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CLOSED - FORM EXPRESSIONS FOR THE OPTIMUM WINDING ANGLES OF FIBRES IN LAMINATED CYLINDRICAL PRESSURE VESSELS SUBJECTED TO INTERNAL PRESSURE, AXIAL FORCE AND TORQUE

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Abstract. The present paper aims at deriving closed-form expressions for the optimum winding angles of fibres in laminated cylindrical pressure vessels subjected to internal pressure, axial load and torque. To achieve this goal, the state of the stress on the surface of the vessel is represented by a composite layered element subjected to general in-plane loading. The symbolic software Mathematica is used to formulate, solve the optimization problem and to generate the data of optimum fibre angles for different loading combinations. The generated data is fitted with simple polynomial expressions capable of accurately predicting the optimum winding angles. The optimization is performed for three candidate composite materials: carbon/epoxy, glass/epoxy, and aramid/epoxy laminates.

1 INTRODUCTION

The demand for composite materials is surging every year with the increase in their applications in various areas such as automotive, aerospace, and other industries [1]. These materials are particularly attractive due to their high-performance physical properties especially the high strength to weight ratios and their ability to withstand the hostile environment such as high radiation and high temperature. However, the anisotropy of the composite laminates makes their analysis and design relatively more complicated than that of other structures [2]. One of the most popular composite materials is the laminated composite where layers are stacked with different fiber orientations. For this type, the sequence of layers should be selected carefully to best withstand the loads and save material.

Optimization of laminated composite plates under different loading and boundary conditions has drawn the interest of research community over the last few decades. The usual optimization object is to design layer thickness, fiber orientations, and stacking sequence which will provide minimum weight or maximum stiffness/strength [3]. Park [4,5] utilized two failure criteria (strain envelope and von mises) as objective functions to design simple symmetric laminates by taking ply orientation angle as the design variable. He concluded with the advantage of using particular classes of laminates (e.g., $[\pm\theta]_s$, $[\theta/0/-\theta]_s$) to obtain the highest strength laminate under in-plane loading. Kim et al. [6,7] used Tsai-Wu failure criterion as the objective function to obtain the stacking sequence and fiber orientations of a symmetrically laminated plate and cylindrical composite shell under various in-plane loading conditions. Song et al. [8] implemented a finite element method based on shear deformation theory to optimize the design of symmetric laminates for maximum strength. They took Tsai-Hill failure criterion as the objective function and ply orientation angles as the design variables to find the optimum sequence for different in-plane and transverse loadings. Groenwold and Haftka [9] compared failure index and strength safety factor as the objective functions for both Tsai-Wu and Tsai-Hill criteria. They concluded that to find the optimum laminate design, it is recommended to directly start by selecting the safety factor as the objective function.

Among the optimization methods, genetic algorithm (GA) is considered to be the most commonly used method to design composite laminates for strength [10,11,12,13]. Other similar methods have been also used such as simulated annealing (SA) [14,15,16].

None of the above cited work attempted to obtain a closed formula for the optimum fiber orientation angles. In the present work, optimum strength design of symmetrically laminated cylindrical pressure vessel under internal pressure, axial tension and torsion is proposed utilizing the classical lamination theory (CLT) along with Tsai-Wu and Tsai-Hill failure criteria. The optimum design, for three different materials (glass/epoxy, carbon/epoxy and aramid/epoxy), is achieved by taking failure criteria as the objective functions and the orientation angles as the design variable. To achieve this objective, first a Mathematica code based on analytical procedure is prepared. This code is capable of finding the global optimum stacking sequence through testing different lay-up combinations. The procedure is performed for a wide range of winding angles, the three material types and different load ratios. The data collected from this parametric analysis is fitted with closed-form expressions for the optimum winding angle. Finally, several numerical examples are presented to verify the accuracy of the developed closed-form solutions.

