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PARTITIONED COUPLING APPROACHES FOR THE SIMULATION OF NATURAL HAZARDS IMPACTING PROTECTIVE STRUCTURES

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Abstract: In recent years, the intensity and frequency of natural hazards such as landslides, debris flow and avalanches have increased significantly due to climate change and global warming. These catastrophic events are responsible for numerous destructions of infrastructures with high economic losses and, even worse, often claim human lives. Therefore, in addition to the prediction, the design and installation of protective structures are of tremendous importance. Due to its hybrid approach of an Eulerian background grid in combination with Lagrangian moving material points, the Material Point Method (MPM) is particularly suited to capture the flow process of those mass movement hazards. For the numerical simulation of protective structures, however, other numerical methods are often preferable. Considering highly flexible structures, which are often utilized due to their high energy absorption capacity classical Finite Element Method (FEM) is best suited to model cable, beam, and membrane elements, while a retaining wall consisting of a few discrete blocks may be preferable modeled by Discrete Element Method (DEM).

Therefore, we are proposing partitioned coupling approaches to combine the advantages of different numerical methods so that the protective structures can be appropriately designed to withstand the impact of those mass movement hazards.

Keywords. Material point method, Discrete element method, Partitioned coupling, Natural hazards

1 INTRODUCTION

Mass-movement hazards such as landslides, debris flow, or avalanches are dynamic soil events that due to their huge moving masses, have tremendous destruction and violence. Due to global warming and climate change, the intensity and frequency of those natural hazards have increased significantly within the last decades. According to the Centre for Research on the Epidemiology of Disasters (CRED) and the UN Office for Disaster Risk Reduction (UNDRR) [1], climate-related disasters within the year 2020 only have been responsible for 15,050 deaths worldwide, with more than 98.4 million citizens affected, and about US171.3 billion economic losses. Hence this topic is more topic than ever. Consequently, in the long term, countermeasures are necessary to counteract climate change effectively. However, in addition, protective structures must be designed and built to protect in short-term vulnerable areas and save human lives perspectively.

The design and dimensioning of such protective structures, however, is a complex task requiring advanced numerical simulation techniques. Classical Finite Element Method (FEM) or other Lagrangian mesh-based methods will likely suffer from mesh entanglement and distortion, requiring computationally expensive re-meshing schemes to model the huge masses flowing down the mountainous region. Therefore, continuum-based particle methods are the natural alternative to simulate those large strain events, including huge topological changes of the material. Among them, the Material Point Method (MPM) is particularly suited, as it combines the advantages of both mesh-less and mesh-based numerical techniques. As initially proposed by Sulsky et al. [2], the physical domain is discretized by Lagrangian moving particles called material points. Each represents a discrete part of the physical domain and carries the history-dependent variables during the calculation. In addition to the material points, discretizing the body, an Eulerian computational background grid is introduced in MPM, which is utilized to solve the governing equations. Hence, inherently MPM brings along many similarities to the established updated-Lagrangian FEM.

For the numerical simulation of the protective structures often, alternative numerical methods are preferable to capture their specific behavior. Considering highly flexible protective structures on the one hand, which mainly consist of a net spanned between steel profiles and fastened by a few additional cables in an uphill direction, FEM is the appropriate choice to model those structures. To design them for the impact of the natural hazards, a partitioned MPM-FEM coupling algorithm was developed by the author in [3, 4] to model the interaction between MPM, representing the flowing mass downhill, and FEM to model the flexible protective structure consisting of membrane, truss and cable elements.

On the other hand, considering a protective structure consisting of heavy blocks stacked on top of each other, the Discrete Element Method (DEM) is the preferable numerical method. Especially due to its ability to accurately calculate contact forces depending on calibrated material parameters, the block's movements and behavior in interaction with the flow can be numerically simulated. Therefore, it is the preferred method to model discrete obstacles, which may be shaped arbitrarily. However, due to the discrete approach, the computational cost increases significantly, with the number of particles limiting its application for large-scale events. Hence, to model the mass-movement flow itself, a continuum-based approach is essential, requiring MPM, while DEM may preferably model discrete obstacles.

