Chapter 17
Incremental Dynamic Analysis and Pushover Probabilistic
Analysis of Buildings. A Comparison

Yeudy F. Vargas, Luis G. Pujades, Alex H. Barbat, and Jorge E. Hurtado

Abstract Capacity-spectrum-based-methods are also used for assessing the vulnerability and risk of existing buildings. Capacity curves are usually obtained by means of nonlinear static analysis. Incremental Dynamic Analysis is another powerful tool based on nonlinear dynamic analysis. This method is similar to the pushover analysis as the input is increasingly enlarged but it is different as it is based on dynamic analysis. Moreover, it is well known that the randomness associated to the structural response can be significant, because of the uncertainties involved in the mechanical properties of the materials, among other uncertainty sources, and because the expected seismic actions are also highly stochastic. These mechanical properties are considered as random variables and the seismic hazard is considered in a probabilistic way. A number of accelerograms of actual European seismic events have been selected in such a way that their response spectra fit well the response spectra provided by the seismic codes for the zone where the target building is constructed. In this work a fully probabilistic approach is tackled by means of Monte Carlo simulation. The method is applied to a detailed study of the seismic response of a reinforced concrete building. The building is representative for office buildings in Spain but the procedures used and the results obtained can be extended to other types of buildings. The main purposes of this work are (1) to analyze the differences when static and dynamic techniques are used and (2) to obtain a measure of the uncertainties involved in the assessment of the vulnerability of structures. The results show that static based procedures are somehow conservative and that uncertainties increase with the severity of the seismic actions and with the damage. Low dam-
age state fragility curves have little uncertainty, while high damage grades fragility curves show great scattering.

1 Introduction

To prevent the seismic risk, it is necessary to assess the vulnerability of existing structures. To do that, several methods have been proposed, starting from different approaches. One is the vulnerability index method by EMS-98 macroseismic intensities and structural behaviour through a vulnerability index [1, 2]. Another highly used method is based on the capacity spectrum and the vulnerability or fragility of the structure. Capacity curves are calculated from an incremental nonlinear static analysis, commonly known as “Pushover Analysis” (PA) [3–5]. Another tool used to evaluate the performance of structures against seismic actions is the Incremental Dynamic Analysis (IDA) proposed by Vamvatsikos & Cornell [6]. The purpose of IDA is to obtain a measure of damage in the structure and make an interesting analogy between PA and IDA as both procedures are based on incremental increases of the loads on the structure and on the measure of its response in terms of a control variable which usually is the maximum displacement among others. Furthermore, IDA allows obtaining the dynamic response of a structure for increasing seismic actions. On the other hand, most of the parameters involved in the structural response are random variables. In this work only the randomness due to the mechanical properties of the materials and the seismic action is considered. The randomness expected in the vulnerability and fragility of the building is analysed by means of Monte Carlo techniques. Therefore, a probabilistic comparison between the PA and the IDA is performed when calculating the fragility and expected damage of an existing reinforced concrete building. The main result of this work is the quantitative assessment of the expected randomness of the structural response, defined by its capacity curve, as well as of the fragility curves and the expected damage, which can be given in terms of mean values and standard errors. The damage assessment through nonlinear static procedures is tested against the results of fully nonlinear dynamic analyses. One of the main conclusions of this work is the importance of measuring the vulnerability of structures taking into account that the variables involved are random. Furthermore, this approach incorporates detailed information about the building and uses powerful tools to analyze the structure such as the PA and the IDA, providing valuable key information that can hardly be obtained with other simplified methods in which the building and the seismic actions are defined by only one parameter.
Fig. 17.1 Picture of the block of buildings omega located in the Technical University of Catalonia (BarnaTech) (above) and sketch of the 2D structural model (below)

2 Building Description

This paper analyzes a reinforced concrete structure, consisting of columns and waffle slabs, which is part of the North Campus of the Technical University of Catalonia in Barcelona, Spain. It has 7 stories and 4 spans, the height is 24.35 m and the width is 22.05 m. Figure 17.1 shows a block of four buildings as the analyzed one. In the first building 5 levels can be clearly seen; the other two stories are under the ground. The fundamental period of the building is 0.97 seconds. This value is higher when compared to that of conventional reinforced concrete buildings, because in the numerical model, the waffle slabs are approximated with beams of equivalent inertia and, therefore, are structural elements wide and flat leading to a reduction of the lateral stiffness of the structure. In the calculation model, the structural elements (equivalent beams and columns) follow an elasto-plastic constitutive law, which does not take into account either hardening or softening. Yielding surfaces are defined by the bending moment-axial load interaction diagram in columns and by the bending moment-angular deformation interaction diagram in beams.
Table 17.1 Parameters defining the Gaussian random variables considered in this work

