

SEISMIC RETROFIT OF EXISTING STRUCTURES WITH ROCKING WALLS

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Abstract. The high occupancy levels in urban multi-story buildings, along with evolving safety standards, necessitate a reassessment of seismic performance objectives for structures. This study aims to explore a retrofitting and rehabilitation approach designed to enhance the resilience of existing buildings, with a broader objective of improving their performance under different conditions and prolonging their lifespan. In light of significant structural damage and documented weak-story failures observed after major earthquakes, this research examines the use of stepping rocking shear walls as a retrofit solution for moment-resisting frames (MRFs). This study begins with an overview of the current literature on rocking wall systems. Nonlinear time-history analyses are then conducted on two structural configurations—a 9-story and a 20-story MRFs—both in their original form and after retrofitting with stepping rocking shear walls. The results reveal that incorporating a rocking shear wall enhances first-mode dominance, leading to a more uniform distribution of interstory drift and significantly reducing the risk of weak-story failure. Additionally, the findings demonstrate that rocking shear walls effectively minimize residual drift, thereby improving post-earthquake operability while preserving structural integrity. Comparisons with FEMA P-58 performance criteria further highlight a decrease in damage probability and repair costs associated with the proposed retrofit method. Overall, the study underscores the potential of rocking shear walls as a viable and efficient retrofit strategy for enhancing the seismic resilience of both existing and new structures.

1 INTRODUCTION

To address significant seismic damage in moment-resisting frames, which sometimes led to weak-story failures, the idea of a rigid core system became increasingly popular [1–6]. The presence of substantial axial loads on shear walls in tall buildings significantly reduces their ductility. This reduction is particularly pronounced under long-duration pulse motions, which impose considerable ductility demands [7, 8]. Moreover, the base of the core wall is susceptible to cyclic degradation during prolonged shaking, often resulting in permanent inelastic deformations. Such inelastic responses can lead to permanent drifts and substantial repair costs which compromising the sustainability of the overall design. An example of this type of failure is illustrated in Fig. 1 (a), depicting a fourteen-story moment-resisting frame with a fixed shear-wall building after the 1964 Anchorage, Alaska earthquake.

After 1994 Northridge, California earthquake followed by 1995 Kobe, Japan earthquake, coherent acceleration pulses (0.8-1.5 sec duration at that time) which result in large monotonic velocity, re-