Hence, combining the advantages of both numerical methods but still preserving the modularity of the involved solvers, a partitioned MPM-DEM coupling scheme was developed by the author in [5]. This approach is extended to more advanced application cases, comparing the numerical solutions to experimental results. These examples demonstrate the application of the proposed method for retaining wall systems and for protective structures consisting of massive blocks stacked on top of each other impacted by continuous flow.

2 Material Point Method (MPM)

Strong form

Considering the Lagrangian moving body $\mathcal{B}$ occupying the domain Ω with regular boundary Γ in the three-dimensional Euclidean space $\mathcal{E}$ the conservation of linear momentum

$$\mathbf{L} = \int_{\Omega} \rho \dot{\mathbf{u}} \mathrm{d}\Omega \tag{1}$$

needs to be satisfied where ρ is the spacial mass density and $\dot{\mathbf{u}}$ the velocity. Furthermore, the symmetry of Cauchy stress tensor $\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$ is assumed, which guarantees the conservation of angular momentum.

Based on that, Cauchy's first equation of motion is derived

$$\rho \ddot{\mathbf{u}} = \mathrm{div}\boldsymbol{\sigma} + \rho \mathbf{b} \tag{2}$$

which holds for each point $\mathbf{x} \in \Omega$ and for all times t. It implies the conservation of mass and assumes an isothermal setting. Within this equation, $\mathbf{b}$ denotes the volume acceleration, while the second material time derivative of the displacement field $\mathbf{u}$ is the acceleration.

This second-order differential equation, which is the governing equation for a continuum body $\mathcal{B}$ is determined by the Dirichlet and Neumann boundary conditions

$$\mathbf{u} = \overline{\mathbf{u}} \quad \text{on} \quad \mathrm{d}\Gamma_D \tag{3}$$

$$\mathbf{p} = \boldsymbol{\sigma}\mathbf{n} = \overline{\mathbf{p}} \quad \text{on} \quad \mathrm{d}\Gamma_N \tag{4}$$

where $(\overline{\bullet})$ denotes the prescribed values.

Weak form

Since, in general, a closed-form solution for the given problem cannot be found, the second-order differential equation is transferred to its weak form by formulating it through the *Principle of Virtual Work* [6]

$$\delta W = -\int_{\Omega} \boldsymbol{\sigma} : \delta \mathbf{e} \mathrm{d}\Omega - \int_{\Omega} \rho \ddot{\mathbf{u}} \cdot \delta \mathbf{u} \mathrm{d}\Omega + \int_{\Omega} \rho \mathbf{b} \cdot \delta \mathbf{u} \mathrm{d}\Omega + \int_{\Gamma_N} \overline{\mathbf{p}} \cdot \delta \mathbf{u} \mathrm{d}\Gamma_N = 0 \tag{5}$$

which equalizes zero for systems in equilibrium. In this equation, $\delta \mathbf{u}$ are the virtual displacements, while $\delta \boldsymbol{\epsilon}$ is the virtual strain arising from the gradient of the virtual displacement field.

Discretization in time and space

To numerically solve the weak equilibrium equation (5), the continuous problem needs to be discretized. Therefore the spatial fields are approximated by nodal values and locally defined basis functions.

Furthermore, the continuous time domain is divided into discrete time steps, and applying the Newmark-β [7] implicit time integration scheme finally leads to a non-linear equation system which iteratively is solved utilizing the Newton-Raphson method.

This derivation is valid for both FEM and MPM distinguishing, however, between the classical FE mesh subdividing the structure itself into non-overlapping elements, while in MPM, the computational background grid covers the complete computational domain, including empty spaces into which the material is expected to move during the computation. Thus the body $\mathcal{B}$ in MPM is usually embedded within the computational background grid and is discretized into a finite number n_p of material points representing each a finite volume Ω_p of the body

$$\mathcal{B} \approx \mathcal{B}^h = \bigcup_{p=1}^{n_p} \Omega_p \; . \tag{6}$$

Those material points carry the historical information during the calculation procedure and move according to the body's deformation.

