An adaptive stabilized finite element method based on residual minimization for unsteady advection-diffusion problems

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ABSTRACT

The most common strategy to solve time-dependent problems is the method of lines. It consists of first approximate the spatial derivatives using some discrete scheme. Then approximate the resulting system using a time-marching technique, obtaining a fully discrete problem to solve.
For advection-dominant problems, classical methodologies for the spatial discretization (e.g., standard FEM) can suffer from numerical instabilities if the mesh is not enough fine, leading to unphysical oscillations. Therefore, stabilized FEM such as Discontinuous Galerkin method (DGM) are more suitable for this purpose.
As a time-marching alternative, explicit methods have the advantage of allowing for fast resolutions, as they usually employ a precomputed preconditioner at every time step when considering a fixed spatial mesh. However, a sufficiently fine discretization will lead to a restrictive CFL condition. Altought implicit schemes can overcome this restriction, they require the resolution of a time-dependent matrix at each time step. Last implies that an excessive computational cost can be required to obtain high-quality advection-dominant solutions capturing physical properties of the analytical solution, for instance, shock formations or internal boundary layers.
Recently, a novel adaptive stabilized finite element method for steady linear problems is introduced in [1]. This method is derived by combining residual minimization idea and the stability offered by the DG method. It provides stable solutions belonging to classical finite element spaces, together with residual representatives, helping to perform on-the-fly adaptive mesh refinements. Consequently, high-resolution solutions of advection-dominant problems become feasible, with a considerable reduction in the computational effort needed for this purpose [2].
In this talk, we will introduce a novel adaptive-stabilized finite element method for unsteady advection dominant problems. Based on [1], we built the discrete scheme by performing a residual minimization at every time step, to an ad-hoc modification of a discrete formulation obtained from the coupling of an implicit time-marching scheme and a DG formulation in space. As a result, we obtain a stable solution and a residual estimative at every discrete time level. As in [1], this residual estimative can be employed to guide mesh refinements, implying a considerable reduction in the computational effort required for implicit schemes.

REFERENCES

[1] Calo, V.M. and Ern, A. and Muga, I. and Rojas, S. *An adaptive stabilized conforming finite element method via residual minimization on dual discontinuous Galerkin norms. Computer Methods in Applied Mechanics and Engineering.*, Vol. **363** , pp. 112891, (2020).

[2] Cier,R.J. Rojas, S. Calo, V.M. *An automatic-adaptivity stabilized finite element method via residual minimization for heterogeneous, anisotropic advection-diffusion-reaction problems*, 2011.11264 arXiv, (2020)