## ADJUSTMENT OF STRESSES IN THE TOP CHORD OF THE DOME-LIKE HYBRID ROOF STRUCTURE

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**Summary.** The roof, considered in the research, consists of a framework and a flexible shell made of polymer membrane. The framework includes rigid beams, steel cables and hinged struts. The membrane is laid on the beams which are arranged in the radial direction. It is pre-tensioned by backstay cables. Non-uniform loads, e.g. snow accumulations on a half of the span, result in high peak stresses in the beams. Passive and active strategies are considered for the stress mitigation. Effect of supplementary ties and spatial structure of the ribs forming the framework is investigated. Having been provided simultaneously, the spatial ribs and the ties result in substantial favorable effect. The drawback is in complication of the construction. Active adjustment strategy, keeping the structure of the roof simple, implies tensioning of the backstay cables for reducing the beam stresses. Separate and simultaneous adjustment approaches are analyzed. Finding the appropriate combination of tensioning values for the cables has been converted into the optimization problem.

## **1 INTRODUCTION**

Hybrid roofs consist of dissimilar structural elements assembled in order to optimally fit operational requirements. They include rigid beams, pin-joint struts, steel cables and membrane cladding.

High-strength cables and polymer membrane materials are effective for buildings with large column spacing. They allow creating light-weight, translucent and architecturally impressive roof structures for public buildings<sup>1</sup>. Membrane constructions have great potential for industrial applications as well<sup>2</sup>. They are an appropriate solution not only for temporary but also for permanent buildings<sup>3</sup>. The cables and the membrane are delivered to the site in compact coils and packages, thus reducing transportation cost<sup>4</sup>.

The beams effectively mitigate structural deformations. They sustain non-uniform and point external loads, thus providing new opportunities for the hybrid constructions<sup>5,6</sup>. Combination with high-strength cables induces bending moments in the beams, which are opposite to the ones brought about by external loads. The resultant stresses become smaller allowing the beams to cover long-span buildings.

Static analysis of hybrid constructions should include interaction between the cables, the beams and the membrane. Matrix approach is proposed for the analysis of the hybrid space structures<sup>7</sup>. The technique to determine the flexibility distribution in the structure is given. The initial beam stresses are obtained by means of the local analysis method.

The limit state approach is considered for the analysis of cable-tensioned steel structures. A plastic-hinge nonlinear formulation is used for simulating pin-joint truss systems. Elastoplastic analysis is applied for rigid frame constructions. The effectiveness of pre-tensioning the cables is investigated and analyzed<sup>8</sup>.

The mixed algorithm is elaborated for geometric nonlinear simulation of cable-truss systems<sup>9</sup>. Stiffness matrices are obtained using lagrangian formulations. The solution is achieved by means of iterative secant method.

Holistic finite element analysis of complex hybrid structures with polymer membrane included can be performed using specialized software, e.g. EASY.2020<sup>10</sup>. The analysis is implemented by iterative minimizing nodal unbalanced forces.

Hybrid structures are far superior to the ordinary building constructions. They fully comply with the concept of sustainable development. On the other hand, non-uniform impacts adversely affect on the hybrid structures. For example, snow accumulated on a half of the span results in significant increasing of bending moments in the beams of the top chord<sup>11</sup>.

To reduce material consumption under various external influences so-called 'adaptable' structures have emerged. Active and passive strategies of adaptation are considered and analyzed<sup>12,13</sup>.

The passive adaptation is implementing the structural compliance into the structural behavior. It helps avoiding or mitigating external impacts by adapting to them<sup>13</sup>. Triangulated top chord composed of flexible cables is proposed for stability enhancement of tensegrity-like cable domes<sup>14</sup>. Slackening of the cables under various external loads is prevented by force redistribution throughout the entire surface. Spatial ribs and supplementary flexible ties mitigate bending moments in the beams of the hybrid roof structure<sup>11</sup>. Reduction of bending moment in a particular plane, e.g. the vertical one, may, however, be accompanied with increasing of the remaining forces and moments, resulting in the overall growth of stresses in the structural element. Thus, the problem of passive stress adjustment in the hybrid roof structures needs further investigation. In addition, spatial ribs and multitude of supplementary ties result in complicating of the structural framework.

The active adaptation strategy is implemented by means of targeted impacts on the construction in order to improve its behavior under external adverse effects. The active adaptation implies real-time stress adjustment by actuators using load cells indications<sup>12</sup>.