MPM update scheme

Due to the discretization of the body into material points in combination with the computational background grid, MPM shows many similarities to the classical finite element updated Lagrangian calculation procedure. This solution procedure, however, is enhanced by continuous inter- and extrapolation material point information and nodal values of the computational background grid. Therefore the MPM procedure can be categorized into three main phases:

1. **Initialization phase**: A search is performed to define the background grid element, which belongs to each material point, before the necessary variables are extrapolated via mass projection to the corresponding nodes as initial conditions.

2. **Lagrangian phase**: Solution of the discretized governing equations equivalently to classical updated Lagrangian FEM.

3. **Convective phase**: Solutions obtained at the nodes of the background grid are interpolated back to the material points, resulting in an update of the material point's position and its kinematic values. Then the background grid is reset to its initial position.

Those three phases of the MPM scheme are illustrated in Figure 1.

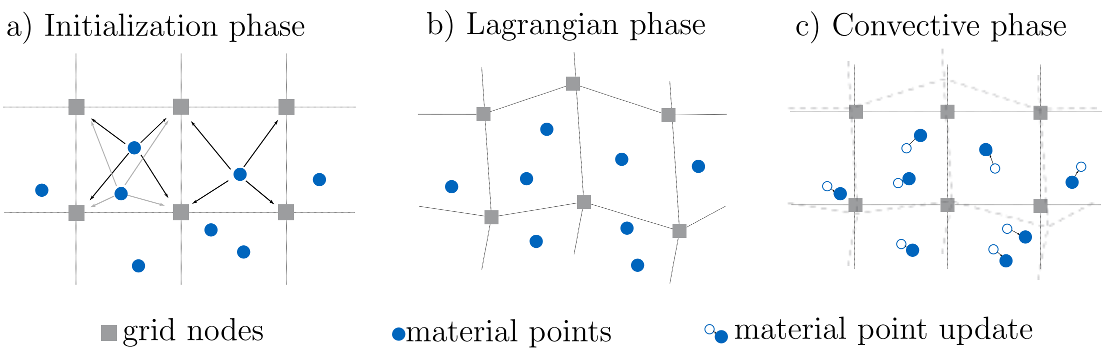

Figure 1: The three phases of MPM a)Initialization phase, b) Lagrangian phase, and c) Convective phase. Square markers identify the grid nodes, while round markers indicate the material points. Adapted from [4]

Boundary conditions

Since the material points, which are discretizing the body, are moving through the Eulerian background grid, the boundary conditions can rarely be imposed directly at the nodes of the computational background grid. Instead, a weak imposition is commonly required, which is particularly a challenging task for the imposition of Dirichlet conditions. This topic is addressed by Chandra et al. [8] introducing the Penalty method for implicit MPM while an alternative approach using Lagrange multipliers can be found in [9] to weakly enforce Dirichlet conditions.

These approaches introduce boundary particles, which track the body's outline during the calculation process and carry the necessary information for the boundary imposition. Specifically, those boundary particles are crucial for the partitioned coupling schemes to track the interface and ensure mutual communication between the partitions involved. While for partitioned coupling with FEM, Dirichlet conditions are imposed within the MPM partition using Penalty augmentation [8] or Lagrange multiplier imposition [9], Neumann conditions are required for the coupling with DEM. Therefore the traction surface integral of equation (5) can be rewritten as

$$\int_{\Gamma_N} N_I \overline{\mathbf{p}} d\Gamma_N \approx \bigcup_{bp=1}^{n_{bp}} N_I \overline{\mathbf{p}} A_{\mathrm{bp}} = \bigcup_{bp=1}^{n_{bp}} N_I \mathbf{F}_{\mathrm{M}} \tag{7}$$

where N_I is the nodal basis function evaluated at the boundary particle location and $\mathbf{F}_{\mathrm{M}}$ is the respective resulting point load at each introduced boundary particle within the MPM partition (indicated by subscript M). A_{bp} represents the current area of the boundary particle, while n_{bp} is the total number of boundary particles discretizing the Neumann interface Γ_N.

Hence, during the MPM calculation process, the point loads $\mathbf{F}_{\text{M}}$ are mapped in the Initialization phase via the basis function to the corresponding nodes of the computational background grid. Then the system is solved in the Lagrangian phase before the kinematic variables at the boundary particles are updated, following the concept of material points in the Convective phase. Therefore the boundary particles move according to the body deformation, which is an important feature in tracking the interface.