2 PROBLEM FORMULATION

2.1 Problem statement

The structure to be optimized is a symmetric laminated composite cylindrical vessel with diameter D and wall thickness h subjected to internal pressure p, axial tensile load F and torque Tas shown in Figure 1. According to the theory of thin-walled cylindrical pressure and CLT, the internal loadings (N_x, N_y, N_{xy}) are obtained in term of the external loadings (P, F, T):

$$N_x = \frac{F}{\pi D} + \frac{PD}{4} \tag{1}$$

$$N_y = \frac{PD}{2} \tag{2}$$

$$N_{xy} = \frac{2T}{\pi D^2} \tag{3}$$

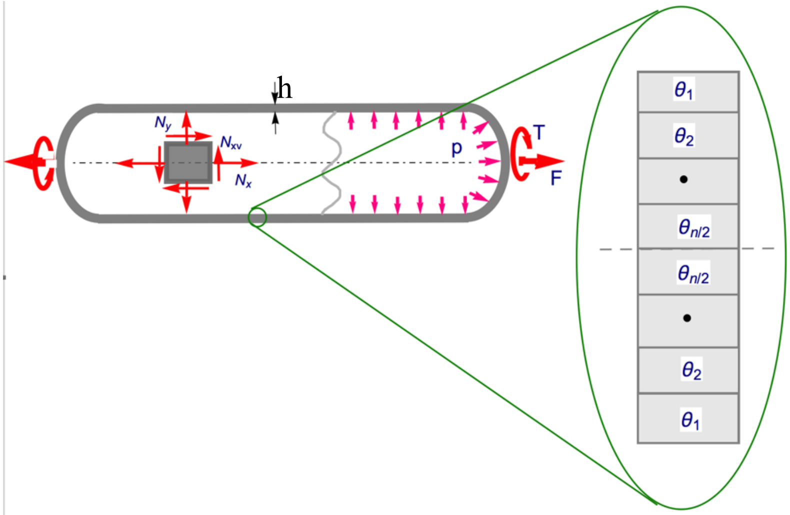

Figure 1: Thin-walled cylindrical pressure vessel under internal pressure, axial tension and torque

With the above simplification, the problem is reduced to the in-plane loaded laminate shown in Figure 2. The laminate has a total thickness h and contains N number of plies, each has a thickness of $t = h/N$ and has its own fiber orientation angle. The plies are assumed to be homogeneous and perfectly bonded. Moreover, the plate is assumed to behave according to the classical lamination theory (CLT).

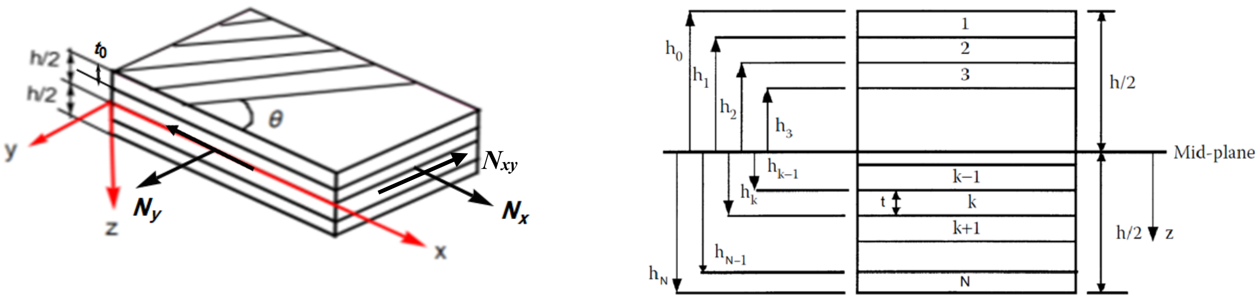

Figure 2: Represented element subjected to in-plane loading and coordinate locations of plies

The ultimate goal is to determine the optimum fiber orientation angle of each ply leading to the maximum strength of the whole laminate based on the first-ply failure theory. No constraint is imposed on the fiber orientations angles of the plies which means that all possible angles in the continuous interval $\{-90,90\}$ are allowed. The two failure criteria of Tsai-Hill and Tsai-

Wu are considered as design constraints and the optimization is performed for three composite systems: glass/epoxy, carbon/epoxy and aramid/epoxy.