The active adaptation concept is used for enhancement of the efficiency of solar energy harvesting. Adaptive facade, which consists of multitude of solar panels tracking the Sun is developed<sup>15</sup>. The resultant energy savings reached 25%. Self-shading problem is mitigated using dynamic facade modules.

Active dampers are used for suppression of vibrations in cable bridges<sup>16</sup>. The tendons which consist of a force sensor and an actuator are used. The approach has proved its robustness for actively controlling nonlinear systems, such as a cable structure and a cable-stayed bridge.

The problems of active structural control are considered<sup>17</sup>. In case of sudden external impacts (e.g. earthquake, displacements of supports or soil deformations), the parameters of the construction are dynamically adjusted for mitigating the outcomes. The parameters include forces in primary structural elements. The adjustment means are jacks and actuators embedded in the construction. The effect is in leveling the stresses throughout the construction thus diminishing the peak values.

#### **2 THE HYBRID ROOF STRUCTURE**

The hybrid roof structure consists of a framework and a flexible shell<sup>11,18</sup> (Fig. 1). The shell is made of polymer membrane or architectural fabric<sup>1,3,19</sup>. The fabric consists of woven threads and a polymer coating. The shell protects the building from rain and solar radiation. In addition, it contributes to stress leveling in the framework.

The membrane is pre-tensioned by backstay cables. Pre-tensioning is needed for avoiding wrinkles and preventing formation of slackened areas<sup>19,20</sup>.



Figure 1: The hybrid roof structure. a - the framework of the roof; b – the flexible shell; 1 – the rib; 2 – the beam of the top chord; 3 – diagonal bearer cable; 4 – hoop cable; 5 – central cable; 6 - flexible polymer membrane; 7 - backstay cable; 8 – hinged strut; 9 – central ring; 10 - edge supporting beam; 11 – fixed support

The framework includes ribs which are arranged in the radial direction. The ribs consist of rigid beams, flexible cables and hinged struts. They are linked together in the center of the roof and attached to the edge fixed supports. The ribs are united in the circular direction by the hoop cable and by the central ring. The ring is a chain-like structure made of pin-joint elements. The hoop equilibrates forces induced by diagonal pre-tensioned cables.

#### **3 BEHAVIOR OF THE FRAMEWORK OF THE ROOF UNDER LOAD**

The following loads influencing the roof are taken into account: structural own weight  $(L_{d,0})$  adopted 0.2 kPa throughout the whole membrane surface, uniformly distributed snow load equal to 1.8 kPa  $(L_{d,1})$ , snow accumulated on one side of the surface  $(L_{d,2})$ , figure 2a) and wind impact  $(L_{d,3})$ , figure 2b).

Having grouped the individual loads, the following load cases are considered and analyzed:  $L_{c,1} = L_{d,0} + L_{d,1}$ ,  $L_{c,2} = L_{d,0} + L_{d,3}$ ,  $L_{c,3} = L_{d,0} + 0.9 \cdot (L_{d,1} + L_{d,3})$ ,  $L_{c,4} = L_{d,0} + L_{d,2}$ , and  $L_{c,5} = L_{d,0} + 0.9 \cdot (L_{d,2} + L_{d,3})$ . Specialized software package EASY.2020<sup>10</sup> is used for non-linear structural simulation.



Figure 2: Non-uniform loads: a – snow load,  $L_{d,2}$ ; b - wind load,  $L_{d,3}$ 

Non-uniformly distributed loads bring about excessive bending of the top chord. Bending moments  $M_v$  in the vertical plane of the beams situated in fully and partially loaded zones are opposite in signs (figure 3a). In addition, large moments  $M_w$  in the horizontal plane are induced in the ribs situated between the load zones (figure 3b).



Figure 3: Diagrams of bending moments in the beams of the top chord under the load case  $L_{c,5}$  (results by EASY.2020 software<sup>10</sup>). a – the moments in the vertical plane,  $M_{\nu}$ ; b - the moments in the horizontal plane,  $M_{\nu}$ 

The normal stress  $\sigma$  in a particular node of the beam is obtained in [MPa] as follows:

$$\sigma = \left(\frac{|N|}{A} + \frac{|M_{v}|}{W_{v}} + \frac{|M_{w}|}{W_{w}}\right) \cdot 10^{-3}, \qquad (1)$$

where the axial force N is given in [kN], while the moments are in [kN · m]; A = 0.00268 m<sup>2</sup> is the cross section area, while  $W_v = 0.000184$  m<sup>3</sup> and  $W_w = 0.0000231$  m<sup>3</sup> are the elastic section moduli of the I-beam, adopted for the top chord of the roof's framework.