3 Discrete Element Method

In contrast to MPM, which belongs to the group of continuum-based methods, DEM is a discrete particle method whose particular strengths lie in efficiently analyzing the motion and interaction of individual particles. The detection of the contact and the subsequent calculation of the respective contact forces are, however, crucial to the DEM calculation procedure. Most efficiently, this can be resolved for spherical particles and therefore is considered herein. To model arbitrarily shaped particles, clustering of particles [10] is applied, which still allows efficient contact detection.

Furthermore, the Double Hierarchy Method, originally proposed by [11], is applied here, providing an efficient way of handling various contact partners simultaneously. This is especially important since, in addition to mutual contact of the particles, the contact of the particles with the boundaries needs to be detected.

Those boundaries are particularly important for the partitioned coupling scheme to ensure communication with the subsequent solver. Therefore, analogously to [12, 13], a wall condition is introduced at the shared interface, representing a Dirichlet condition with stiffness properties of the MPM counterpart for the DEM particles. The geometry of this wall condition can be created similar to a FE mesh, consisting of vertices, edges and faces in 3D and therefore requiring the contact detection between spherical particles and geometric entities.

As soon as contact is detected, the respective contact forces are calculated. Various contact laws can be applied, while a Hertz-Mindlin spring-dashpot (HM+D) [14] model is used in this work. All interacting forces are then assembled to derive the forces $\mathbf{F}_i$ and torque $\mathbf{T}_i$ on each particle i.

Finally, after the contact force evaluation, the DEM solution process proceeds to the integration of motion, following Newton's second law of motion. While the mass m relates the translational acceleration $\ddot{\mathbf{u}}$ to the forces $\mathbf{F}$

$$\mathbf{F} = m\ddot{\mathbf{u}} \tag{8}$$

the inertia tensor $\mathbf{I}$ is used to calculate the moments (torques) $\mathbf{T}$

$$\mathbf{T} = \mathbf{I}\ddot{\boldsymbol{\omega}} \tag{9}$$

via the rotational acceleration $\ddot{\boldsymbol{\omega}}$. Within this work, a second-order Velocity-Verlet (central differences) scheme is used to integrate the translational degrees of freedom.

4 Partitioned MPM-DEM coupling scheme

Exploring the strengths of DEM and MPM but still preserving the solvers' modularity, a partitioned approach is developed, treating the partitions involved as black-box solvers while the interaction is shifted to their shared interface

$$\Gamma_{\mathrm{MP}} = \Gamma_{\mathrm{M}} \cap \Gamma_{\mathrm{P}}. \tag{10}$$

Herein, the subscript M indicates the MPM domain Ω_{M} and interface Γ_{M} and the subscript P indicates the DEM domain Ω_{P} with interface Γ_{P}. At this interface, the interface transmission conditions need to be fulfilled, enforcing the kinematic and dynamic interface conditions.

Due to the partitioning, the numerical models for the MPM and DEM are created independently of each other. At the shared interface, which usually coincides with the outline of the MPM body, a wall condition is defined in the DEM partition, whereas boundary particles are introduced in the MPM counterpart.

To solve the non-linear interface equations, which arise due to the partitioning, a fixed-point iteration that sequentially executes the solvers is utilized, applying a weak coupling scheme, as illustrated in Figure 2.

