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FINITE ELEMENT QUANTITATIVE ANALYSIS AND DEEP LEARNING QUALITATIVE ESTIMATION IN STRUCTURAL ENGINEERING

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Abstract. *In the past two decades, finite element method (FEM) has been widely used to study mechanics of solids, fluid–structure interactions, and building construction strategies. FEM has been rapidly grown all over the world due to development of computer technology. Computer has much more powerful computing capability than humans. However, structural engineering education not only focused on teaching engineers to use FEM as computation tool, but also concentrated on cultivating engineers' capability of experience-based qualitative analysis. In addition, artificial intelligence techniques have been rapidly developed in recent years. It is demonstrated that human experience-based capabilities might also be replaced by deep learning methods in various game-playing areas. Thus, this study aims at exploring what role artificial intelligence techniques will play in the futural structural analysis area. In this paper, several finite element analyses are carried out for three representative boundary value problems, such as tightly stretched wires under loading, soil seepage, and plane stress. Corresponding deep neural networks are trained using FEM simulation data to quickly and accurately predict results of relevant problems. It is indicated that to some extent artificial intelligence technique might replace human experience-based qualitative analysis as a surrogate of FEM.*

1 INTRODUCTION

As a numerical method for solving partial differential equations (PDEs), finite element method (FEM) is widely used in structural analysis, solid mechanics, seepage, fluid dynamics and other engineering problems. Rapid development of neural networks in recent years has provided researchers with new directions for solving these problems. For example, one of approaches is to directly solve the PDEs equations using neural networks based on physical constraints[1, 2]. Raissi et al.[3] proposed physics-informed neural networks (PINNs), and PDE equations are combined with loss function of neural network, in which collocation method was used to randomly select residual points in the domain for training. PINNs were applied to solve problems of fluid mechanics and material mechanics[4, 5]. Lu et al.[6] developed the Python library for PINNs, DeepXDE, which could be used as an analytical tool for solving computational engineering problems. Samaniego et al.[7] found that in variational format of

PDEs, corresponding functional physically represents energy. Training process of neural network was to minimize loss function. And the energy form could be used as loss function of neural network to solve relevant problems. Another approach was data-based method relying on results of FEM. Nie et al.[8] randomly generated a large number of two-dimensional finite element samples, in which each pixel represented a four-node element. Geometry, boundary and loading information were input in digital form, and stress could be predicted using convolutional neural networks. In addition, Jiang et al.[9] used conditional generative adversarial networks to study relatively complex two-dimensional mechanical problems. Nourbakhsh et al.[10] proposed a general surrogate model for characterizing stress in 3D trusses, in which parametric dome, wall, and slab truss structures were used as dataset. In the surrogate model, input values were 25 features of an individual truss member, including 22 nodal features and 3 member features, and output value was stress. It is demonstrated that artificial intelligence techniques have been rapidly developed in recent years.

However, studies focused on exploring what role artificial intelligence techniques will play in the futural structural analysis area are limited. In this paper, several finite element analyses are carried out for three representative boundary value problems, such as tightly stretched wires under loading, soil seepage, and plane stress. Different material parameters, geometric dimensions, and loading are used as features to create the surrogate-based model. Corresponding deep neural networks are proposed and trained using FEM simulation data to quickly and accurately predict results of relevant problems. Accuracy of the proposed neural is demonstrated. It is indicated that to some extent artificial intelligence technique might replace human experience-based qualitative analysis as a surrogate of FEM.

2 DEEP NEURAL NETWORKS

One of the simplest deep neural networks is the multilayer perceptron, also known as the forward neural network (FNN). The multilayer perceptron consists of input layers, hidden layers, and output layers. Figure 1 presents an example of a network with a single hidden layer, where $\boldsymbol{x}$ denotes input data, i the number of input units, $\boldsymbol{h}$ hidden layer, j the number of hidden layer units, $\boldsymbol{y}$ output data, and k the number of output units.