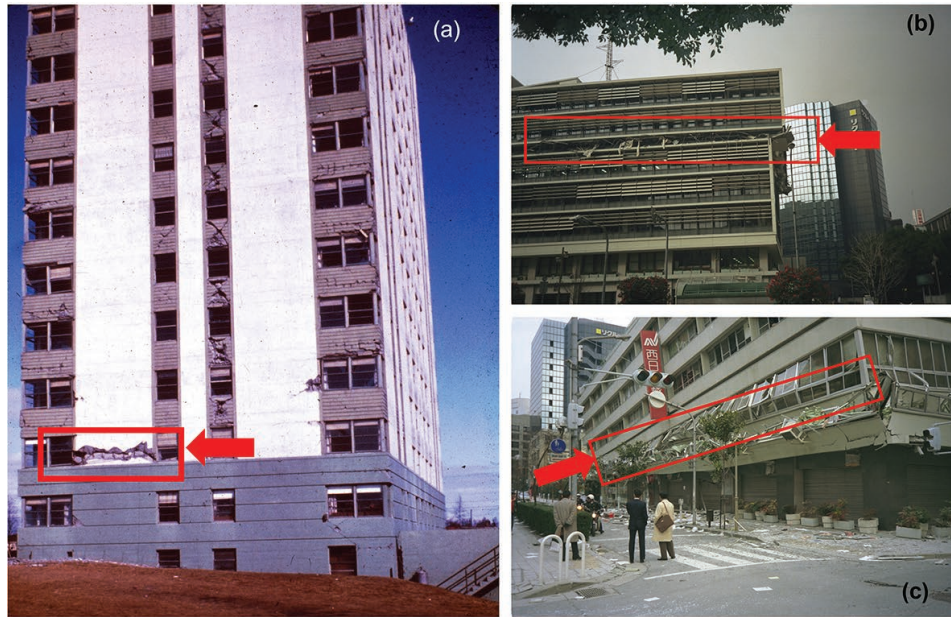


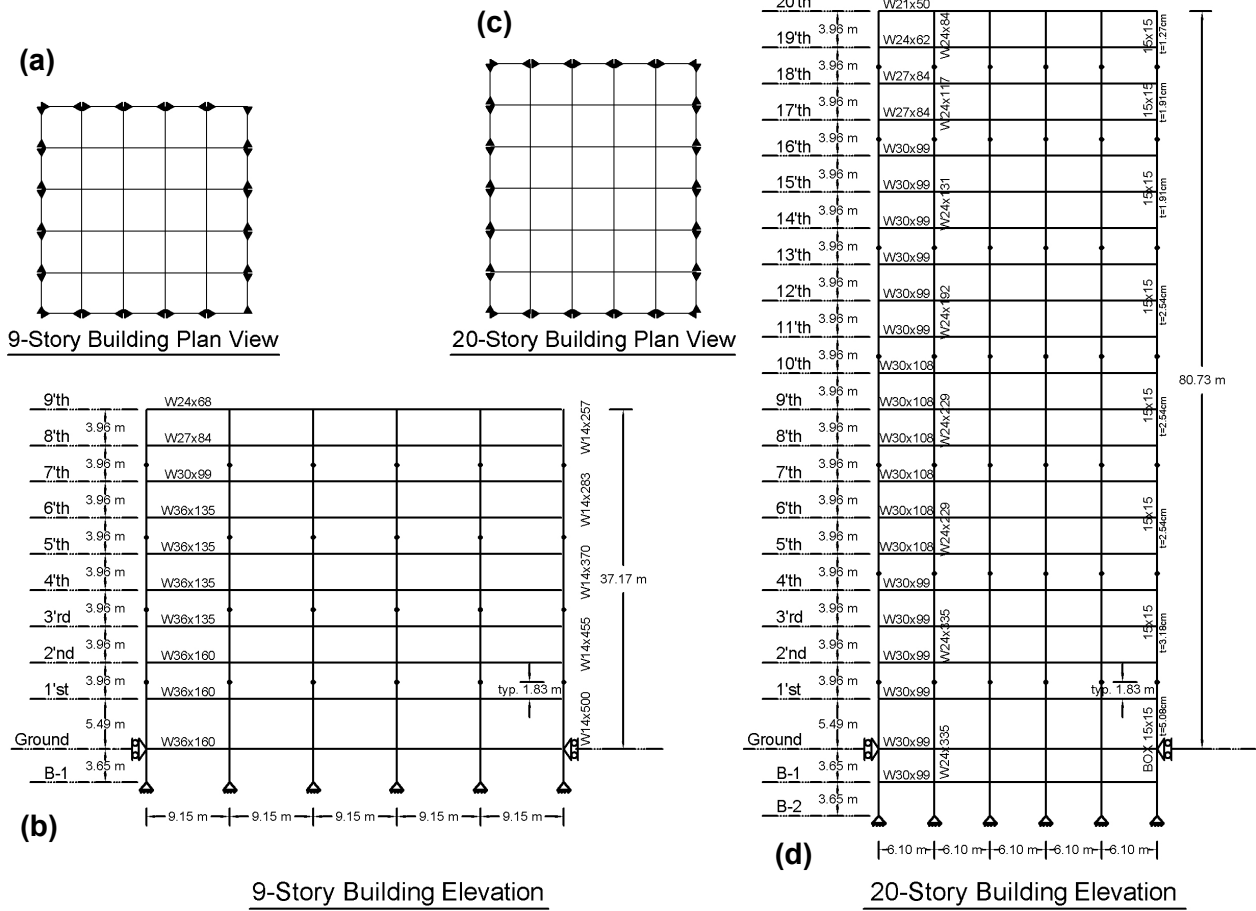
Figure 1: (a) Shear wall damage of a fourteen-story reinforced concrete apartment building in Anchorage, Alaska, severely during the 1964 earthquake. (Image courtesy of the U.S. Geological Survey, <https://library.usgs.gov/> [11]). (b and c) Weak-story failure in the upper stories of buildings following the 1995 Kobe, Japan earthquake. (Image courtesy of the National Oceanic and Atmospheric Administration [12]).

ceived revived attention. Makris [9], and Alavi and Krawinkler [10] studied the destructive potential of pulse-like ground motions recorded near the causative fault of earthquakes. Notably, several tall moment-resisting frames, designed according to existing seismic code provisions, exhibited weak-story failures, sometimes occurring several stories above the ground (see Fig. 1 (b) and (c)).