The maximum normal stresses all over the beams of the top chord (figure 4) are obtained as follows:

$$\sigma_m = \max(\sigma_{i,i}), \tag{2}$$

where  $\sigma_{i,j}$  is the normal stress (1) in node  $j \in [0...8]$  of the beam  $B_i$ ,  $i \in [1...5]$  (figure 5).



Figure 4: Graph of maximum normal stresses all over the beams of the top chord of the hybrid roof



Figure 5: The top chord of the roof. Designation of the beams ( $B_1...B_5$ ), nodes (0...8) and the backstay cables ( $C_1...C_4$ )

The figure 4 shows steady growth of the beam stresses with the increasing of the load nonuniformity. Thus, the load case  $L_{c,5}$ , inducing the highest stress and comprising snow accumulations  $L_{d,2}$  and the wind impact  $L_{d,3}$ , is adopted for evaluation of effectiveness of stress adjustment techniques in the present study.

### **4 STRUCTURAL IMPROVEMENT OF THE FRAMEWORK**

In order to diminish the bending moments in the top chord of the roof, installation of supplementary ties and transition from plane to spatial ribs are proposed<sup>11</sup> (figure 6).



Figure 6: General structure of the improved embodiments of the framework of the roof: a – the framework of plane ribs; b – the spatial framework;  $t_1$  and  $t_2$  - supplementary flexible ties

The ties and the spatial ribs reduce bending moments in the vertical plane  $M_v$ . They, however, do not result in the structural enhancement by themselves. The overall stresses in the beams of the top chord become higher due to the increasing of the bending moments  $M_w$  in the horizontal plane (figure 7). The figure shows, that only the combination of the ties  $t_1$  and  $t_2$  with the spatial structure of the ribs gives favorable effect providing reduction of the peak beam stress from 508 MPa to 219 MPa.



Figure 7: Analysis of the improved embodiments of the framework of the roof: a - graphs of maximum absolute bending moments in the top chord; b - graphs of maximum stresses in the top chord

Structural improvement of the roof is an efficient passive strategy for the stress reduction. On the other hand, it results in complicated structure of the roof.

Thus, active adjustment of the stresses is proposed and analyzed below.

#### **5** ACTIVE STRESS ADJUSTMENT IN THE TOP CHORD OF THE ROOF

#### 5.1 General considerations

The roof composed of flat ribs, which has no supplementary ties (figure 1), is considered for investigation of active adjustment of stresses in the top chord.

Diminishing the stresses is proposed by means of tensioning the backstay cables  $C_1...C_4$  (figure 5). The tensioning  $\Delta L$  is the difference between the geometrical length of the cable  $L_g$  and the initial length  $L_0$ :

$$\Delta L = L_g - L_0. \tag{3}$$

Negative tensioning (slackening,  $\Delta L < 0$ ) is introduced for the cables  $C_1$  and  $C_2$  situated in the fully loaded zone (figures 2 and 5). It allows reducing the total load influencing the surface. Positive tensioning ( $\Delta L > 0$ ), in contrast, is applied for the cables  $C_3$  and  $C_4$ , situated in the partially loaded area, thus providing additional impact on the membrane, which sums up with insufficient external load.

Position of the load zones and arrangement of the cables  $C_1...C_4$  are detected by the signs of the moments  $M_{\nu}$  (figure 3). Positive moments are in the beams which are situated in the fully loaded area of the surface, while negative bending moments point out the partially loaded zone.

Separate and simultaneous strategies for the stresses adjustment are examined. The first one is in separate tensioning the cables  $C_1...C_4$  for achieving local effect. Simultaneous cable tensioning is in diminishing the stresses in all the beams of the top chord.

#### 5.2 Separate tensioning the backstay cables

Adjustment of the backstay cables is first examined separately. The following slackening is considered for the cables  $C_1$  and  $C_2$ :  $\Delta L_1 = \Delta L_2 = -0.125$  m, while the cables  $C_3$  and  $C_4$  are positively tensioned by the values:  $\Delta L_3 = \Delta L_4 = +0.125$  m.

The influence of the separate cable adjustment on the bending moments in the top chord is assessed using the following ratio (Table 1):

$$K_i = \frac{M_i}{M_0},\tag{4}$$

where  $M_0$  is the bending moment in the reference state (no adjustment is provided), while  $M_i$  is the bending moment which is altered by deliberate tensioning or slackening the corresponding cable  $C_i$ ,  $i \in \{1, 2, 3, 4\}$  (figure 5).

