^2}+C_{22}\frac{\partial^2 w}{\partial x_2^2}+\sum_{q=1}^{N}\alpha_2^{(q)}H_q\right)\right|_{x_2=l_2}=-\frac{12}{h^3}M_2^{(2)},\\
&\left.w\right|_{x_1=0}=W_1^{(1)}\left(x_2,\tau\right),\quad \left.w\right|_{x_1=l_1}=W_2^{(1)}\left(x_2,\tau\right),\\
&\left.w\right|_{x_2=0}=W_1^{(2)}\left(x_1,\tau\right),\quad \left.w\right|_{x_2=l_2}=W_2^{(2)}\left(x_1,\tau\right),\\
&\left.H_q\right|_{x_1=0}=H_{q1}^{(1)}\left(x_2,\tau\right),\quad \left.H_q\right|_{x_1=l_1}=H_{q2}^{(1)}\left(x_2,\tau\right),\\
&\left.H_q\right|_{x_2=0}=H_{q1}^{(2)}\left(x_1,\tau\right),\quad \left.H_q\right|_{x_2=l_2}=H_{q2}^{(2)}\left(x_1,\tau\right).
\end{aligned}
$$

CONCLUSION

We can come to the following conclusions. The mathematical model of elastic diffusion unsteady vibrations of a Timoshenko rectangular orthotropic plate is constructed using the d'Alembert variational principle (the generalized principle of virtual displacements). This model has described the relationship between mechanical and diffusion fields in a continuum. The initial-boundary value problem of a simply supported plate bending is formulated. The model considers the relaxation diffusion effects that determine the final speed of propagation of diffusion perturbations. The transition to the classical Kirchhoff-Love plate model is checked.

ACKNOWLEDGMENTS

This work was supported by the Russian Science Foundation (project No 20-19-00217)

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