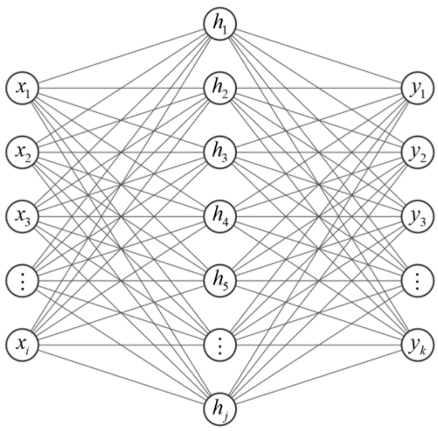


Figure 1: A simple neural network.

Formula of the forward propagation of the neural network is given as

$$
\begin{aligned}
h_j &= \sigma\left(\omega_{ji} x_i + b_j\right) \\
y_k &= \omega_{kj} h_j + b_k
\end{aligned}
\tag{1}
$$

where ω_{ji}, ω_{kj} denote weights, b_j, b_k the biases, and σ the activation function.

Most of activation functions are nonlinear. Common activation functions have *relu* function, *sigmoid* function, *tanh* function, and *softplus* function, as shown in Figure 2.

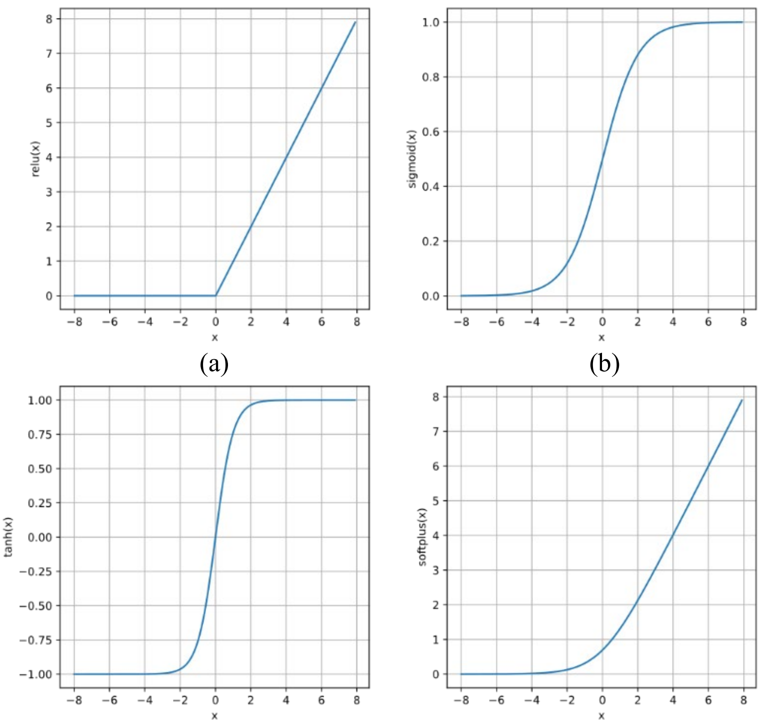
The network uses a loss function to measure error between predicted values y and known values $\bar{y}$. Commonly used loss function, the mean squared error (MSE), is given as

$$
MSE = \frac{1}{n}\sum_{k=1}^{n}\left(y_k - \bar{y}_k\right)^2 \tag{2}
$$

Symmetric mean absolute percentage error (SMAPE), the other loss function used to evaluate overall quality of neural network predictions, is defined as

$$
SMAPE = \frac{1}{n}\sum_{k=1}^{n}\left(\frac{\left|y_k - \bar{y}_k\right|}{\left|y_k\right| + \left|\bar{y}_k\right|}\right) \times 100\% \tag{3}
$$

Gradients of parameters are calculated by neural network through backpropagation. And value of the loss function is reduced through optimization algorithm until predicted values are close to known values.



(c) (d)

Figure 2: Common activation functions: (a) *relu* function, (b) *sigmoid* function, (c) *tanh* function, and (d) *softplus* function.

Convolutional neural networks (CNNs) can be used to process multidimensional matrices. They have been widely used in image processing[11]. Usually, an image consists of two-dimensional pixels, in which each pixel might contain multiple channels. CNNs are mainly composed of convolutional layers and pooling layers. The convolutional layer integrates information of all channels by convolution kernels. If multiple convolution kernels are used, multiple channels might be regarded as output. Pooling layer aggregates input values to reduce sensitivity of convolutional layer into location of features[12].