2.2 Analysis Formulation for Laminated Structures

Based on the classical lamination theory (CLT), each ply is assumed to be under plane stress condition, i.e., the out of plane stresses components $(\sigma_z, \tau_{xz}, \tau_{yz})$ are neglected. Furthermore, due to the symmetry of the laminate and the restriction of loading to in-plane direction, the curvature terms become zero. As a result, the strain components $(\epsilon_x, \epsilon_y, \gamma_{\mathrm{xy}})$ are uniform throughout the thickness and therefore, the laminate mechanical response become independent of the stacking sequence. To derive the laminate constitutive equations, consider a single "lamina" of continuous fiber arranged in a rectangular array where the principal material axes 1 is coinciding with the fibers direction, 2 is normal to the fibers direction and 3 is completing the right-handed-rule system of coordinates as shown in Figure 3.

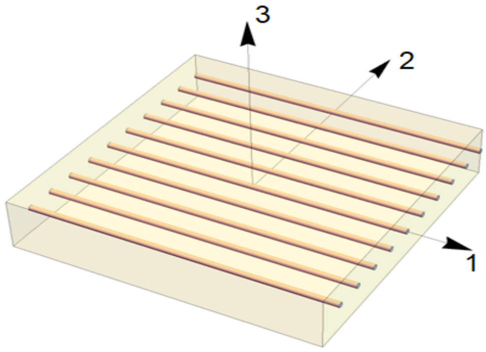

Figure 3: Typical lamina and the principal coordinate axes

The stress - strain relations for this individual layer k referred to the principle (local) laminate coordinate system (1,2) can be expressed as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix}_k \tag{4}$$

where Q_{ij} are the stiffness coefficients which are expresed in term of engineering constants $(E_1, E_2, G_{12}, \upsilon_{12}$ and $\upsilon_{21} = \upsilon_{12}\frac{E_1}{E_2})$ as

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}},$$
$$Q_{12} = \frac{E_1\nu_{21}}{1-\nu_{12}\nu_{21}},$$
$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \text{and}$$
$$Q_{66} = G_{12}.$$

Using the transformation matrix $[T]$ for a lamina with an orientation angle (θ_k):

$$[T]_k = \begin{bmatrix} C^2 & S^2 & 2\mathrm{CS} \\ S^2 & C^2 & -2\mathrm{CS} \\ -\mathrm{CS} & \mathrm{CS} & C^2 - S^2 \end{bmatrix}_k \tag{5}$$

where C and S denote $\cos\theta_k$ and $\sin\theta_k$, respectively.

The stress - strain relations for an individual layer k (located between $z = h_{k-1}$ and $z = h_k$) referred to the global laminate coordinate system (x, y) are given by:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{\mathrm{xy}} \end{bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{\mathrm{xy}} \end{bmatrix}_k \tag{6}$$

where $\bar{Q}_{\mathrm{ij}}$ are the elements of the transformed reduced stiffness matrix, which are given by:

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}C^4 + Q_{22}S^4 + 2(Q_{12} + 2Q_{66})C^2S^2 \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})C^2S^2 + Q_{12}(C^4 + S^4) \\ \bar{Q}_{22} &= Q_{11}S^4 + Q_{22}C^4 + 2(Q_{12} + 2Q_{66})C^2S^2 \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})C^3S - (Q_{22} - Q_{12} - 2Q_{66})CS^3 \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})CS^3 - (Q_{22} - Q_{12} - 2Q_{66})C^3S \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})C^2S^2 + Q_{66}(C^4 + S^4) \end{aligned}$$

The resultants forces per unit length are obtained by the integration of the stresses, i.e.:

$$\begin{bmatrix} N_x \\ N_y \\ N_{\mathrm{s}} \end{bmatrix} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{\mathrm{xy}} \end{bmatrix}_k \mathrm{d}z \; = t \sum_{k=1}^{N} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{\mathrm{xy}} \end{bmatrix}_k \tag{7}$$

Substituting Eq. (6) into Eq. (7) yields the following matrix equation relating the in-plane forces (N_x, N_y, N_{xy}) to the midspan strains $(\epsilon_x, \epsilon_y, \gamma_{\mathrm{xy}})$.