<table>
<thead>
<tr>
<th></th>
<th>Mean Value (kPa)</th>
<th>Standard deviation (kPa)</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$</td>
<td>25000</td>
<td>2500</td>
<td>0.1</td>
</tr>
<tr>
<td>$f_y$</td>
<td>500000</td>
<td>50000</td>
<td>0.1</td>
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</tbody>
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3 Damage Index Based on Pushover Analysis

Pushover analysis is the tool more often used to evaluate the behaviour of the structures against seismic loads. This numerical tool consists in applying horizontal loads to the structure, according to a certain pattern, until the structural collapse is reached. The result is a relationship between the displacement at the roof of the building and the base shear, called capacity curve. In this article, due to the probabilistic approach, the PA is performed repeatedly, therefore, generating automatically the horizontal load limit. To do that, Satyarno [7] proposes the adaptive incremental nonlinear analysis that establishes the horizontal load limit as a function of the tangent fundamental frequency, i.e., the frequency associated with the first mode of vibration, which is calculated in each step. A detailed manual of the program Rauumoko [8] is available for calculating the static and dynamic response. As mentioned above, the mechanical properties of the materials are considered as random variables. The impact of epistemic uncertainties in the structural response has been treated by Crowley et al. in [9] by considering the variation of the ground floor storey height, column depth and beam length, among others. The aim of that article is to generalize the results for a structural typology. In the present study, the aim is to obtain a measure of the uncertainties in the structural response for one building and, for this reason, we consider only the epistemic uncertainties associated to the compressive strength of concrete and the yield strength of steel. Thus, the values used in the structural design for concrete compressive strength $f_c$, and the tensile strength associated with steel yielding strength $f_y$, are considered as random variables assuming they follow a Gaussian probability function whose parameters are shown in Table 17.1. For the Monte Carlo analysis 1000 random samples are generated by means of the inversion method of the cumulative probability distribution curve. This method warranties the homogeneous distribution of the samples. Figure 17.2 shows the capacity curves obtained by means of the PA analysis.

The capacity curves shown in Fig. 17.2 are transformed into capacity spectra, which relate the spectral displacement to spectral acceleration by means of the following equations [10]:

$$
s_{d_i} = \frac{\delta_i}{PF_i}; \quad s_{a_i} = \frac{V_i/W}{\alpha_i}
$$

(17.1)

The subscript $i$ in Eq. (17.1) refers to the applied load increments on the structure during the PA; $s_{d_i}$ is the spectral displacement; $\delta_i$ is the displacement at the roof of
the building; $P_i$ is the modal participation factor of the first mode of vibration; $sa_i$ is the spectral acceleration; $V_b$ is the base shear; $W$ is the weight of the building and $\alpha_i$ is the modal mass coefficient of the first mode of vibration.

On the other hand, the capacity spectrum can be represented in a bilinear form, which is defined completely by the yielding $(D_y, A_y)$ and ultimate $(D_u, A_u)$ capacity points. As we will see later on, this simplified form is useful for defining damage state thresholds in a straightforward manner; see also [5]. Assumptions to build the bilinear capacity spectrum are: (1) the area under the bilinear curve must be equal to the area of the original curve; (2) the coordinates of the point of maximum displacement must be the same in both curves; (3) the slope of the initial branch should be equal in both curves. Figure 17.3 shows an example of the bilinear representation of the capacity spectrum. Different studies have been proposed to calculate the damage of the structure from the definition of damage states ($ds$), which are a description of
the damage in the structure for a given spectral displacement. For example, FEMA [11] and Risk UE [12], define 4 $ds$, namely slight, moderate, extensive and complete. Description of the damage states depends on the type of structure. For instance, according to FEMA [11], in the case of reinforced concrete structures, the $ds$ slight is described as: “beginning of cracking due to bending moment or shear in beams and columns”. Collapse state considers that the structure reaches an imminent risk of collapse. Risk UE defines the damage state capacity spectrum in its bilinear representation.