Over the past few decades, there has been an increasing effort to highlight the unique advantages with allowing main vertical structural elements (such as piers in bridges or shear wall in buildings) to uplift and rotate above their foundation to mobilize a lower failure mechanism by design. The advantages of this approach is that the failures associated with cyclic degradation are essentially avoided; while, permanent displacements remain small due to the inherent recentering tendency of the rocking mechanism [13–15].

Early studies, including Housner’s foundational work on rocking blocks [16] and investigations on rocking shear walls and steel frames, demonstrated reduced seismic demands compared to fixed-base systems [17, 18]. Despite these advancements, rocking systems remained underutilized until the PRESSS (PREcast Seismic Structural Systems) program in the 1990s, which reintroduced post-tensioned precast rocking walls and experimentally validated their performance, though discrepancies were noted in unloading phases and drift predictions [19–22]. Further studies explored rocking walls with tendon reinforcements and damping mechanisms, highlighting their ability to reduce drifts, improve resilience, and minimize residual displacements [23–25]. Rocking walls with pinned connections were proposed to mitigate near-fault effects in moment-resisting frames, demonstrating improved seismic performance [26]. Additionally, self-centering rocking systems were shown to effectively recenter structures after seismic events [27–30].

Subsequent research introduced refinements such as propped rocking walls, controlled rocking braced frames, and retrofit strategies incorporating dampers and inerters, improving seismic response and energy dissipation [31–37]. Aghagholizadeh and Makris further investigated the dynamics of



illustrated in Fig. 2, where beams and columns consist of wide-flange sections made of structural steel with yield strengths of 248 MPa (36 ksi) and 345 MPa (50 ksi), respectively.

The nine-story building, shown in Figs. 2 (a) and 2 (b), has a total height of 37.19 m and a square footprint measuring 45.73 m by 45.73 m. Lateral resistance is provided by two perimeter MRFs in each principal direction, while interior framing supports gravity loads without contributing to lateral stiffness. The structure includes a basement (3.65 m), a ground floor (5.49 m), and eight upper floors (each 3.96 m). Typical bay widths in both directions are 9.15 m. Columns are monolithic, with splice joints placed every two floors, 1.83 m above the centerline of the floor beams. For structural analysis, the lateral load-resisting system in one principal direction is modeled in OpenSees, using fiber elements for beams and columns to capture material nonlinearity. The twenty-story building, depicted in Figs. 2 (c) and 2 (d), has a total height of 80.73 m and a rectangular footprint of 36.58 m by 30.48 m. Similar to the nine-story structure, lateral resistance is provided by two perimeter moment-resisting frames (MRFs) in each principal direction, while interior framing supports only gravity loads. The building features two basement levels (each 3.65 m), a ground floor (5.49 m), and 19 upper floors (each 3.96 m). The typical bay width is 6.1 m, and columns are monolithic, with splice joints positioned every two floors, 1.83 m above the floor beam centerline.

The calculated natural vibration periods for the first three modes of the 9-story building are 2.26 s, 0.874 s, and 0.488 s and for the 20-story building are 3.83 s, 1.33 s, and 0.769 s. This aligns with values reported in prior studies [46–48].