3 NUMERICAL EXAMPLES

3.1 Tightly stretched wire under loading

A tightly stretched wire under loading is held at both ends, as shown in Figure 3. Total length of the wire is L, internal tension is T, and deflection of the tightly stretched wire could be determined based on external loading. The tightly stretched wire is equally divided into 100 elements with 101 nodes. For the sake of simplicity, it is assumed that loadings act only on nodal points. Nodal loadings are randomly generated in the range of 5 N to 50 N. Three different loadings are considered as follows.

Case A. There is totally only one loading subjected on any random nodal point;
Case B. There are totally 101 loadings subjected on all nodal points;
Case C. Random generation of any number of loadings from 1 to 101 subjected on any random nodal point.

To clearly visualize nodal displacements during simulation, all nodal displacements are enlarged in certain degree, 100 times in Cases B-C, and 1000 times in Case A.

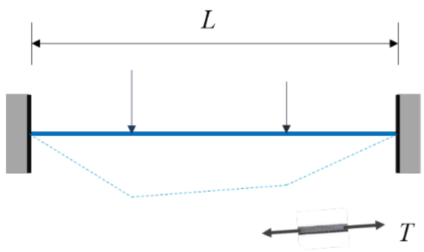


Figure 3: A tightly stretched wire under loading.

While loading is considered as the only feature, length of wire L is assumed as 10 m, and tension of wire T 6000 N. Loadings are randomly generated in different samples. For each case, there are 10000 samples, in which 8000 samples are used as the training set, and 2000 the testing set.

A single-layer perceptron is used without hidden layer. Input vector of the network consists of values of nodal loading, or 0 if there is no load on the nodal point. MSE is used as loss function. Adam algorithm is used as optimization algorithm, where batch size is 10.

After the neural network is trained, solutions of three cases all have good accuracy. MSE of all tests are less than 0.01. Here stiffness matrices used to calculate nodal displacements are the same in different examples. While biases are neglected, weight matrices in the neural network represents flexibility matrices in FEM physically.

While both loading and tension are considered as features, tension in the wire T is randomly selected in the range from 6000 N to 10000 N. It is indicated that stiffness is different for each sample. Total length of the wire is given as 10 m. 10000 samples are randomly generated for every case.

Two single-layer perceptron are built, where input data of NN1 consists of 101 loadings, and output data 101 node displacements; input data of NN2 consists of T and 101 loadings, and output data 101 node displacements. Other assumptions of the neural network are the same as above.

Figure 5 shows MSE of the test set after the neural network is trained. It is indicated that accuracy of NN2 is higher than that of NN1 especially in case B, where MSE decreases about 89%. In the other two cases, MSE decreases slightly. When tension is taken as a component of the input vector, it is difficult to find weights and biases using single-layer networks for cases of linear relationship between tension and displacement.

To improve accuracy of neural network predictions, a new neural network architecture is proposed. It takes the information of every node as input data, including coordinates, boundary conditions, nodal loadings, and tension. If there are n nodes, the vector is x coordinate of all nodes of $n\times 1$ in size. In the vector of boundary conditions, 1 indicates constrained boundaries and 0 free ones. The tension vector is obtained by multiplying its value by an all-ones matrix with the shape of $n\times 1$. Thus, the input data of the network contains 4 channels, and its shape is $4\times n\times 1$. Batch normalization used after the input layer is helpful to achieve convergence of results. To integrate various information of nodes, a convolutional layer is set up. Size of the convolution kernel is 3×3, in which padding is 1. There are totally 3 convolution kernels. The next layer is the convolution layer of $3\times n\times 1$ in size. The third one is the fully connected layer. The last is output layer consisting of n nodal displacements. Figure 4 shows the convolutional neural network scheme for the tightly stretched wire problem, called NN3, where the number of fully connected layer units is 202.