$$\begin{bmatrix} N_x \\ N_y \\ N_{\mathrm{s}} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{\mathrm{xy}} \end{bmatrix} \tag{8}$$

where A_{ij} are the extensional stiffnesses that relate in-plane loads to in-plane strains which are given by:

$$A_{\mathrm{ij}} = \; t \sum_{k=1}^{N} \bar{Q}_{\mathrm{ij}} \tag{9}$$

Given the in-plane forces, the global strain components $(\epsilon_x, \epsilon_y, \gamma_{\mathrm{xy}})$ can be obtained from Eq. (8). The results can then be used in (6) to find the global stresses components $\left(\sigma_x^k, \sigma_y^k, \tau_{xy}^k\right)$. Finally, the principal stress component can be obtained from the following transformation:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}_k = [T]_k \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{\mathrm{xy}} \end{bmatrix}_k \tag{10}$$

2.3 Static Failure Criteria

The main objective function is to maximize the strength capacity of the laminated plate. This function is represented by the strength ratio (SR) which is a measure of how much the load can be increased if the lamina is safe (SR > 1) or how much the load should be decreased if the lamina has failed (SR < 1). The strength ratio can be computed by one of the composite failure criteria. In this study, Tsai-Wu and Tsai-Hill criteria are assumed. Both of these criteria provide an analytical expression that takes into consideration the stresses interactive effects. The formulation of SR for both criteria is given below.

2.3.1 Tsai–Wu criterion

Tsai-Wu is a quadratic failure criterion is based on Beltrami theory of total strain energy failure [17]. For this criterion, SR is given by:

$$SR_{TW} = \frac{-b + \sqrt{b^2 + 4a}}{2a} \tag{12}$$

where:

$$a = F_{11}{\sigma_1}^2 + F_{22}{\sigma_2}^2 + F_{66}{\tau_{12}}^2 + 2F_{12}\sigma_1\sigma_2 \tag{13}$$

$$b = F_1\sigma_1 + F_2\sigma_2 \tag{14}$$

The strength coefficients $F_{11}, F_{22}, F_{66}, F_{12}, F_1$ and F_2 are related to the strength properties of the laminate by:

$$\begin{aligned} F_1 &= \frac{1}{X_t} + \frac{1}{X_c}, & F_2 &= \frac{1}{Y_t} + \frac{1}{Y_c} \\ F_{11} &= \frac{1}{X_t X_c}, & F_{22} &= \frac{1}{Y_t Y_c} \\ F_{66} &= -\frac{1}{2}\sqrt{F_{11}F_{22}}, & F_{66} &= \frac{1}{S^2} \end{aligned} \tag{15}$$

where the symbols "X" and "Y" represent the ultimate longitudinal strengths along the fiber direction (axis 1) and normal to it (axis 2), respectively; the subscripts "t" and "c" symbolize the tensile and compressive; S is the ultimate in-plane shear strength.

2.3.2 Tsai–Hill criterion

Tsai-Hill theory is an extension of the distortion energy failure theory (Von - Mises yield criterion) for isotropic materials as applied to anisotropic material theory [17]. The strength ratio based on Tsai-Hill criterion is given by:

$$SR_{TH} = \frac{1}{\sqrt{\left(\frac{\sigma_1}{X_t}\right)^2 + \left(\frac{\sigma_1\sigma_2}{X_t X_t}\right) + \left(\frac{\sigma_2}{Y_t}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2}} \tag{16}$$

2.4 Materials

The three most common fiber-reinforced polymer composites namely, glass/epoxy (Gl/Ep), graphite/epoxy (Gr/Ep) and aramid/epoxy (Ar/Ep) are considered in this study. Table.1 lists the material and strength properties for a unidirectional lamina made of each of the three materials [17].