Based on the values ($Dy$, $Ay$) and ($Du$, $Au$), the spectral displacements for the four damage states threshold $ds_1$ are obtained:

$$ds_1 = 0.7 \times Dy$$
$$ds_2 = Dy$$
$$ds_3 = Dy + 0.25 \times (Du - Dy)$$
$$ds_4 = Du$$

(17.2)

Therefore, after calculating the capacity spectrum in bilinear representation and applying Eq. (17.2), it is possible to obtain the damage states thresholds as random variables, as is shown in Fig. 17.4. The mean, standard deviation and coefficient of variation of the damage states are shown in Table 17.2. It is worth noting how the co-

<table>
<thead>
<tr>
<th>Variable</th>
<th>$ds_1$ (cm)</th>
<th>$ds_2$ (cm)</th>
<th>$ds_3$ (cm)</th>
<th>$ds_4$ (cm)</th>
</tr>
</thead>
<tbody>
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<td>12.3</td>
<td>15.2</td>
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</tr>
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<td>$\sigma_{ds}$</td>
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<td>0.38</td>
<td>1.00</td>
<td>3.25</td>
</tr>
<tr>
<td>c.o.v.</td>
<td>0.03</td>
<td>0.03</td>
<td>0.06</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Fig. 17.5 Fragility curves as random variables

The efficient of variation of the damage state 4 is greater than those of the input variables. This effect is due to the fact that this type of system is not robust, mainly because of the nonlinearity of the problem. In addition, these results show the importance of the probabilistic approach in this type of analysis as the output can be greater than those of the input variables. After obtaining the damage states as random variables it is also possible to calculate the fragility curves which, for each damage state, represent the probability of reaching or exceeding the corresponding damage state. Fragility curves are represented as a function of a parameter representing the seismic action, for instance spectral displacement, PGA, etc.

The following simplified assumptions to construct fragility curves from damage states thresholds are considered: (1) the probability that the spectral displacements in each damage state threshold, $d_{s_i}$, equals or exceeds the damage state is 50%; (2) for each damage state $d_{s_i}$, the corresponding fragility curve, follows a lognormal cumulative probability function described by the following equation:

$$P[d_{s_i} / sd] = \phi \left[ \frac{1}{\beta_{d_{s_i}}} \ln \left( \frac{sd}{d_{s_i}} \right) \right]$$

where $sd$ is the spectral displacement and $\beta_{d_{s_i}}$ is the standard deviation of natural logarithm of the damage state $d_{s_i}$; (3) the expected seismic damage in buildings follows a binomial probability distribution. Figure 17.5 shows all fragility curves calculated after applying the described procedure.

Since the probabilities of occurrence of each damage state are easily obtained from the fragility curves, one can calculate the expected damage index, $DI$, which is the normalized mean damage state. $DI$ can be interpreted as a measure of the overall expected damage in the structure.

$$DI = \frac{1}{n} \sum_{i=0}^{n} i P(d_{s_i})$$
where \( n \) is the number of damage states considered, in this case 5 (four non-null) and \( P(ds_i) \) is the probability of occurrence of \( ds_i \). Figure 17.6 shows the DI calculated from the fragility curves of Fig. 17.5. The curves of Fig. 17.6 must be interpreted as random vulnerability curves. These curves are an important result of this work as they allow linking PA and IDA procedures by comparing the obtained results.

### 4 Damage Index Based on the Incremental Dynamic Analysis

Dynamic analysis allows obtaining the time history of the response of a structure to an earthquake action. In IDA, the earthquake is scaled to various PGA, allowing obtaining the maximum response as a function, for instance, of PGA. As mentioned above, the main purpose of this article is to compare the results obtained with PA and IDA. An important element of the uncertainty related to the seismic response of structures is the random variability in the ground-motion prediction, whose influence has been studied in [13]. According to the probabilistic approach it is necessary to obtain the seismic action as a random variable. To do that, 20 earthquakes have been selected from two databases, one from Spain and the other from Europe [14], whose elastic response spectra are compatible with elastic response spectrum taken from EuroCode 8 (EC8) [15]. In this case, the elastic spectrum type 1 and soil D is selected. This spectrum corresponding to great earthquakes and soft soils has been chosen in order to submit the structure to strong enough seismic actions to obtain significant damage. Figure 17.7 shows the spectra of the selected earthquakes, their median value, and the spectrum type 1 soil D, taken from EC8. After selecting the accelerograms, the dynamic response of the structure is calculated for different PGA increasing at intervals of 0.04 g, until the value that causes the collapse. This value is 0.8 g. In each run of the nonlinear dynamic analysis, the damage index proposed by Park & Ang [16] and the maximum displacement at the roof of the building are calculated, allowing comparing these results with those of the PA analysis. Figure
Fig. 17.7 Selected spectra of the accelerograms that are compatible with spectrum type 1 soil D of Eurocode 8