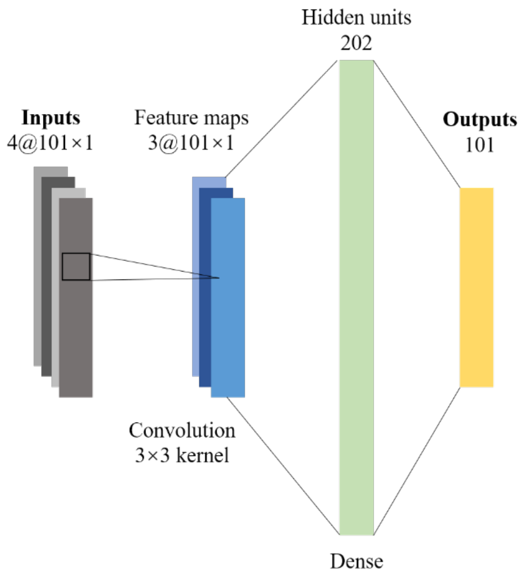


Figure 4: The convolutional neural network scheme for the tightly stretched wire problem.

For the sake of comparison, the same dataset is used for NN3. Figure 5 shows errors of three neural networks of the numerical example. For all of three cases, NN3 has the smallest error superior to the other two networks, because the convolution layer fully connected with NN3 is able to consider nonlinear relationship between input and output. It is indicated that for small number of loadings or the sparse loading vector, it is disadvantageous for the neural network to find relationship among loading, tension and displacement. For case C, the number of loadings is random, and the MSE of NN3 is 0.61. Thus, it is indicated that the neural network has a certain generality.

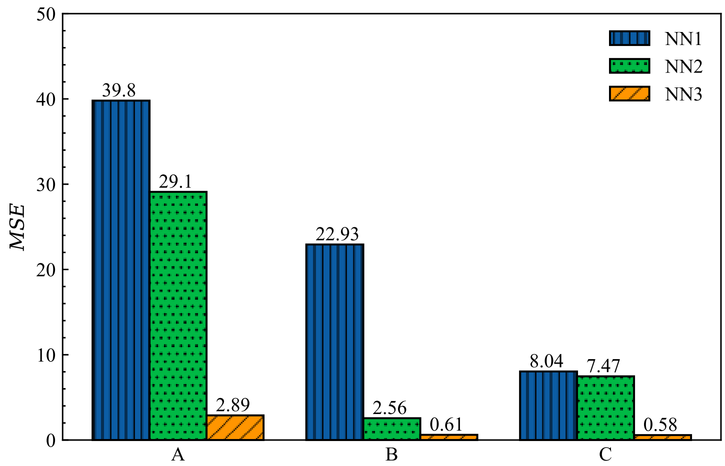


Figure 5: For three cases (A, B, C), MSE of the test set for different neural networks.

While length, loading and tension are considered as features, length of the wire L is randomly generated in the range from 10 m to 100 m. Likewise 100 elements are divided equally. Loadings of case C are applied. Tension in the wire T is randomly generated in the range of 6000 N to 10000 N. 10000 simulations are carried out using FEM.

NN3 is used for training, and the batch size is set to 128. After 250 epochs, the MSE is basically stable, about 22.8 and SMAPE is about 6.9%. Figure 6 shows the top three maximum errors on the test set, and the results of the neural network are very close to the results of FEM generally. In addition, it is also found that although the overall error of some samples is small, the shape has a large deviation, as shown in Figure 7. Deviation might appear in cases in which loadings are relatively sparse. And the corresponding displacement curve might be vibrated.