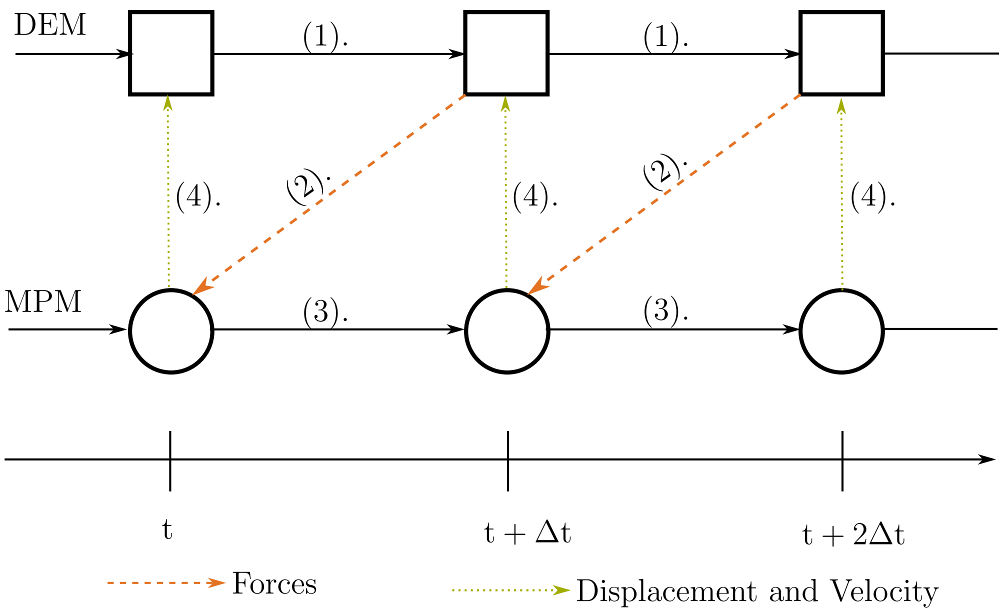

Figure 2: Partitioned weak MPM-DEM coupling scheme. Adapted from [5]

Hence the DEM partition is solved first with given displacements and velocities at the nodes of the discretized wall condition. During the DEM calculation process, possible contact between the DEM particles and the wall condition is detected, resulting in contact forces $\mathbf{F}_{\mathrm{P}}$ at the corresponding nodes of the discretized wall condition. Those forces are then transferred with an interpolation-based mapper [15] in a second step to the boundary particles in the MPM partition as external forces $\mathbf{F}_{\mathrm{M}}$.

Then, in the third step, the MPM partition is solved with the external forces at the boundary particles as input, resulting in a kinematic update of the material points. Consequently, also the position and the velocity of the shared interface, discretized by the boundary particles, is updated.

This kinematic update is finally mapped back [15] to the DEM partition, updating the nodal displacements and velocities of the DEM wall condition accordingly.

Since a weak coupling scheme is applied, the DEM solver advances in time and solves the DEM model with the updated wall condition. Those steps are repeated within each time step until the end of the simulation.

A detailed description of this coupling scheme can be found in [5].

5 Numerical Example

The MPM-DEM coupling approach is applied to simulate large deformation and post-failure behavior of soil and retaining wall blocks. The numerical solution of a two-dimensional simulation is compared to the experimental results conducted by Bui et al. [16]. Within the experiment, Aluminium bars with a length of 5cm were used as the model ground to simulate the two-dimensional conditions.

The segmental retaining wall within this study consists of six identical Aluminium blocks stacked on top of each other with an overlapping of 1.2cm as depicted in Figure 3.

