The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5 – 9 June 2022, Oslo, Norway

Tangential differential calculus for curved, linear Kirchhoff beams with systematic convergence studies

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Keywords: *Kirchhoff beam, IGA, tangential differential calculus, convergence studies.*

We propose a reformulation of linear Kirchhoff beams in two dimensions based on the tangential differential calculus. The rotation-free formulation of the Kirchhoff beam is classically based on curvilinear coordinates, see, e.g., [1] and [2]. However, for general applications in engineering and sciences that take place on curved geometries embedded in a higher-dimensional space, the tangential differential calculus [3] enables a formulation independent of curvilinear coordinates and, hence, is suitable also for implicitly defined geometries. The geometry and differential operators are formulated in global coordinates related to the embedding space. Therefore, a parametrization is not required, so the TDC formulation is more general. Furthermore, the reformulation is often more intuitive as quantities like Christoffel symbols are not required. This work follows our outlines for linear shells in [4, 5] where reformulations with the TDC are also emphasized; we also note relations to [6].
The Kirchhoff beam, being the curved variant of the Euler-Bernoulli beam, requires at least C_1-continuity for the finite element shape functions. Using a standard FEM based on Lagrange elements does not furnish higher-order continuity. Therefore, isogeometric analysis (IGA) is employed for the generation of shape functions in the numerical analysis. The boundary conditions are enforced using Lagrange multipliers.
We emphasize systematic convergence studies for established and new test cases by investigating residual errors. That is, as a post-processing step, the obtained FE solution is inserted into the strong form of the governing equations and the error is then integrated over the domain in an L_2-sense. For sufficiently smooth physical fields, higher-order convergence rates in the residual errors are achieved. For classical benchmark test cases with known analytical solutions, we also confirm optimal convergence rates of p+1 in the displacements.

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