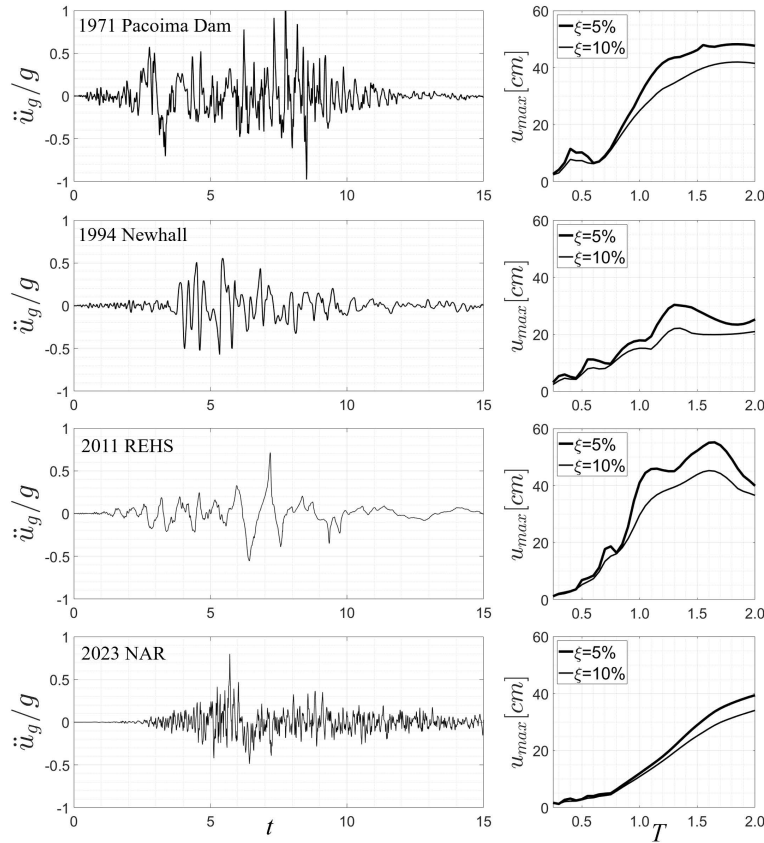


Figure 3: Recorded time histories and elastic response spectra for a damping ratio of $\xi = \frac{c}{2m\omega_o}$ with values of 5% and 10% for the four ground motions used in the response analysis presented in this study.

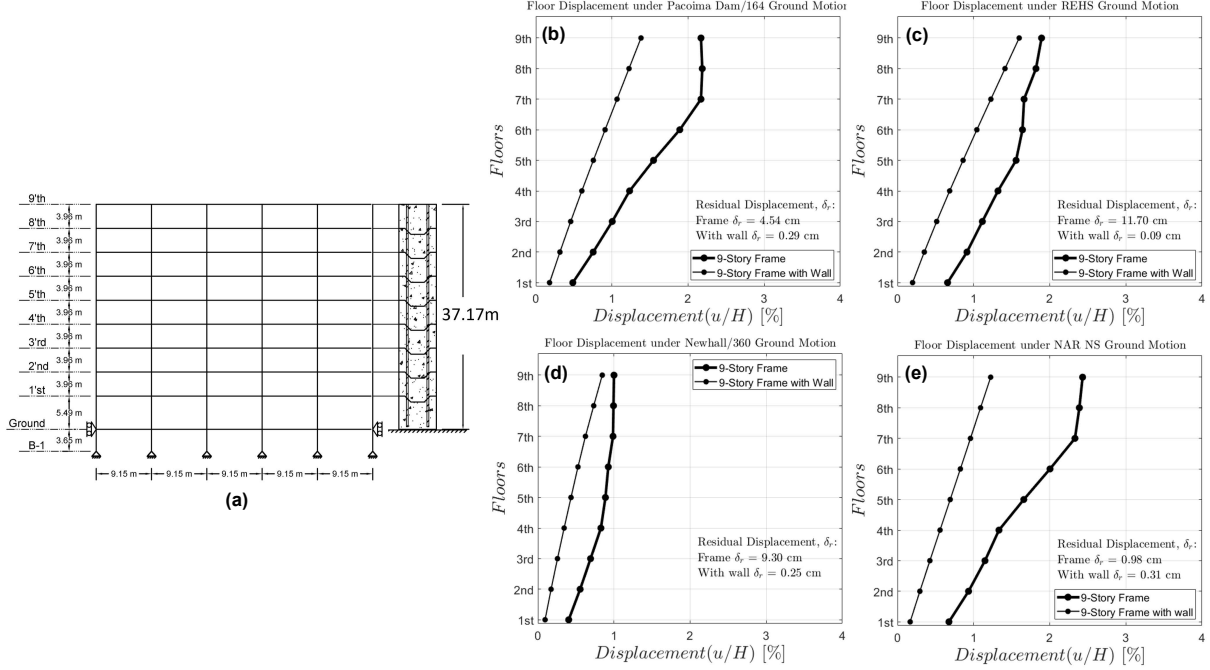


Figure 4: Maximum floor displacement (normalized by the total height of the building) for the 9-story moment-resisting frame (MRF) without a wall (heavy line) and the MRF with a stepping rocking wall (light line) under various earthquake excitations. The residual top-floor displacement, δ_r , at the end of each seismic excitation is indicated in each plot.

