14th International Conference on Structural Safety and Reliability - ICOSSAR'25
1-6 June, 2025 - Los Angeles California, USA
R. Ghanem, E. Johnson, S. Masri and A. Olivier (Eds)

VARIANCE GAMMA PROCESS PARAMETER ESTIMATION FOR STRUCTURAL CAPACITY DEGRADATION

T. MICIC

Department of Engineering, City St George's, University of London
London EC1V 0HB, UK
E-mail t.micic@citystgeorges.ac.uk

Key words: variance gamma process, capacity degradation, non-monotonous

Abstract. Variance gamma process is proposed for modelling structural capacity loss for exisitng structures and benefits of this approach are identified. Variance gamma process (VGP) is a jump Levy process where the process increments follow gamma distribution. The VGP modelling can be expressed as a difference of two gamma processes, thus increasing the number of parameters that can be used to describe the status of the structure and potential degradation scenarios, while retaining relative simplicity that gamma distributions offer. This paper proposes that standard methods for parameter estimation can be implemented for variance gamma process parameters. With rapidly increasing data availability from diverse infrastructure monitoring technologies the VGP approach presents tool that can improve maintenance planning. Beyond maintenance planning, a major benefit from variance gamma process implementation would be in cases where natural hazards, such as earthquakes or floods, create a disruption point in the service life profile.

1 INTRODUCTION

Quantitative measures of structural ageing are increasingly needed due to growing requirements for automation in assessment and target performance requirements. Key aspect of structural assessment is compliance with the relevant assessment standard (when available) or in relation to the design standard, however, irrespective of the approach, the lack of data was historically a challenge. Now, however, with increasing options for monitoring and automation of inspection, new types of data are becoming available in large quantities. It has been shown that structural ageing itself can be represented through capacity loss where the loss is non-decreasing, right continuous function, Van-Noortwijk (2009), Ohadiand Micic (2011). For example, in order to model ageing of a reinforced concrete elements, continuous gamma process was introduced as an appropriate representation for flexural capacity loss where parameters could be estimated from inspection outcomes. However, while such models benefit from simplicity of the formulation, it is not convenient for the parameter estimation with 'jump' changes such as strengthening, disruption or change in the maintenance pattern, natural hazards impact, etc. These aspects are becoming increasingly important as the environmental factors, natural hazard exposure, variable maintenance management strategies have significant impact on the existing infrastructure ageing process that could lose its monotonic trend. Furthermore, plentiful data is increasingly available from new monitoring technologies that could provide higher accuracy. At the same time, new performance expectations such as compliance with changes to imposed loads or predicted climate scenarios are emerging and requiring quantitative estimates of the exposure within any time interval.

Thus, it would be helpful to develop a sophisticated practical models for capacity loss for infrastructure that would include independent increments and non-monotonic degradation paths. For continuous degradation processes there are several alternative approaches relying on stochastic process modelling such as, VanNoortwijk, (2009) and Frangopol et al. (2004), however, a more general Lévy process would be useful Belhaj Salem et al. (2022) . Such process, while continuous, should have infinite variation in every nonempty time interval.

In this paper we are considering modelling for the capacity loss for an existing infrastructure, such as reinforced concrete or structural steel elements with the following assumptions:

- Infrastructure component's performance has been acceptable in the past, thus it is compliant with design standards of the time
- Gross errors associated with design and construction can be ignored
- Diverse physical measurements or expert judgements in relation to structural components can be obtained
- Loss of component capacity is site specific
- Distinct change in operational conditions might have occurred
- Repair that might have occurred since construction is site specific

2 STRUCTURAL PERFORMANCE CHARACTERISATION

In most general sense, every structural component will deteriorate in some way over the service life. This process is continuous, site specific, most likely non linear and thus, uncertain. Many well known mechanisms , such as fatigue, corrosion, etc influence the structural performance over the life cycle. At present we are increasingly concerned that environmental factors could also significantly impact the service life of the structural components by accelerating the gradual deterioration processes. In addition, the components can be subject to an abrupt and discrete damage as illustrated in Figure 1, that would result in significantly changed service life. With reference to Figure 1 we can consider some benchmark profile of increasing capacity loss (1). However, over the life cycle the structure could be subject to impact, natural disasters such as earthquakes, etc. resulting in significantly shorter service life (2). In the same manner, the structure could be subject to a range of maintenance and repair actions as well as potential strengthening (3), resulting in significant extension in the service life for the structure.