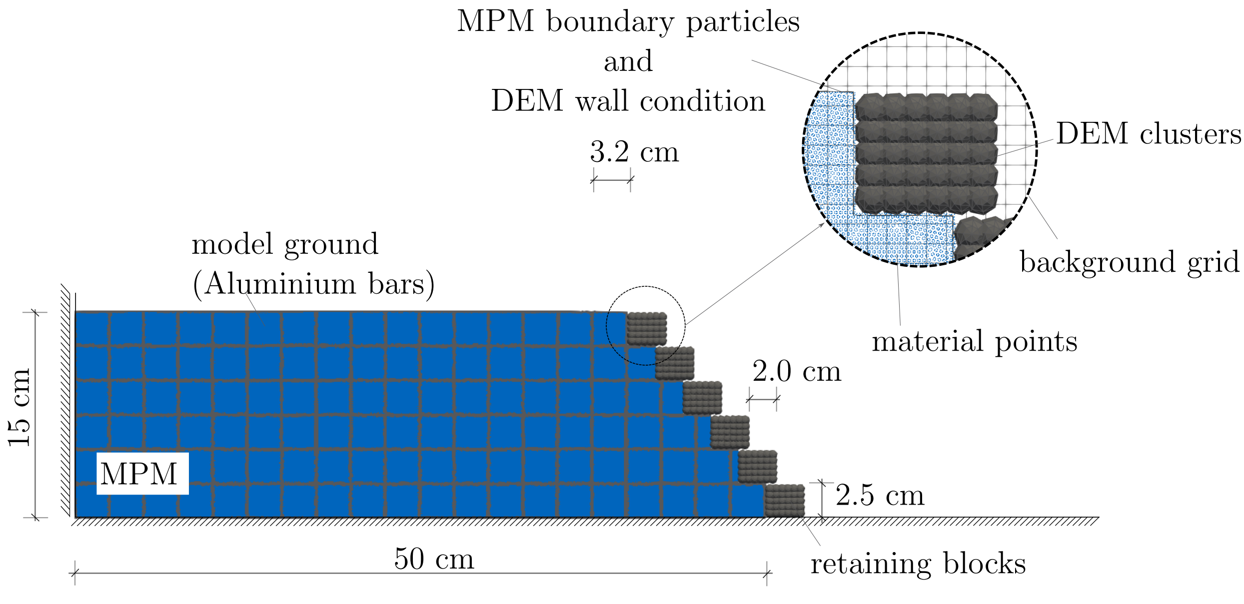

Figure 3: Retaining wall system with a detailed view of the discretization.

Those rectangular blocks have a width of 3.2cm, height of 2.5cm, and length of 5.0cm. The material properties of the blocks and those of the model ground are taken from [16].

While the retaining wall blocks are modeled with DEM by clustering 6×5 spherical particles to obtain the rectangular shape, the model ground is simulated by MPM using

Mohr-Coulomb plane strain material law. As boundary conditions for the model ground, a fixed support is assumed at the bottom while a slip condition is imposed in the lateral direction.

Figure 4 shows the comparison of the numerical and experimental results at specific times for the failure process of the retaining wall block system. Due to the pressure forces of the backfilled soil, the retaining wall blocks start to move rightwards, causing the collapse of the retaining wall system. While the block at the bottom slides only horizontally, triggering the system's failure, translational and rotational motion can be observed for the other blocks, which are in very good agreement with the experimental results.

A great benefit is that modeling the blocks by DEM inherently allows the separation of the blocks while still a continuum-based description can be used to model the backfilled soil. Especially for the numerical investigation of large-scale events, such as avalanches, mud-flow, and landslides, interacting with retaining wall systems, these properties are essential, and the partitioned approach is required for the numerical investigation.

In [5], additional validation and application examples for the partitioned MPM-DEM coupling scheme can be found. Among them, the scheme's application to simulate the impact of gravity-driven flow onto several blocks, which are stacked on top of each other, is shown. Also, in this study, a very good agreement with experimental results is obtained, demonstrating that the proposed scheme can as well be applied to simulate the impact of mass-movement hazards onto barriers consisting of heavy blocks.

6 Conclusion

The proposed MPM-DEM coupling scheme is particularly beneficial for the numerical investigation of soil or granular material interacting with discrete obstacles, which may be shaped arbitrarily. Since DEM provides the accurate calculation of contact forces, the post-failure behavior of blocks or other barriers impacted by a gravity-driven flow can be simulated efficiently with the proposed scheme.

MPM is particularly well suited to capture the flow process with large strains and time-dependent material behavior of mass-movement hazards flowing down a mountainous region. Protective structures, however, are preferably modeled with other numerical methods. While retaining walls or walls consisting of rigid blocks are preferably modeled with DEM since mutual contact needs to be considered, classical FEM outperforms other numerical methods for calculating flexible structures.

Thus, with a generous interface description, MPM can be coupled with DEM or other numerical methods, treating the involved sub-systems as black-box solvers. In [3, 4] a partitioned MPM-FEM coupling scheme is applied for the numerical investigation of a gravity-driven flow impacting a highly flexible protective structure.

