

SPDE-Net: Neural Network based prediction of stabilization parameter for SUPG technique

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Numerical techniques for solving Singularly Perturbed Differential Equations (SPDE) suffer low accuracy and high numerical instability in presence of interior and boundary layers. Stabilization techniques are often employed to reduce the spurious oscillations in the numerical solution. Such techniques are highly dependent on user chosen stabilization parameter. Streamline Upwind Petrov Galerkin (SUPG) technique is one such residual based stabilization technique. Here we propose *SPDE-Net*, a novel neural network based technique to predict the value of optimal stabilization parameter for SUPG technique. The prediction task is modeled as a regression problem and is solved using Artificial Neural Network(ANN). Three training strategies for the ANN have been proposed i.e supervised, L^2 error minimization (global) and L^2 error minimization (local). It has been observed that the proposed method yields accurate results, and even outperforms some of the existing state-of-the-art ANN-based partial differential equation (PDE) solvers such as Physics Informed Neural Network (PINN). The training is based on error between the Finite Element Method(FEM) solution and the analytical solution of SPDE. Global and local variants of stabilization parameter τ are demonstrated. Experiments on a benchmark case of 1-dimensional convection diffusion equation show a reasonable performance of L^2 error minimization (global) as compared to the conventional supervised training.

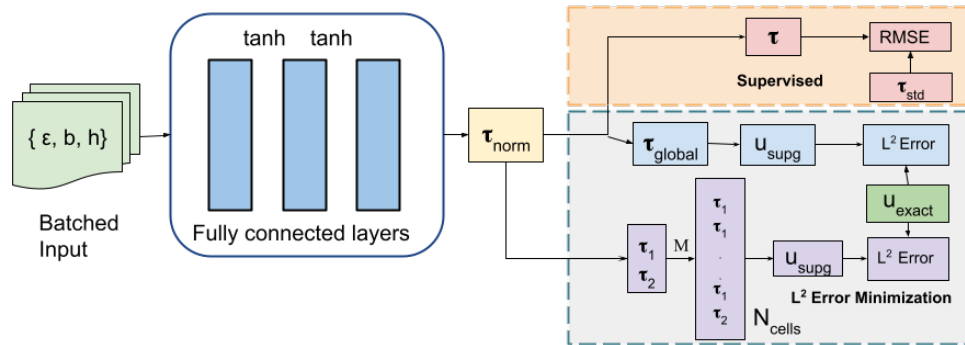


Figure 1: Schematic diagram of *SPDE-Net*

REFERENCES

- [1] Sangeeta Yadav and Sashikumaar Ganesan, (2021). *SPDE-Net: Neural Network based prediction of stabilization parameter for SUPG technique*. Proceedings of The 13th Asian Conference on Machine Learning, Proceedings of Machine Learning Research 157:268-283 <https://proceedings.mlr.press/v157/yadav21a.html>.