 Table 1: Effect of separate tensioning/slackening the backstay cables on the bending moments denoted in figure 3

Bending moment	K -ratio			
	$K_1$	$K_2$	<i>K</i> <sub>3</sub>	$K_4$
$M_{v,1}$	0.94	0.99	0.94	0.89
$M_{v,2}$	0.98	0.98	0.96	0.58
$M_w$	0.96	0.82	0.38	0.63

Table 1 shows, that negative tensioning (slackening) of the backstay cables ( $C_1$  and  $C_2$ ) in the loaded zone mitigates bending moments in the top chord of the roof negligibly.

Tensioning of cables  $C_3$  substantially mitigates the moment in the horizontal plane  $M_w$ . The effect of the cable  $C_4$  includes diminishing the moments both in the horizontal and in vertical planes.

Reduction of bending moments in the top chord beams by means of tensioning the backstay cables separately is illustrated in figures 8a and 8b. The best results are achieved for the beams situated between fully and partially loaded zones (beams  $B_3$ ), as well as in the middle of the partially loaded zone (beam  $B_5$ ).

On the other hand, tensioning the cables  $C_3$  and  $C_4$  separately results in significant growth of bending moments  $M_w$  and axial forces N in beams  $B_4$  (figures 8c and 8d).

Figure 8c, however, shows that the graphs of bending moments  $M_w$  brought about by tensioning the cables  $C_3$  and  $C_4$  are almost symmetrical. Thus, simultaneous adjusting the backstay cables would be much more efficient in terms of overall stress mitigation in the beams.



Figure 8: Tensioning the backstay cables separately. Diagrams of bending moments and axial forces in the top chord beams of the roof. a – absolute values of moments  $M_w$  in the beam  $B_3$ ; b – absolute values of moments  $M_v$  in the beam  $B_5$ ; c – moments  $M_w$  in the beam  $B_4$ ; d – absolute values of axial forces N in the beam  $B_4$ ; 1 – reference state (no adjustment); 2 – tensioning the cables  $C_4$  ( $\Delta L_4 = 0.125$  mm); 3 – tensioning the cables  $C_3$  ( $\Delta L_3 = 0.125$  mm)

#### 5.3 Simultaneous tensioning the backstay cables

Finding the appropriate combination of tensioning values for cables  $C_3$  and  $C_4$  is converted into the optimization problem as follows:

$$\sigma_m(\vec{X}) \to \min, \tag{5}$$

where  $\sigma_m$  is the maximum normal stress in all the considered nodes of the top chord of the roof (2);  $\vec{X} = (\Delta L_3 \quad \Delta L_4)^T$  is a vector of unknown structural parameters which belong to the permissible domain:

$$\Delta L \in [\delta L_{\lim,1} \quad \dots \quad \delta L_{\lim,2}]. \tag{6}$$

The coordinate descent method is used for obtaining the optimal solution for the problem (5). Having adopted the initial guess vector  $\vec{X} = \vec{X_0}$ , the resultant vector  $\vec{X_{res}}$  is reached in an iterative way.

Iteration k ( $k \ge 1$ ) consists of the following steps:

1. Determination of the increment  $\vec{\Delta X}_k$  for the  $\vec{X}$  -vector:

$$\vec{\Delta X}_k = \alpha_v \cdot \vec{e_v}, \qquad (7)$$

where v is the index of the structural parameter to be modified,  $v \in [1...n]$ , where n = 2 is the number of parameters considered;  $\vec{\alpha}$  is the vector of step size;  $\vec{e_v}$  are the direction vectors:

$$\overrightarrow{e}_1 = \begin{pmatrix} 1\\ 0 \end{pmatrix}$$
 and  $\overrightarrow{e}_2 = \begin{pmatrix} 0\\ 1 \end{pmatrix}$ . (8)

The index v of the parameter to be applied at the current iteration is obtained as follows:

$$\nu = k - (t-1) \cdot n \,, \tag{9}$$

where *t* is the integer number, belonging to the interval:

t

$$\in [0.5 \cdot k, 0.5 \cdot (k+1)].$$
 (10)

2. Determination of the new vector of the parameters:

$$\vec{X}_{new} = \arg\min(\sigma_m(\vec{X}), \sigma_m(\vec{x}_L), \sigma_m(\vec{x}_R)),$$
(11)

where

$$\vec{x}_{L} = \vec{X} - \vec{\Delta X}_{k}, \qquad (12a)$$

$$\vec{x}_R = \vec{X} + \vec{\Delta X}_k .$$
 (12b)

The components of  $\vec{x_L}$  and  $\vec{x_R}$  vectors must belong to the permissible domain, satisfying the condition (6).