The framework of partitioned coupling, in combination with robust interface descriptions within the solvers involved, provides great flexibility in bringing together different numerical methods to combine their respective advantages. Therefore the proposed coupling schemes are to be further improved and validated in the near future so that, finally, they can be applied for real-scale mass-movement hazards involving complex multi-phase

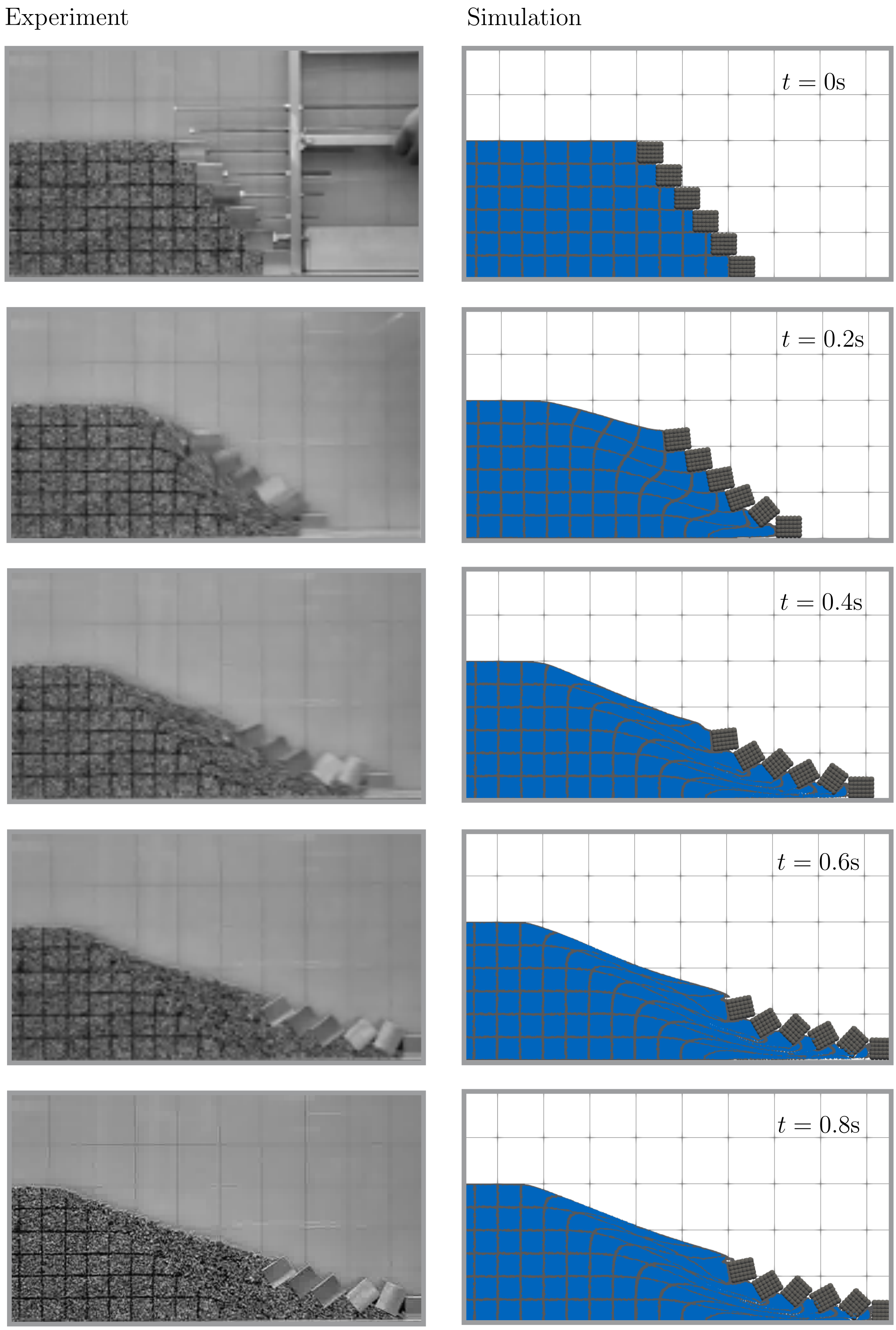

Figure 4: Simulation results in comparison to the experimental results conducted by Bui et al. [16].

flows which are impacting different types of protective structures.

Code Availability

For this work, the open-source multiphysics software *KRATOS*[17, 18, 19] has been used which is written in C++ and offers a Python interface.

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