Fig. 17.8 Damage index obtained with static and dynamic procedures

Figure 17.8 shows the results obtained. It is important to note the large scatter in both cases, showing the importance of assessing the vulnerability of structures from a probabilistic perspective, whichever procedure is used.

Figure 17.8 shows that the damage index obtained with the procedure based on the PA is conservative. However, for extreme cases when the damage index is close to 0 and 1, which correspond to the null and collapse damage states, similar values are obtained with both procedures. On the other hand, it can be seen that the curves obtained with the PA procedure are somehow conservative, as the structural damage begins for a smaller spectral displacement. PA based curves are shifted with respect to the IDA based curves. This behavior can be attributed to the fact that the damage state thresholds $ds_1$ and $ds_3$ in Eq. (17.2) are based on expert opinion. A little change in these values would avoid this shift. The use of constant coefficients, namely of 0.7
and 0.25, in these equations are useful for massive large scale assessments. In this type of studies [5] a great amount of buildings is evaluated based on the use of simplified structural typologies owing to the difficulty to obtain specific capacity curves and coefficients for each building. This approach leads to reasonably good results in average sense. A new method for estimating the damage state thresholds is proposed here. This method is based on an accurate analysis of the variation of the slope of the capacity curve, namely of its derivative. It is worth noting too that, as we will see below, the new procedure of assessing the $d_{S1}$ and $d_{S3}$ thresholds avoids the shifting between PA and IDA based damage curves of Fig. 17.8.

Figure 17.9 shows an example of the derivative function of the capacity spectrum plotted in Fig. 17.10. In both figures the new damage states thresholds are shown.

In fact, the derivative function is related to the degradation of the stiffness as it gives the actual stiffness of the structure as a function of the spectral displacement
Fig. 17.11 Derivative functions of all capacity spectra

caused by lateral load increases in the pushover analysis. Then, $ds_1$ is defined by the spectral displacement where the lateral stiffness start to decrease; in other words, the point where the damage starts to increase. At this stage of the method, $ds_2$ has been defined as the spectral displacement corresponding to a reduction of 50% of the initial stiffness. $ds_3$ is defined by the spectral displacement where the derivative tends to be constant, indicating the end of the degradation of the stiffness which remains almost constant till the displacement of collapse. Finally, $ds_4$ is maintained as the spectral displacement corresponding to the ultimate point. It is worth noting that the shapes, but not the values, of the derivative functions are very similar for all the 1000 capacity spectra analyzed. See Fig. 17.11. Therefore, the new damage states based on the stiffness degradation and the damage states calculated via Risk UE approach, which will be called $ds_{i-S}$ and $ds_{i-R}$ respectively, are compared. In order to characterize the statistical properties of the distribution of the old (see Fig. 17.4) and new defined damage states, the Kolmogorov–Smirnov test [17] has shown that the damage states calculated with both approaches follow a Gaussian distribution. Figure 17.12 shows the comparison between both probability density functions. For the damage states different to extensive and collapse, the mean values and the standard deviations of the $ds_{i-S}$ are higher than those of $ds_{i-R}$. Then, the procedure described above for obtaining the fragility curves and damage indices was applied again by using the new damage states. Figure 17.13 shows the obtained results. For comparison purposes, the damage indices obtained by means of the dynamic analyses are also plotted in this figure. This figure allows comparing new and old damage index functions as well as each of these functions with the results of the dynamic analyses.

Concerning PA results, blue and black colour curves, a clear shift towards increasing spectral displacements of the new damage functions can be seen, indicating that the Risk UE choice is somehow conservative. Furthermore, new black curves fit better the IDA results (red points).

In order to quantitatively improve this comparison, Fig. 17.14 and Fig. 17.15 compare respectively the first and second moments of these distributions. These sta-
Fig. 17.12 Comparison between damage states based on Risk UE and stiffness degradation approach.