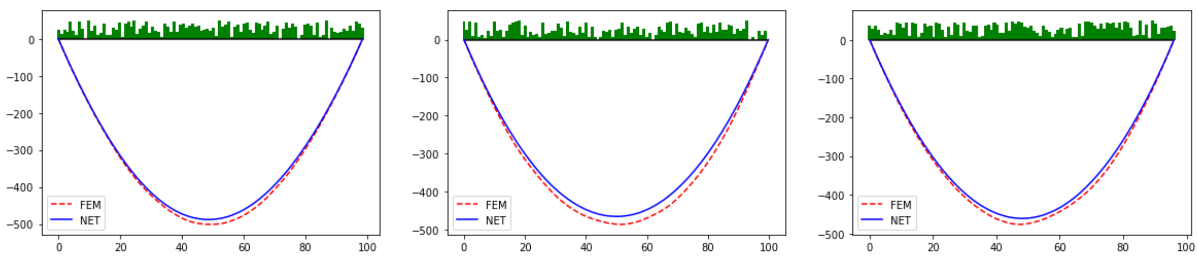


Figure 6: The three samples with the maximum error in the test set (The green bar chart above the X-axis represents the magnitude and location of loads).

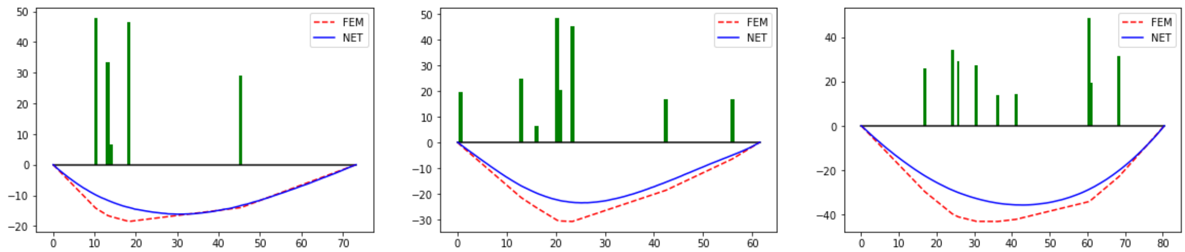


Figure 7: Notable samples in the test set (The green bar chart above the X-axis represents the magnitude and location of loads).

3.2 Flow through porous media

As shown in Figure 8, a concrete gravity dam has a sheet pile in the upstream face that can help to reduce the uplift pressure. Area of the domain is 60 m × 60 m, and boundary condition of the bottom side is assumed as the impervious rock. The soil area is divided into 250 three-node elements with a total of 154 nodes in FEM analysis, where D denotes depth of the sheet pile, B the width, R permeability of soil is assumed as, and Φ the upstream head. Table 1 presents minimum and maximum values of FEM parameters given in the flow through porous media problem.

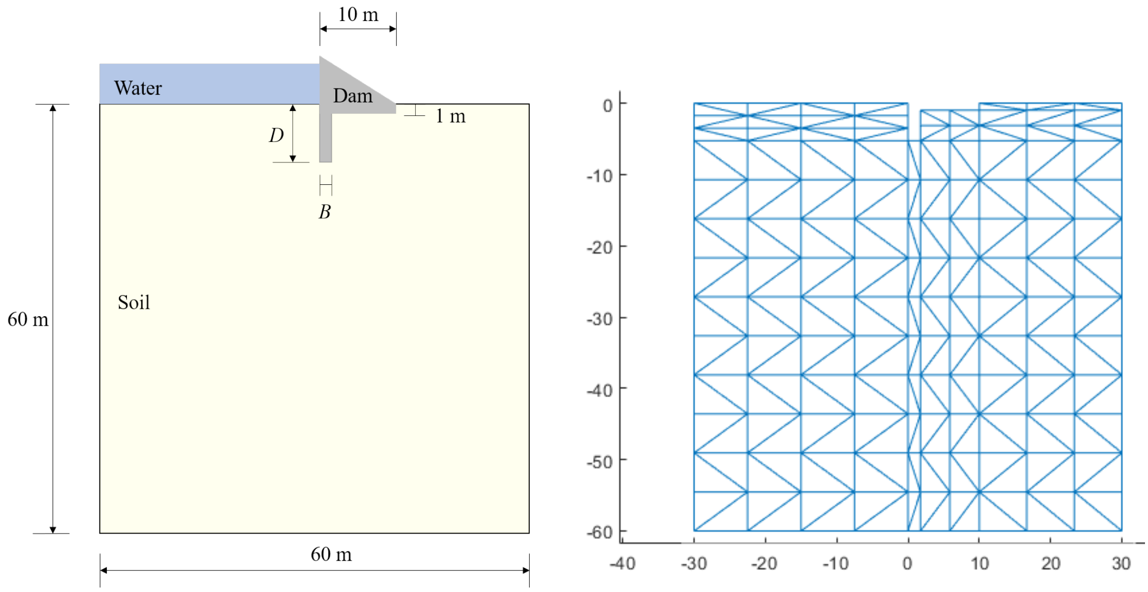


Figure 8: A concrete gravity dam and mesh in FEM.