In this context, any stochastic process model for the capacity prediction for an existing structural component over the life cycle will need to address multiple issues:

- Design uncertainties associated with the time of construction and site specific environmental conditions,
- Quality control procedures associated with inspections and maintenance actions
- Human error effects
- Changes in environmental conditions due to pollution, urbanization, climate change, etc.
- Changes in relevant regulation
- Capturing uncertainties associated with cumulative effects that are site specific in respect to capacity, prior repair, maintenance, retrofit, etc.

The stochastic processes such as Gamma process have been recognised to be appropriate to represent temporal degradation over the service life, Pandey (2009), Van Noortwijk (2009), Frangopol et

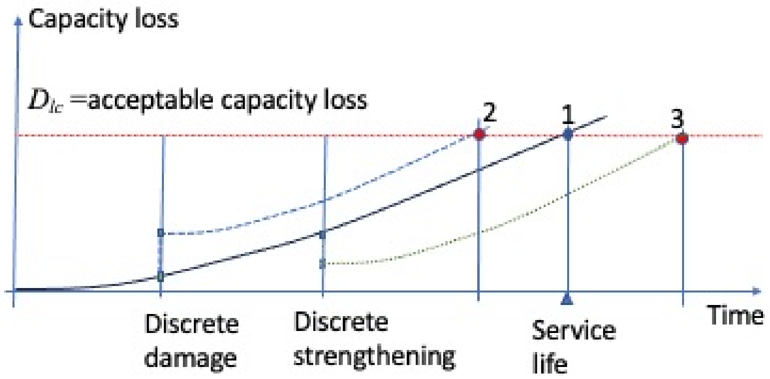

Figure 1: Deterioration scenarios for a sample structural component

al. (2004) and have been used to model degradation of structural components and industrial systems, Belhaj Salem et al. (2022). However, as illustrated in Figure 1 these processes cannot represent non-monotonic (incremental) behaviour. It can be observed that within the general degradation process, discrete damage and strengthening actions over all discrete time intervals are not sufficiently well represented using functions such as Gamma process. In order to be able to include the non-monotonous events within the degradation process we need to introduce a stochastic process model that would represent the range of uncertain service life profiles.

Following Belhaj Salem et al. (2022) who have considered degradation models for centrifugal pumps we propose Variance Gamma Process as degradation model for structural component capacity loss and demonstrate the potential application. Firstly, the properties of the Variance Gamma Process (VGP) will be introduced together with parameter estimation.

3 VARIANCE GAMMA PROCESS

As Belhaj Salem et al. (2022) confirm, VGP is a pure jump process, having infinite number of jumps in any interval of time. The sum of absolute variations of VGP is finite which permits it to be written as a function of two increasing processes. From Madan et al. (1998) the VGP can be presented as a difference of two gamma processes. This is considered as a significant advantage, due to gamma process simple representation. In this context the gamma processes can represent two different manifestations. One can represent the increase in the structural component capacity loss (degradation) and the other can represent the uncertainty associated with a range of events such as climate change scenarios, maintenance processess, etc.

Madan et al. (1998) have considered the variance gamma process (VGP) for option pricing. This process, is defined by evaluating Brownian motion (with constant drift and volatility) at a random time change given by a gamma process. They have considered that for each unit of calendar time may be viewed as having an economically relevant time length, given by an independent random variable that has a gamma density with unit mean and positive variance. These properties can be used for degradation modelling, with a site specific gamma process specification to reflect exposure parameters.

3.1 VGP formulation

Firstly, we consider the Brownian motion, $(W(t))_{t \geq 0}$. Where $W(0) = 0$ and the non-overlapping increments are independent and normally distributed;

$$W(t) - W(s) \sim N(0, (t - s)) \tag{1}$$

the process, $(W(t))_{t \geq 0}$ is a standard Brownian motion.