Fig. 17.13 Comparison between damage indices obtained with all methodologies.

tistical properties, namely the mean values and standard deviations are computed for each spectral displacement by using the corresponding random ordinates. It can be clearly seen in Fig. 17.14 how the mean of the random variable obtained with the derivative approach fits quite well the mean of the damage index obtained via nonlinear dynamic analysis. Figure 17.15 shows that, for spectral displacements in the range 0.1 to 0.3 m, the standard deviation corresponding to PA results is lower than one corresponding to IDA results. This effect is attributed to the fact that PA results do not consider the seismic actions leading to lower uncertainties. To consider the uncertainties of the seismic action, we use a simplified method allowing obtaining the expected displacement as a function of PGA for a given seismic input, represented by the 5% damped elastic response spectrum. Obviously, the building in this analysis is defined by its capacity spectrum. In this procedure, the elastic response spectrum is reduced based on the ductility of the building which is calculated from
the capacity spectrum as the ratio between the spectral displacements of the ultimate capacity point ($Du$) and that of the yielding point ($Dy$) (see Fig. 17.3). An extended explanation of this technique can be found in [9] and has been also used in [18], it has been initially proposed by [19] and its development has been reviewed in [20]. In this way, increasing the PGA at intervals of 0.04 g between 0.04 and 0.8 g, as in the IDA, a relation between the PGA and the spectral displacement, $sd$, is obtained for each spectrum corresponding to each of the 20 accelerograms used and for each of the 1000 capacity spectra. Therefore, a total of 20000 relations between $sd$ and PGA are obtained.

Figure 17.16 and Fig. 17.17 show the mean and the standard deviation curves of the damage indices as a function of PGA for PA, by using the new defined damage states, and IDA results.
It can be seen how, in the range between 0 and 0.4 g, the mean values and the standard deviations show a good agreement. Note that now the uncertainties in the seismic actions are included in both curves. For greater values, standard deviations in the new PA approach are larger than for the IDA approach but both decrease because damage indices greater than one were not allowed. The fact of the better agreement between the PA and IDA results, when using the new damage states thresholds, indicates that this proposal based on the stiffness degradation, obtained from the derivatives of the capacity curves, should be preferred to the expert-opinion based one as proposed in the Risk UE approach. Furthermore, the damage index calculated in this way is able to represent, not only the expected damage obtained via nonlinear dynamic analysis, but also the uncertainties associated to the mechanical properties of the materials and the seismic action. Finally, it is important to note that in the case study building analysed here the Risk UE approach is a little conservative.
as the damage appears before the new approach. This is so because the new damage states thresholds are greater. Obviously the spectral displacements of the damage states thresholds can coincide but if the new defined grades are smaller, the Risk UE approach may underestimate the expected damage. In any case, the new approach to determine damage state thresholds capture better the degradation of the buildings strength as indicated by the agreement with the IDA results.

5 Conclusions

In this work, the vulnerability of a real reinforced concrete structure, with columns and waffle slabs has been assessed, taking into account that the input variables are random. Only the randomness of the concrete compressive and the steel yielding strengths has been taking into account but the seismic action has been also considered in a stochastic way. Two approaches to evaluate the expected damage of the building have been used. The first one is based on the pushover analysis and the second one is based on the incremental dynamic analysis. An important conclusion is that, despite working with advanced structural analysis, these procedures show significant uncertainties when taking into account the randomness of the variables associated with the problem. It should be emphasized that in this work relatively small coefficients of variation for input variables have been considered taking into account the uncertainties that may exist in older structures that did not have quality control and have not been designed according to the earthquake-resistant criteria. The results obtained give support to the idea that static procedures are conservative when compared with the dynamic analysis. Furthermore, for expected damage analysis, a new procedure has been proposed to define the damage states thresholds. The technique is based on the degradation of the stiffness which can be observed in the derivative function of the capacity curve. The results using this new approach show a better agreement with the dynamic analysis than the obtained ones when using damage states thresholds based on expert-opinion.

Probably one of the most relevant conclusions of this work is that whichever procedure is used to evaluate the expected seismic damage of a structure, the input parameters of the structural problem to be treated, must be considered as random variables. We have seen how the probabilistic consideration of a few of these parameters produces significant uncertainties in the seismic response. Simplified deterministic procedures based on characteristic values usually lead to conservative results but some abridged assumptions on the definition of the seismic actions and on the estimation of the seismic damage states and thresholds can lead also to underestimate the real damage that can occur in a structure.

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References