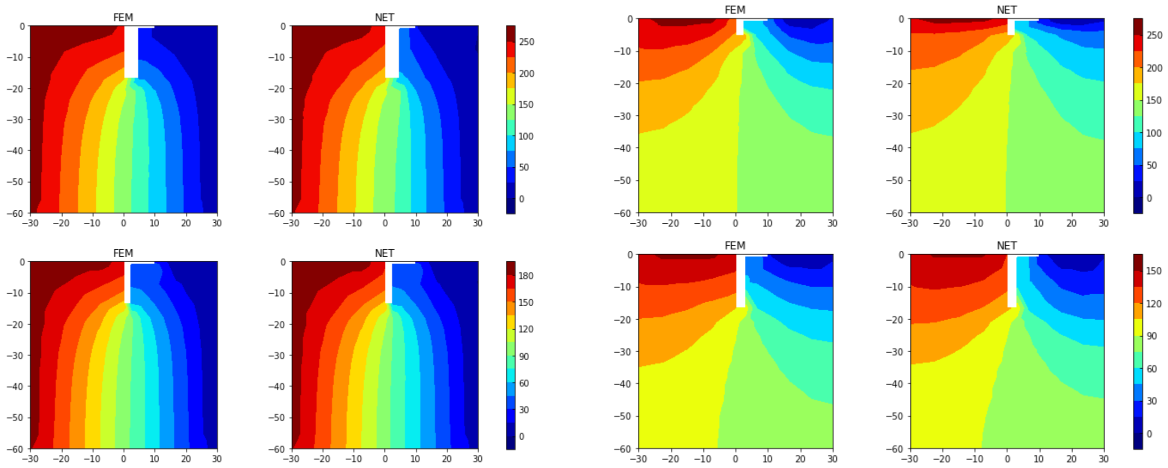
Table 1: Values of FEM parameters given in the flow through porous media problem

	D (m)	B (m)	R (m/day)	Φ (m)
Minimum	5	1	0	5
Maximum	20	5	0.5	30

For the flow through porous media problem, two boundary conditions are given. One has impervious boundaries in the upstream and downstream, and the other one has constant heads in the upstream. In each boundary condition, 10000 samples are generated, in which 8000 samples are selected as the training set and 2000 samples the test set.

Neural network is shown in Figure 4. It is noted that the input layer for this problem has 5 channels, such as x coordinates of nodes, y coordinates of nodes, boundary conditions, hydrostatic heads on boundary, and soil permeability. For the vector of the boundary condition, 1 denotes constrained boundary nodes, and 0 free nodal points. Size of the input data is 5×154×1. There are 5 convolution kernels and 308 units of the fully connected layer. Length of the output layer is 154.

After the neural network is trained, MSE of the test set is around 7.9, and SMAPE about 7.1%. Figure 9 shows four samples randomly chosen in the test set. It can be seen that results obtained from the neural network are very similar to solutions obtained from FEM.



(a) (b)

Figure 9: Four samples randomly chosen in the test set of (a) constant heads, and (b) impervious boundaries.

3.3 Plane cantilever beam

Figure 10 shows a two-dimensional cantilever beam subjected to a uniform downward loading at the free end, where H represents its height, L its length, and q uniformly distributed loading at the end. The elastic modulus of the material is assumed as E and the Poisson's ratio v. To get stress component σ_{xx}, domain of the beam is discretized into 64 three-node elements with 45 nodes in FEM model. Minimum and maximum values of parameters are given in Table 2. 10000 samples of are generated for artificial neural network analyses.