Now, Brownian motion process with drift $\theta \in \mathbb{R}$ and volatility $\sigma > 0$ is annotated:

$$B(t; \theta, \sigma)_{t \geq 0} = \theta t + \sigma W(t) \tag{2}$$

As previously indicated the Gamma process $\gamma(t; \mu, \nu)_{t \geq 0}$ is a stochastic process with independent non-negative increments having gamma distribution with a constant scale parameter and a time dependent shape function for modelling gradually accumulating damage in a sequence of small increments, VanNoortwijk, (2009). We consider a variable with gamma distribution characterised with shape factor $\alpha = \frac{\mu^2}{\nu} > 0$ and scale parameter $\beta = \frac{\mu}{\nu} > 0$, Ohadi and Micic (2011). The Variance Gamma Process at time t can be expressed:

$$X(t; \sigma, \nu, \theta) = B(\gamma(t; \mu, \nu); \theta, \sigma) \tag{3}$$

From Maden et al. (1998) the above process can be represented as a difference between two gamma processes:

$$X(t; \sigma, \nu, \theta) = \gamma_p(t; \mu_p, \nu_p) - \gamma_n(t; \mu_n, \nu_n) = \theta\gamma(t; 1, \nu) + \sigma b(\gamma(t; 1, \nu)) \tag{4}$$

where, μ_p, μ_n, ν_p and ν_n are given by Madan et al. (1998)

$$\mu_p = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} + \frac{\theta}{2}$$

$$\mu_n = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} - \frac{\theta}{2}$$

$$\nu_p = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} + \frac{\theta}{2}^2 \nu$$

$$\nu_p = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} - \frac{\theta}{2}^2 \nu$$

While Madan et al. (1998) proposition has been widely used in financial analysis there are emerging applications in system engineering and maintenance programme developments, Belhaj Salem et al. (2022). The latter is particularly important for structural component deterioration applications as VGP can model independent increments (in deterioration) and non monotonic paths that can be observed where the structure undergose strengthening, impact damage, advers environmental impact, etc.

3.2 Parameter estimation

For existing structural components certain physical properties, such as cover to reinforcement for reinforced concrete elements are likely to have higher variability than at the time of construction. This variability is coupled with site specific external effects, such as the rate of corrosion, spaling, impact, repair, etc. Statistical samples for site specific parameters are small, however, due to increasing use of sensors, this status is likely to change rapidly in the near future. Similar pattern of change is likely to emerge for the load estimates for structures in general.

In most cases reported in literature, the parameter estimation for VGP used the simulated data and using standard approaches such as maximum likelihood method. Belhaj Salem et al. (2022) report on different approaches such as expectation-maximization algorithm. For gamma process parameter estimation, the range of approaches is well documented and several references have detailed descriptions for evaluation of these parameters, such as, VanNoortwijk, (2009). In this paper, for simplicity, parameter estimates for gamma processes are obtained using the method of moments.

4 VARIANCE GAMMA PROCESS APPLICATION

Historically, the structural capacity loss was estimated using deterministic approach, while in more recent times the random variable model and stochastic process models have been implemented. This is inevitable as we now have sophisticated models for environmental impact such as climate projection models UKCP18 and we have an increasing availability of data due to increasing use of sensors. From the perspective of infrastructure owners, it would be ideal to have in place the capacity loss prediction model that includes climate scenarios. It is evident from Figure 2 that for consideration

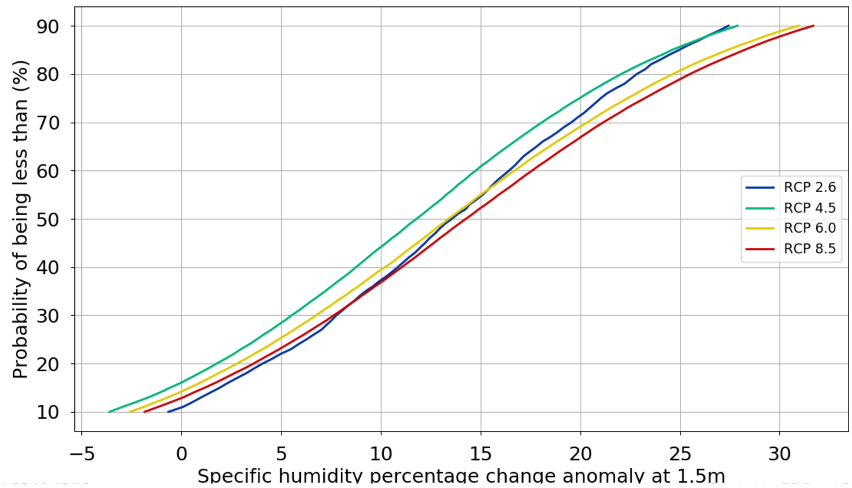

Figure 2: UKCP18 Sample climate scenario for 2050 (Hadley Centre)