To model a rocking shear wall in OpenSees, a zero-length fiber section is defined at the base of the wall, representing the interface between the wall and the ground surface. The cross-section of this zero-length element is modeled using a fiber section with a nonlinear elastic-no-tension (ENT) material, which is available in OpenSees [50, 51]. For the 9-story MRF a shear wall with height of 37.17 m and width of 6.20 m is selected (width/height = $\tan \alpha \approx 1/6$). Similarly, for the 20-story building a shear wall with a height of 80.73 m and a width of 8.0 m is chosen, corresponding to a width-to-height ratio of $\tan \alpha \approx 1/8$.

3 TIME-HISTORY ANALYSIS

This section presents the dynamic time-history analysis of the 9-story and 20-story frames, both with and without a stepping rocking wall. The selected ground motions include the Pacoima Dam/164 record from the 1971 Imperial Valley earthquake, the Newhall/360 record from the 1994 Northridge earthquake, the REHS record from the 2011 Christchurch earthquake, and the North-South component of the NAR station record from the 2023 Kahramanmaraş, Türkiye earthquake (see Fig. 3). These ground motions were chosen due to their distinct coherent pulses, which vary in duration and influence the inelastic structural response differently at various pre-yielding periods.

Figs. 4 and 5 illustrate the normalized maximum floor displacement and interstory drift ratio for the 9-story moment-resisting frame (MRF) without a wall (heavy black line) and with a stepping rocking wall (light line). Fig. 4 demonstrates that the incorporation of a stepping rocking wall effectively reduces floor displacements across all cases. Additionally, the displacement profiles indicate that the rocking wall promotes a predominantly first-mode response, mitigating weak-story failure. The residual top-floor displacement (δ_r) at the end of each seismic event is also depicted, revealing significantly lower residual displacements for structures with a rocking wall, thereby enhancing their recentering

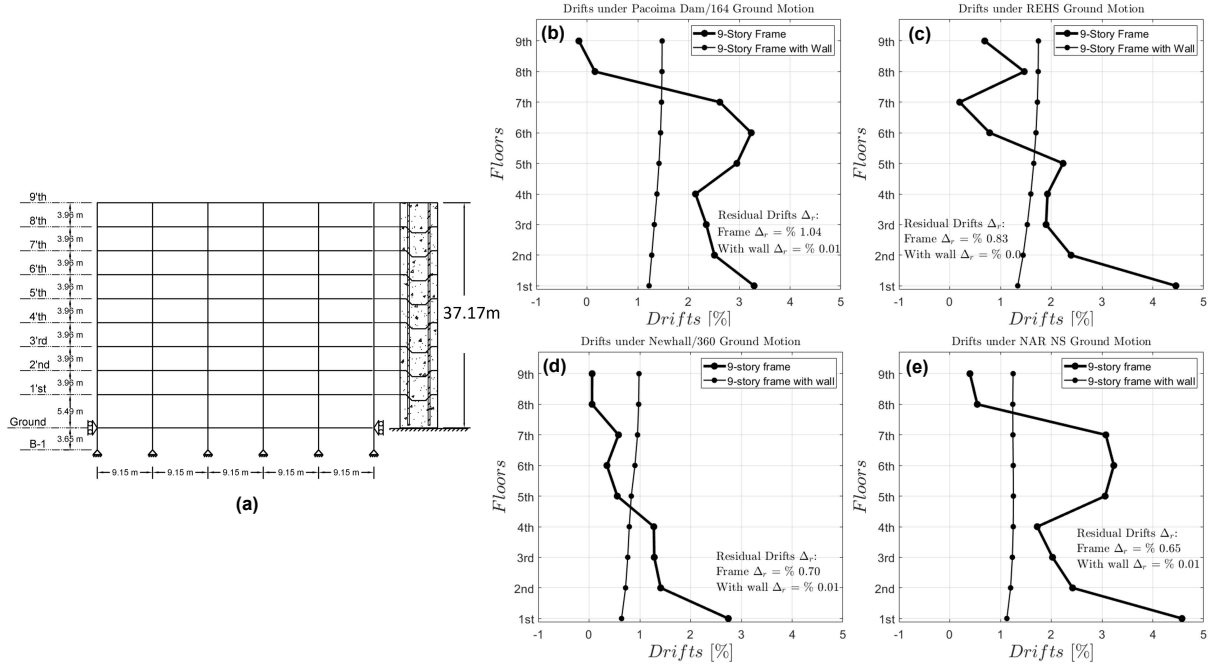


Figure 5: Interstory drifts of the 9-story moment-resisting frame (MRF) without a wall (heavy line) and the MRF with a stepping rocking wall (light line) under various earthquake excitations. The residual drift, Δ_r , at the end of each seismic excitation is indicated in each plot.

capability.