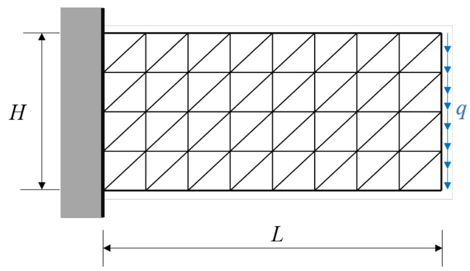


Figure 10: A two-dimensional cantilever beam subjected to a uniform downward loading at the free end.

The neural network architecture is shown in Figure 4. The input layer for the 2D elastic problem has 8 channels, such as x coordinates of nodes, y coordinates of nodes, boundary conditions at x direction, boundary conditions at y direction, loadings at x direction, loadings at y direction, elastic modulus, and Poisson's ratio. There are 8 convolution kernels and 90 units at the fully connected layer. Stress component σ_{xx} is taken as output.

Table 2: Minimum and maximum values of FEM parameters for the cantilever beam.

	H (m)	L (m)	q (N/m)	E (Pa)	ν (-)
Minimum	10	$2H$	0	1×10^{6}	0
Maximum	50	$3H$	100	1×10^{7}	0.5

After the neural network is trained, MSE of the test set is about 53.1, and SMAPE approximately 4.3%. Figure 11 illustrates four samples randomly chosen from the test set. Results of the neural network are in great agreement with ones from FEM. It is indicated that the proposed neural network can predicts contours of stress field very well. For some sample with large MSE, there is a certain deviation of stress, but the general trend is almost the same.

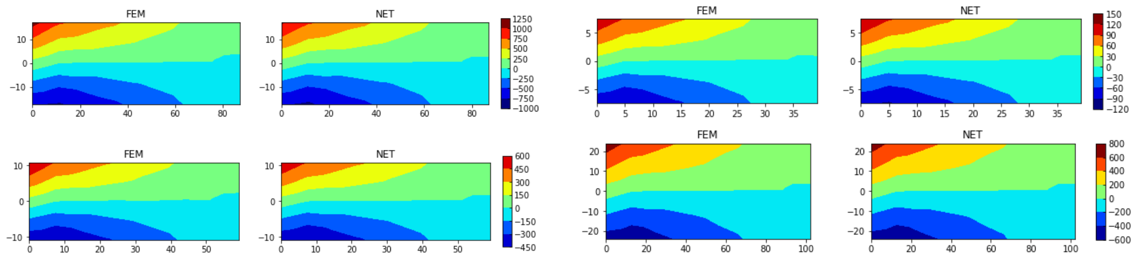


Figure 11: A comparison between solutions from FEM and ones of neural network for the cantilever problem

4 CONCLUSIONS

In this paper, the deep neural network is proposed to qualitatively analyze three representative boundary value problems. The proposed neural networks consist of convolutional and fully connected layers. Information of each node is regarded as input data, such as nodal coordinates, boundary conditions, loading, and material parameters. For the tightly stretched wire problem, while wire length, loading, and tension of the wire are randomly generated, SMAPE of the neural network is about 6.9% for the test set. For the flow through porous media problem, while pile size, upstream hydrostatic head, and soil permeability are randomly generated, SMAPE of the test set is about 7.1%. For the plane cantilever beam problem, height, length, loading, elastic modulus, and Poisson ratio are randomly generated, SMAPE of the test set is about 4.3%. It is demonstrated that neural networks can make good qualitative analysis for general boundary value problems. It is indicated that to some extent artificial intelligence technique might replace human experience-based qualitative analysis as a surrogate of FEM.

In this paper, it has been proved that results obtained from the surrogate is basically consistent with the finite element solution, but it is just the first step. There are still several interesting questions remaining unknown. For instance, under what scenario could the surrogate model be implemented? How much computational time is saved? What is scope of the artificial neural network? Can it become a surrogate of overall structural optimization design? In addition, some mathematical investigation should be carried out. For example, what is effect of selection of basis functions on accuracy of structural analysis? Why can all problems be solved with high accuracy by adding hidden layers? The above questions are expected to be further discussed in futural researches.

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