of different scenarios (RCP2.6, to RCP 8.5) for a sample structure such as a reinforced concrete component, corrosion progress the continuous gamma process to represent capacity loss would be limited as it could only represent one scenario presented in Figure 2. Considering the complexity of the climate model with its numeros factors and scenarios and the site specific parameters, the number of degradation pathways is infinite, thus capturing this uncertainty using the Wiener process at the time of assessment is an attractive proposition. In practice, at any point of the lifecycle, the rate fo the capacity loss would be influenced by a range of specific parameters such as the precipitation profile over time, together with temperature, wind, humidity, etc. Thus, VGP could be particularly useful for consideration of climate change effects considering long time frame and the nature of its defining parameters.

It has been demonstrated by Micic (2017) that it is feasible to obtain valid information from inspection outcomes to establish the site-specific continuous gamma process model for the progress of capacity loss for reinforced concrete components. For a sample reinforced concrete deck capacity models have been established and climate scenarios selected to represent different corrosion initiation times and rates. The Gamma process parameters the shape factor α and the scale factor β obtained for a selected inspection time and target projections times are shown in Table 1. Application of gamma process for such, site specific information, are illustrated in Figure 3. Gamma process functions are shown for an existing component that was inspected in year 24 and subject to four site specific scenarios for 30 and 50 years projection. It is evident from Figure 3 that site specific conditions can have significant effect on progress of capacity loss. It is also evident that using Gamma process the forward projection is to some extent constrained by the prior degradation profile.

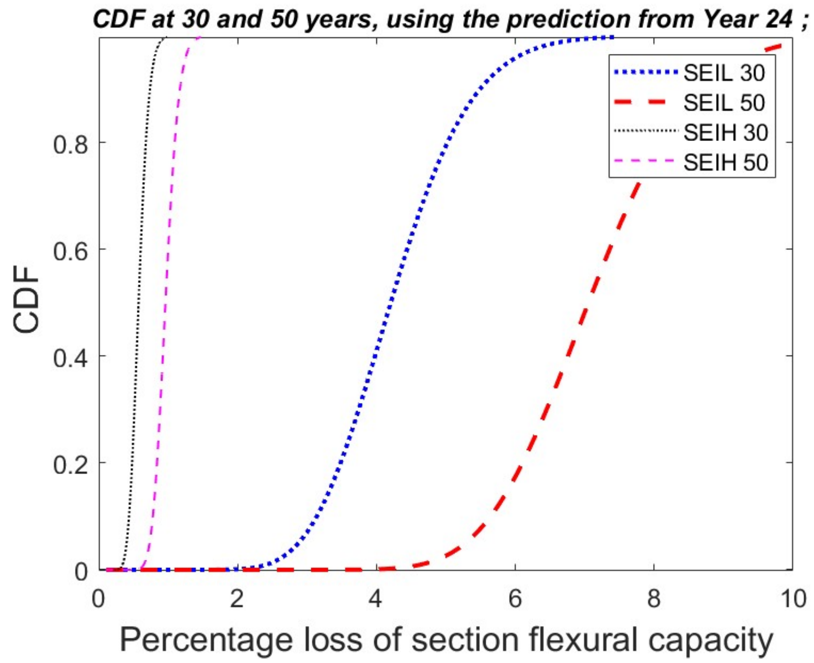

Figure 3: Comparison of probability distribution functions at assessment time for capacity loss for two corrosion scenarios and for 30 and 50 years time horisons

Thus, here we introduce the Wiener process, where degradation increments are gamma distributed,using the formulation of the Equation [3]. In order to convolute the impact of discrete effects (either sudden deterioration or strengthening) we introduce the Brownian motion with drift for forward predictions that include the impact of such changes, Figure (4).