Table 1: Typical mechanical properties of a unidirectional lamina

Material Properties	Symbol	Unit	(Gr/Ep)	Gl/Ep)	Ar/Ep)
Longitudinal elastic modulus	E_1	GPa	181	38.6	76
Transverse elastic modulus	E_2	GPa	10.3	8.27	5.5
Shear modulus	G_{12}	GPa	7.17	4.14	2.3
Major Poisson's ratio	ν_{12}	—	0.28	0.26	0.34
Ultimate longitudinal tensile strength	X_t	MPa	1500	1062	1400
Ultimate longitudinal compressive strength	X_c	MPa	1500	610	235
Ultimate transverse tensile strength	Y_t	MPa	40	31	12
Ultimate transverse compressive strength	Y_c	MPa	246	118	53
Ultimate in-plane shear strength	S	MPa	68	72	34

3 COMPUTER IMPLEMENTATION

In order to find the optimum fiber orientation angle in each ply, one needs to run a huge number of computations depending on the number of angles selected in the interval $\{-90,90\}$ and the number of laminas. As an example, if the angles are selected in an increment of 1°, i.e., 180 angles, then the number of possible stacking sequences $= 180^{N/2}$, where N is the number of laminas. However, after running the code for limited number of angles, the optimum sequence was always of the pattern $[\pm\theta_{N/2}]_S$ for pure normal in-plane loading (no shear) and $[(\theta_1,\theta_2)_{N/4}]_S$ for the general in-plane loading including the shear. The algorithm of Mathematica code performing the optimization procedure is given by the flow chart in Figure 4.