3. Adoption of the new solution:

$$\vec{X} = \vec{X}_{new}, \text{ if } \vec{X} \neq \vec{X}_{new},$$
(13)

or reduction of the step size otherwise:

$$\alpha_{\nu} = \alpha_{\nu} \cdot \lambda \,, \tag{14}$$

where  $\lambda \in (0...1)$  is a reduction factor.

4. Examining the criterion of finishing the iteration process:

$$|\alpha| < \varepsilon, \tag{15}$$

where  $\varepsilon$  is the precision parameter.

5. If the condition (15) is met, then  $\vec{X}_{res} = \vec{X}$ ,  $\sigma_{res} = \sigma_m(\vec{X}_{res})$  and finish. Otherwise, incrementing the iteration number k = k+1 and transition to step 1.

The following parameters are adopted in order to illustrate the algorithm. The permissible domain for the pre-tensioning values is  $\Delta L \in [0 \dots 0.25]$ m. The initial guess is taken in the middle of the domain:  $\vec{X}_0 = \begin{pmatrix} 0.125 \\ 0.125 \end{pmatrix}$ m. The step size vector is adopted the following:  $\vec{\alpha} = 0.1 \cdot \vec{X}_0$ , while the reduction factor is  $\lambda = 0.75$  and the precision parameter is taken  $\varepsilon = |\vec{\alpha}|/4 = 0.0044$  m.

The optimization process is illustrated in figure 9. The resultant pre-tensioning  $\vec{X} = \begin{pmatrix} 0.0607 \\ 0.0344 \end{pmatrix}$  m yields in the maximum normal stress in the top chord of the roof:  $\sigma_m = 237$  MPa.



Figure 9: Obtaining the pre-tensioning values  $\Delta L_3$  and  $\Delta L_4$ . a - illustration of the optimization process; b -  $\sigma_m$ -surface given the pre-tensioning values

## **6 RESULTS AND DISCUSSION**

Diagrams of stresses in the beams of the top chord of the hybrid roof are in figure 10.



Figure 10: Diagrams of normal stresses in the top chord of the hybrid roof. 1 - reference state (no adjustment of the stresses); 2 - stress adjustment by means of simultaneous tensioning of the backstay cables; 3 - stresses in the roof's framework composed of the spatial ribs with the ties

The reference state of the roof is illustrated along with two stress adjustment strategies: by means of simultaneous tensioning of the backstay cables  $C_3$  and  $C_4$  (marked in the figure 5) and using spatial ribs with additional ties  $t_1$  and  $t_2$  (figure 6b).

Both the adjustment strategies result in substantial mitigation of the maximum stresses in the beams. The peak stresses become 237 MPa and 219 MPa versus the reference value

508MPa. Thus, the top chord can be made of ordinary structural steel, instead of high-strength alloy needed for the reference construction.

Active stress adjustment is applied for the hybrid roof of a simple structure. No spatial ribs or supplementary ties are needed in contrast to the passive approach. The active strategy, however, needs appropriate equipment for controlling the stresses and for implementing the adjustment in real time. This is a prospective field of the future research and development.

## CONCLUSIONS

- Passive and active strategies are considered for adjustment of stresses in the top chord of the hybrid roof structure.
- The passive approach implies application of spatial ribs and installation of supplementary flexible ties. Spatial ribs and the ties, provided simultaneously, result in substantial favorable effect reducing the peak beam stress from 508 MPa to 219 MPa.
- The active adjustment strategy implies targeted tensioning of the backstay cables.
- Negative tensioning (slackening) results in negligible favorable effect and, thus, is not reasonable.
- Positive tensioning of the cables situated in the partially loaded zone is investigated. Separate and simultaneous adjustment approaches are analyzed. The first approach reduces bending moments in particular beams only. It does not result in the overall stress mitigation.
- Simultaneous cable adjustment task is converted into the optimization problem. Coordinate descent method is used for finding the solution. Numerical example illustrates achieving the peak stress 237 MPa in the beams. It is 2.14 times as small as the reference peak value. So, the top chord can be made of ordinary structural steel, instead of high-strength alloy needed for the reference construction.
- The work contributes to the field of adaptability of hybrid roof structures. It demonstrates the possibility of mitigating the beam stresses by means of adjusting the tensioning of particular structural elements. The next step of the research is in transition from numerically obtained forces and moments to load-cell indications collected from a real construction.

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