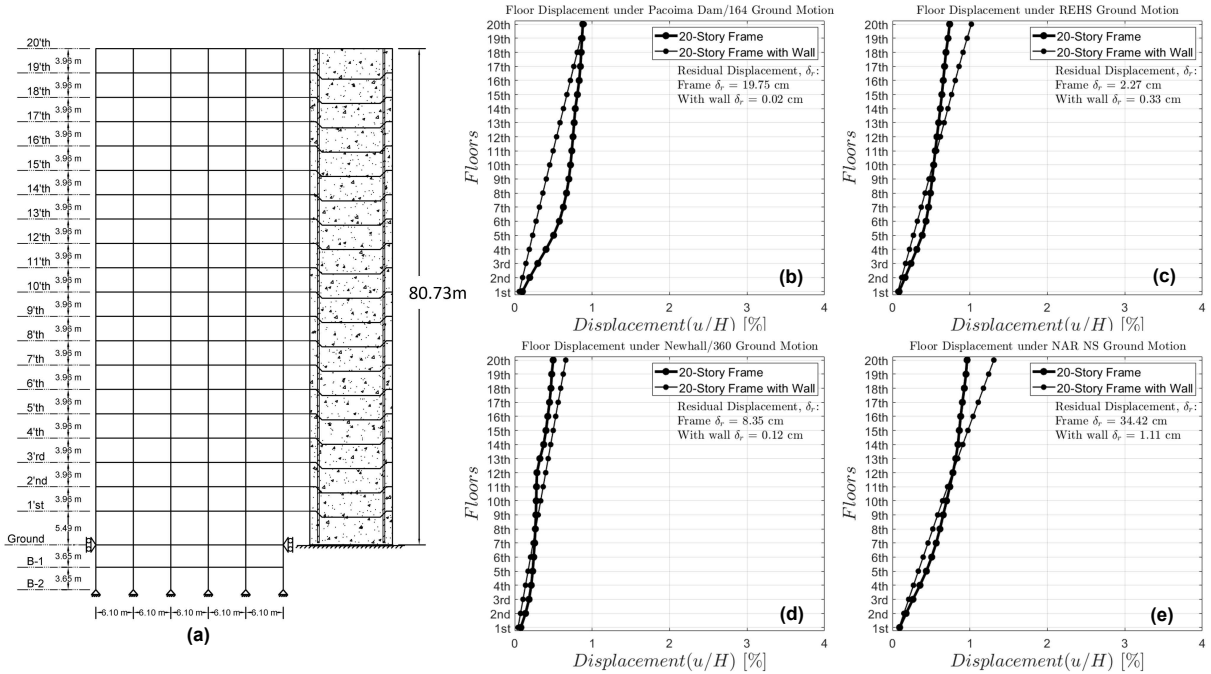


Figure 6: Maximum floor displacement (normalized by the total height of the building) for the 20-story moment-resisting frame (MRF) without a wall (heavy line) and the MRF with a stepping rocking wall (light line) under various earthquake excitations. The residual top-floor displacement, δ_r , at the end of each seismic excitation is indicated in each plot.