The particular benefit appears to be in the opportunity to establish the maintenance strategy that is informed by the current time interval, rather than generic long term behaviour. Figure 5 presents an illustration of the outcome for Variance gamma modelling for degradation using constant drift and volatility between scenarios. Some useful outcomes can be identified. Due to the increase in available

Table 1: Gamma process parameters for alternative scenarios

Scenario	β	α_{30}	α_{50}	$Note$
SEIL	0.199	21.459	35.764	early corrosion initiation slow rate of growth
SEIH	0.024	24.215	40.358	early corrosion initiation high rate of growth
SMIH	0.012	7.496	12.493	y10 corrosion initiation high rate of growth
SMIL	0.111	7.226	12.043	y10 corrosion initiation slow rate of growth
SMIM	0.029	7.316	12.193	y10 corrosion initiation medium rate of growth

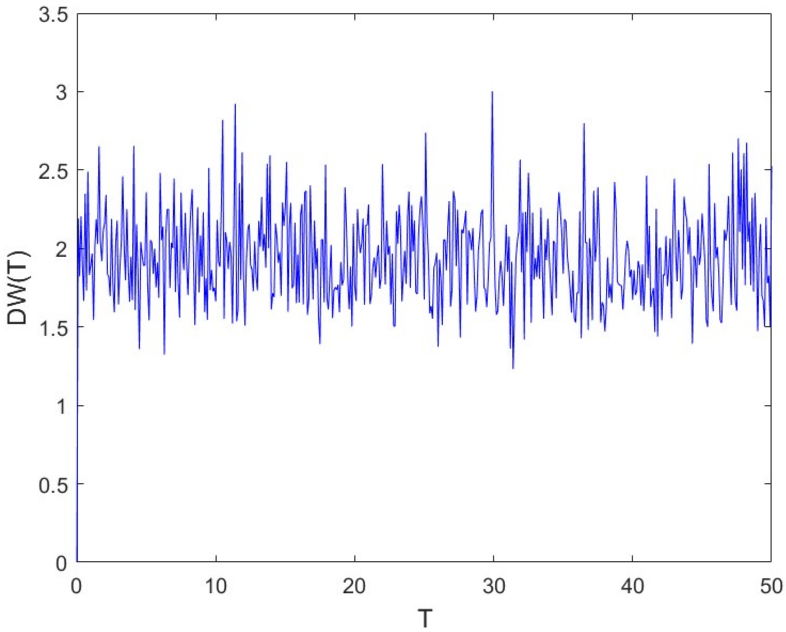

Figure 4: Brownian Motion simulation at assessment time for capacity loss for SEIH corrosion scenario and for 50 years time horizon

parameters, in comparison with the Gamma process, the proposed representation allows for more refined modelling of the site specific parameters. The evaluation of drift and volatility parameters remains to be investigated in greater depth as there is a need to establish how to reconcile diverse sources of uncertainty that are specific for structural applications.

5 CONCLUSIONS

This paper has considered a new formulation for stochastic process modelling for structural component capacity loss. With ultimate aim to enable inclusion of site specific environmental effects, strategies are needed to develop long term projections for the structural capacity. The continuous gamma process, with its two parameters, shape and scale namely, has been identified as an efficient approach to represent capacity loss due to loss of reinforcement area for monotonous increasing processes. Here, in order to include 'discrete' events during service life Variance gamma process is considered for modelling of the capacity loss. It is identified that additional parameters of Variance Gamma Proces enable more refined, site specific modelling. The representation effectively includes