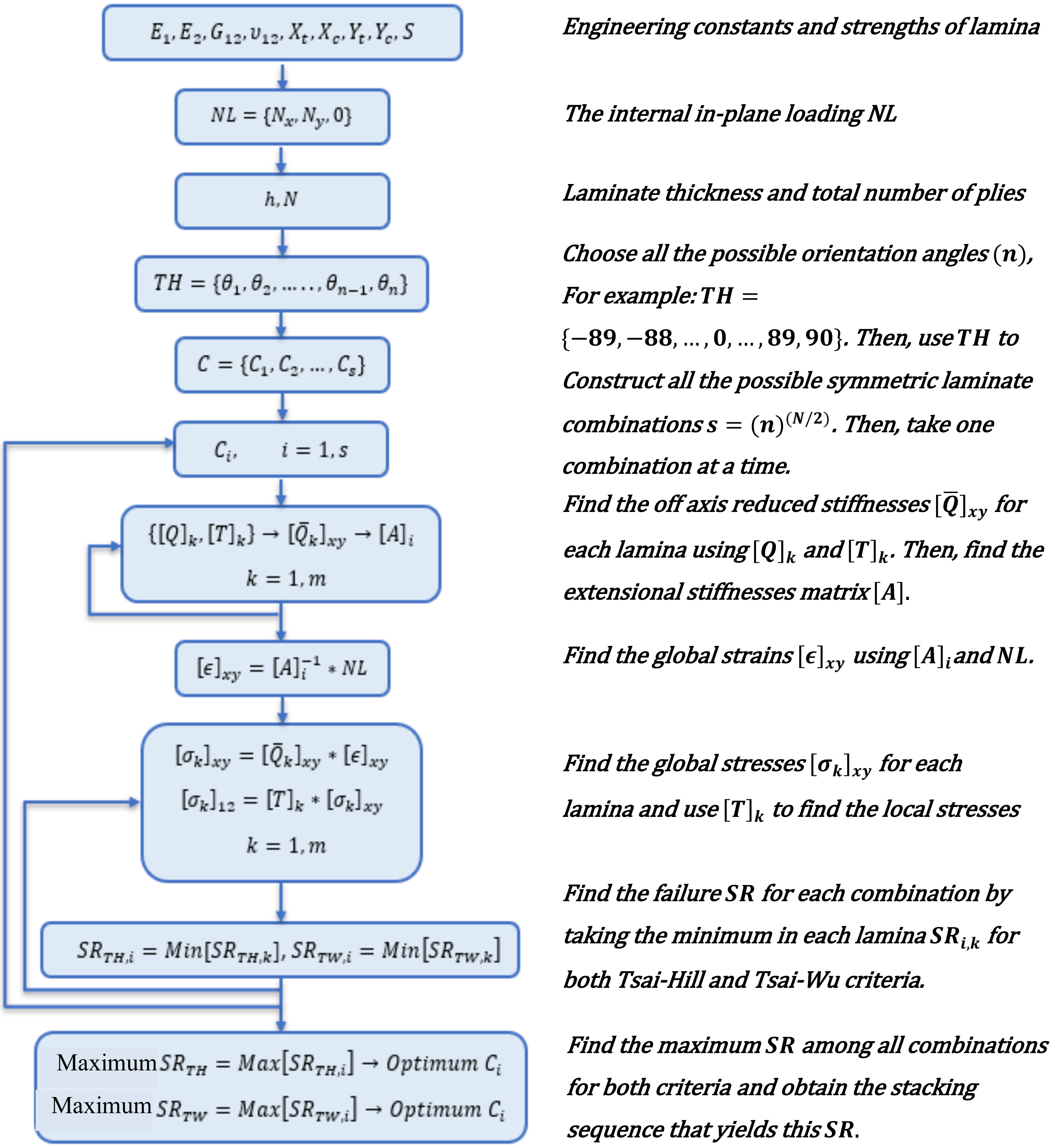

Figure 4: Code flow chart for finding the optimum sequence and corresponding strength ratio

4 OPTIMIZATION RESULTS AND DISCUSSION

The developed Mathematica code is used to generate data of optimum angles for different loading ratios $\beta = N_y/N_x$ and $N_{xy} = 0$. The obtained data are then fitted with the simple power-law functions given in Table 3. For cases involving shear, i.e. $\gamma = \frac{N_{xy}}{N_x} \neq 0$, the results can be obtained by stress transformation.

Table 2: The optimum angle closed formula for each material based on both failure criteria

Material	Failure Criteria	$\theta_{optimum}$
Gr/Ep	Tsai-Hill	$\begin{cases} 0, & 0 \le \beta \le 0.003 \\ -5.785 + 65.5\,\beta^{0.4} - 14.714\,\beta, & 0.003 < \beta \le 1 \\ 95.785 - 65.5\left(\frac{1}{\beta}\right)^{0.4} + \frac{14.714}{\beta}, & 1 < \beta \le 334 \\ 90, & \beta > 334 \end{cases}$
	Tsai-Wu	$\begin{cases} 5.325 + 94.246\,\beta^{0.7} - 54.866\,\beta, & 0 \le \beta \le 1 \\ 84.675 - 94.246\left(\frac{1}{\beta}\right)^{0.7} + \frac{54.866}{\beta}, & \beta > 714 \end{cases}$
Gl/Ep	Tsai-Hill	$\begin{cases} 0, & 0 \le \beta \le 0.0014 \\ 16.6 + 28.56\,\beta^{0.5} - 1.0336\,\text{Ln}(\beta)^2, & 0.0014 < \beta \le 1 \\ 73.4 - 28.56\left(\frac{1}{\beta}\right)^{0.5} + 1.0336\,\text{Ln}\left(\frac{1}{\beta}\right)^2, & 1 < \beta \le 714 \\ 90, & \beta > 714 \end{cases}$
	Tsai-Wu	$\begin{cases} 0, & 0 \le \beta \le 0.032 \\ 112.23 - 69.33\left(\frac{1}{\beta}\right)^{0.2} + \frac{1.7688}{\beta} - \frac{0.02955}{\beta^2}, & 0.032 < \beta \le 1 \\ -22.23 + 69.33\,\beta^{0.2} - 1.7688\,\beta + 0.02955\,\beta^2, & 1 < \beta \le 32 \\ 90, & \beta > 32 \end{cases}$
Ar/Ep	Tsai-Hill	$\begin{cases} 0, & 0 \le \beta \le 0.0007 \\ -0.56 + 104.505\,\beta^{0.6} - 61.242\,\beta + 1.8354\,\beta^2, & 0.0007 < \beta \le 1 \\ 90.56 - 104.505\left(\frac{1}{\beta}\right)^{0.6} + \frac{61.242}{\beta} - \frac{1.8354}{\beta^2}, & 1 < \beta \le 1428 \\ 90, & \beta > 1428 \end{cases}$
	Tsai-Wu	$\begin{cases} 0, & 0 \le \beta \le 0.0103 \\ -59.74 + 95.64\,\beta^{0.1} + 22.055\,\beta - 13.475\,\beta^2, & 0.0103 < \beta \le 1 \\ 149.74 - 95.64\left(\frac{1}{\beta}\right)^{0.1} - \frac{22.055}{\beta} + \frac{13.475}{\beta^2}, & 1 < \beta \le 97 \\ 90, & \beta > 97 \end{cases}$