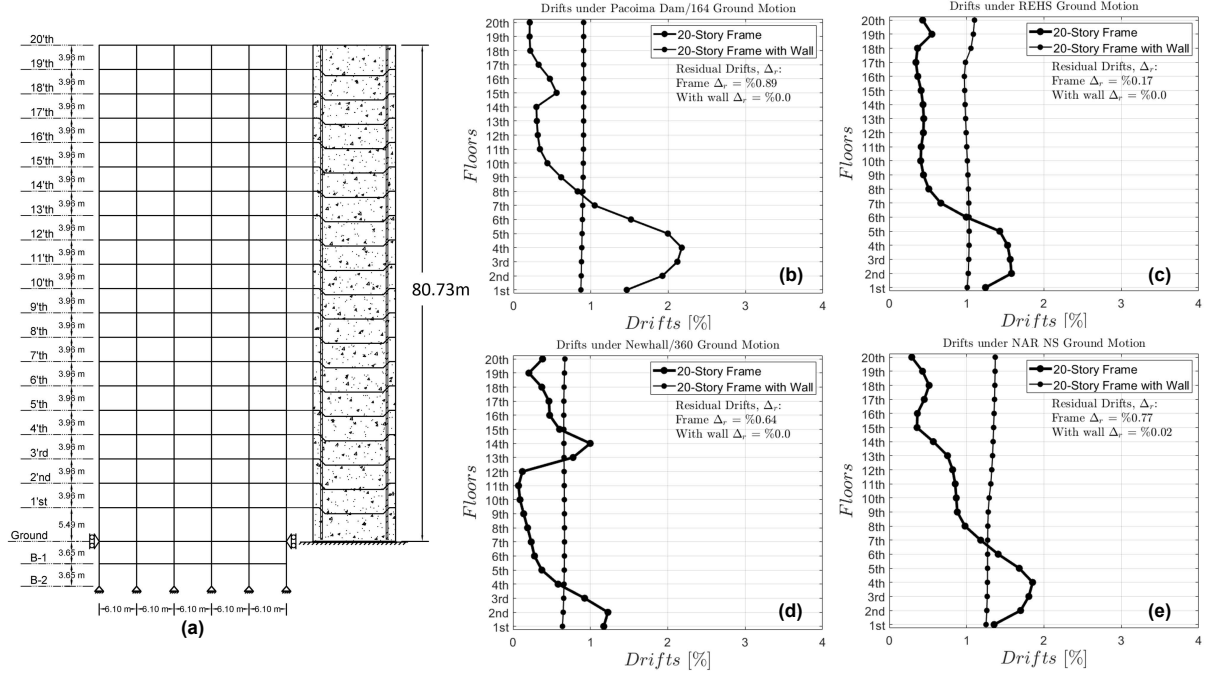


Figure 7: Interstory drifts of the 20-story moment-resisting frame (MRF) without a wall (heavy line) and the MRF with a stepping rocking wall (light line) under various earthquake excitations. The residual drift, Δ_r , at the end of each seismic excitation is indicated in each plot.

Fig. 5 compares the interstory drift ratios for different seismic excitations. The results indicate that the stepping rocking wall leads to a more uniform drift distribution, reducing the likelihood of weak-story mechanisms. Furthermore, the residual drift ratio (Δ_r) at the end of each event, shown in Fig. 5, is a critical parameter in post-earthquake reparability assessments based on FEMA P-58 [52]. According to FEMA guidelines, buildings with residual drift below 0.5% are highly repairable, while those exceeding 1.0% require extensive retrofitting or demolition. The fragility curve in Fig. 8 illustrates the correlation between reparability and residual drift, emphasizing the importance of minimizing permanent deformations. The results confirm that the 9-story MRF with a stepping rocking shear wall significantly reduces residual drift, improving seismic resilience and post-earthquake operability.

Similarly, Figs. 6 and 7 present the normalized maximum floor displacement and interstory drift

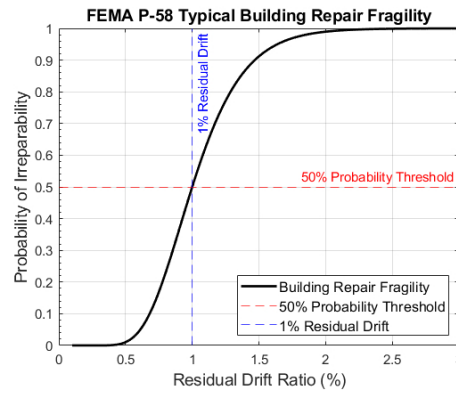


Figure 8: Typical building repair fragility curve as a function of residual drift ratio based of FEMA P-58.