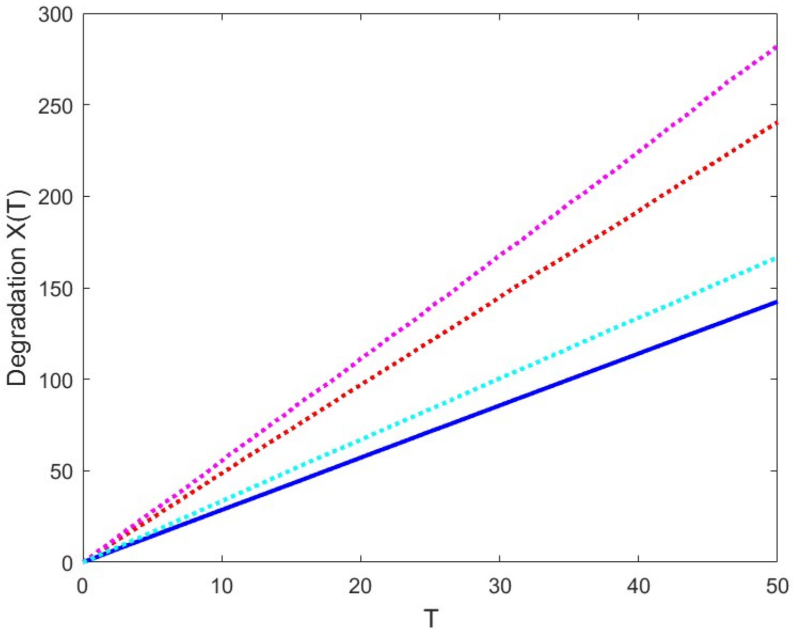

Figure 5: Cumulative degradation representation for four corrosion scenarios and for 50 years time horizon

function of two continuous gamma processes and presents, thus retaining the relative simplicity of gamma process models. While increasing availability of data lends itself for parameter estimation for gamma process parameters, it remains to be investigated if the method of moments as the simplest or the maximum likelihood method are sufficient to avoid introducing additional modelling uncertainties. Over the service life, the progress of capacity loss would be quite different between sites and application of the presented approach would be a major step in parameter estimation and tailor-made maintenance planning. It is evident that Variance gamma process model for capacity loss is intuitively very powerful, however it is necessary now to create a set of benchmarks to establish a comprehensive facility for capacity projections. The method is suitable for shorter term predictions than continuous gamma process models. As a result, more effective inspection intervals and maintenance strategies can be established. A major benefit from variance gamma process model would be in cases where natural hazards, such as earthquakes or floods, create the discrete point in the service life profile.

REFERENCES

[1] Belhaj Salem, M, M. Fouladirad and E. Deloux; *Variance Gamma Process for Degradation for prognosis and imperfect maintenance of centrifugal pumps*; Reliability Engineering and System Safety, Vol 223, paper 108417, (2022)

[2] Frangopol, D.M., Maarten, M., Kallen, J., and Van Noortwijk, J.M., *Probabilistic models for life-cycle performance of deteriorating structures: Review and Future directions* Journal of Structural Engineering Material, 6, 197-212, (2004)

[3] Haldar, A., and Mahadevan, S., *Probability, reliability and statistical methods in engineering design*, John Wiley and Sons. New York, (1999).

[4] Marsh, P., and Frangopol, D.M., *Reinforced concrete bridge deck reliability model incorporating temporal and spatial variations of probabilistic corrosion rate sensor data*', Journal of Reliability Engineering and System Safety, 93, 394-409, (2008).

[5] Ohadi, A.R, Micic, T.,*Stochastic process deterioration modeling for adaptive inspection*, International Conference of Applications of Statistics and Probability in Civil Engineering, Proceeding of ICASP11, Taylor and Francis Group, (2011).

[6] Pandey, M.D., Yuan, X.X., and Van Noortwijk, J.M., *The influence of temporal uncertainty of deterioration on life-cycle management of structures*, Journal of Structure and Infrastructure Engineering, 15, 145-156, (2009).

[7] Van Noortwijk, J.M., *A survey of the application of gamma processes in maintenance*, Journal of Reliability Engineering and System Safety, 94, 2-21, (2009).

[8] Madan, D. B., Carr, P. P. and Chang, E. C. *The variance gamma process and option pricing*, Review of Finance, 2(1), 79-105, (1998).

[9] UKCP18; *https://www.metoffice.gov.uk/research/approach/collaboration/ukcp*

[10] Micic, T; *Structural capacity formulation of ageing infrastructure for projected climate effects*; in Proc. of the 12th International Conference on Structural Safety and Reliability, 12th ICOSSAR; Eds. Bucher, C., Ellingwood, B. and Frangopol, D.M. ; pp. 1915-1925; Vienna, (2017)