5 VERIFICATION OF THE CLOSED-FORM SOLUTIONS

In order to verify the accuracy of the developed closed-form solutions, several cases involving different materials, loading combinations and failure criteria are assumed and the optimum fiber angle(s) are computed using both the code (which yields the exact value) and the closed form expressions given in Table 2. The results of the comparison are given in Tables 3 to 5 for the three different composite materials. The results in the three tables clearly show the excellent agreement between the two solutions which confirms the accuracy of the developed closed-form solutions.

Table 3: Optimum fiber orientations for Gr/Ep

$\{N_x, N_y, N_{xy}\}$	β	γ	θ_{opt}			
			Tsai-Hill		Tsai-Wu	
			Code	Formula	Code	Formula
{5,0,0}	0	0	0°	0°	5.6°	5.3°
{5,2.5,0}	0.5	0	37°	37°	36°	36°
{5,5,0}	1	0	45°	45°	45°	45°
{5,10,0}	2	0	53°	54°	54°	54°
{5,2.5,2.5}	0.5	0.5	(9°/54°)	(9°/54°)	(10°/54°)	(10°/54°)
{5,5,5}	1	1	(45°/45°)	(45°/45°)	(45°/45°)	(45°/45°)
{5,10,7}	2	1.4	(55°/55°)	(53°/57°)	(48°/62°)	(48°/62°)

Table 4: Optimum fiber orientations for Gl/Ep

$\{N_x, N_y, N_{xy}\}$	β	γ	θ_{opt}			
			Tsai-Hill		Tsai-Wu	
			Code	Formula	Code	Formula
{5,0,0}	0	0	0°	0°	0°	0°
{5,2.5}	0.5	0	36°	36°	36°	36°
{5,5,0}	1	0	45°	45°	45°	45°
{5,10,0}	2	0	54°	54°	54°	54°
{5,2.5,2.5}	0.5	0.5	(8°/55°)	(8°/55°)	(10°/53°)	(10°/53°)
{5,5,5}	1	1	(45°/45°)	(45°/45°)	(45°/45°)	(45°/45°)
{5,10,7}	2	1.4	(55°/55°)	(55°/55°)	(55°/55°)	(55°/55°)

Table 5: Optimum fiber orientations for Ar/Ep

$\{N_x, N_y, N_{xy}\}$	β	γ	θ_{opt}			
			Tsai-Hill		Tsai-Wu	
			Code	Formula	Code	Formula
$\{5,0,0\}$	0	0	$0°$	$0°$	$0°$	$0°$
$\{5,2.5,0\}$	0.5	0	$38°$	$38°$	$37°$	$37°$
$\{5,5,0\}$	1	0	$45°$	$45°$	$45°$	$45°$
$\{5,10,0\}$	2	0	$52°$	$52°$	$53°$	$53°$
$\{5,2.5,2.5\}$	0.5	0.5	$(9°/55°)$	$(8°/55°)$	$(10°/54°)$	$(13°/50°)$
$\{5,5,5\}$	1	1	$(45°/45°)$	$(45°/45°)$	$(45°/45°)$	$(45°/45°)$
$\{5,10,7\}$	2	1.4	$(51°/59°)$	$(52°/58°)$	$(55°/55°)$	$(55°/55°)$

6 CONCLUSIONS

The paper utilizes symbolic computing to derive closed-form formulas for the optimum fiber orientation angles of a composite pressure vessel subjected to internal pressure, axial tension and torque. The optimization is based on the maximum strength ratio criterion and the closed-form expressions are obtained for three different composite materials: carbon/epoxy, glass/epoxy, and aramid/epoxy assuming two failure criteria: Tsai-Hill and Tsai-Wu. The numerical examples confirm the accuracy of the developed closed-form formulas.

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