Table 1: Damage States for Residual Story Drift Ratio (FEMA P-58 ([52]) Vol. 1, Table C-1)

Damage State	Residual Story Drift (%)	Description
DS0	$\Delta_{\text{res}} < 0.2\%$	Negligible residual drift, fully functional.
DS1	$0.2\% \leq \Delta_{\text{res}} < 0.5\%$	Minor residual drift, repairable with minimal intervention.
DS2	$0.5\% \leq \Delta_{\text{res}} < 1.0\%$	Moderate residual drift, significant repairs required.
DS3	$1.0\% \leq \Delta_{\text{res}} < 2.0\%$	Severe residual drift, questionable repair feasibility.
DS4	$\Delta_{\text{res}} \geq 2.0\%$	Extensive damage, building likely irreparable.

ratios for the 20-story MRF. Fig. 6 highlights significant higher-mode effects in the response of the 20-story frame without a rocking wall, leading to an irregular displacement profile that may cause localized damage and increased residual deformations. When coupled with a stepping rocking wall, the displacement pattern closely follows a first-mode shape, effectively suppressing higher-mode contributions. The results also indicate that the incorporation of the rocking wall minimizes the residual top-floor displacement (δ_r).

As shown in Fig. 7, the interstory drift ratios for the 20-story MRF are more evenly distributed when a stepping rocking wall is included, reducing the probability of weak-story failure. In contrast, the frame without a rocking wall exhibits more pronounced variations in drift ratios, increasing the likelihood of localized damage. Furthermore, the residual drift ratio (Δ_r) confirms that the 20-story MRF with a stepping rocking wall experiences negligible residual deformations, ensuring continued functionality post-earthquake. The substantial reduction in permanent deformations highlights the effectiveness of the rocking wall in enhancing seismic resilience and repairability.

The residual drift ratio (Δ_r) at the end of each seismic event is displayed on the drift profile figures. According to FEMA P-58 [52], the residual drift ratio is a key parameter in determining a building's repairability following an earthquake. Table 1 outlines the correlation between the residual drift ratio and the associated Damage State (DS) classifications, while Fig. 8 illustrates the relationship between building repairability and residual drift.

The FEMA P-58 building repair fragility curve, presented in Fig. 8, provides a probabilistic framework for assessing the likelihood of repairability as a function of residual drift. According to this model, buildings with a residual drift ratio below 0.5%, exhibit a high probability of being repairable with minimal interventions, whereas structures exceeding 1.0% residual drift face significant repair challenges, often requiring extensive retrofitting or demolition. The fragility curve further highlights that as residual drift increases, the probability of exceeding an irreparable damage threshold rises sharply, emphasizing the importance of designing systems that limit permanent deformations. The results from the 9-story and 20-story moment-resisting frames demonstrate that the inclusion of a stepping rocking shear wall significantly reduces the residual drift ratio, thereby lowering the damage state and enhancing the building's post-earthquake operability. By limiting residual drift to values within the repairable range defined by FEMA P-58, the rocking shear wall improves seismic resilience, ensuring that the structure remains serviceable after a major seismic event with minimal post-earthquake intervention.

4 CONCLUSIONS

The growing demand for seismically resilient infrastructure calls for innovative and efficient retrofit strategies, particularly for multi-story moment-resisting frames (MRFs), which are vulnerable to seismic-induced damage. This research investigates the potential of stepping rocking shear walls as an advanced retrofit solution aimed at improving the seismic performance of MRF buildings. Specifically,

the study focuses on addressing weak-story failure mechanisms, controlling drift demands, and minimizing residual deformations that can compromise post-earthquake functionality.

The primary conclusions drawn from this study are as follows:

- Integrating stepping rocking shear walls into MRF systems promotes a predominantly first-mode response, resulting in a more uniform distribution of interstory drifts across the building height that prevents weak-story failures.
- The presence of stepping rocking shear walls substantially decreases residual drift ratios (Δ_r), which are critical indicators of post-earthquake repairability as defined by FEMA P-58 guidelines. Reduced residual drifts increase the likelihood that the structure remains immediately operable following an earthquake, while also lowering potential repair and downtime costs.
- The self-centering behavior inherent to the stepping rocking wall configuration prevents permanent, inelastic deformations after seismic events. This capability eliminates the need for extensive post-event repairs or structural replacement, thereby enhancing the sustainability and re-usability of the building in the aftermath of earthquakes.

In summary, the outcomes of this research highlight the effectiveness and practicality of stepping rocking shear walls as a retrofit solution. Their ability to control drift, mitigate failure mechanisms, and enable self-centering behavior positions them as a promising strategy for enhancing the seismic resilience, repairability, and long-term sustainability of buildings.

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